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OPTIMAL ENERGY CONTROL IN PNEUMATIC DRIVE OF PROSTHESIS

K. Nazarczuk

Abstract

A method of so called economical control of double-acting pneumatic cylinder used in prostheses is given.

The method lies in supplying the compressed gas from the part of the cylinder which is to be emptied to the part which is to be inflated, providing there exists necessary difference of pressures on both sides of the piston. Thus the gas is not directly vented to atmosphere.

A diagram of control system which ensures optimal distribution of pneumatic energy during quasistatic movements is given. The paper also gives the analytical estimation of profits of this method comparing with traditional methods. Experiments proved the theoretical results.

Introduction

Difficulties connected with miniaturization of supply sources constitute at present one of the main obstacle restricting further development of active prostheses of upper limbs. Even in case of simple prostheses with one or two degrees of freedom, a stock of energy carried by a cripple is usually sufficient for only one day. In this connexion, any attemps to increase the efficiency of drives applied in prostheses are of essential significance.

In pneumatic drives, an increase of the efficiency is endevoured after above all by ensuring the installation tightness and decreasing the clearance volume of cylinders. The author pays attention to another not hitherto used possibility to save compressed gas in pneumatic double-acting cylinders. There is such a possibility when performing such activities during which pressure in the cylinder's chamber being emptied is higher than in the chamber being inflated. It consists in that instead to lead gas to the chamber being inflated and simultaneously vent it to the atmosphere, we can joint the two chambers with each other thus ensuring the flow of gas from the chamber being emptied to that being inflated.

This observation inclined the author to searching the way of control ensuring the optimal utilization of working medium. The estimation of gas savings possible to obtaining thanks to such control is carried out in the work / 1 / in relation to bellows actuators.

This paper presents theoretical grounds as well as a scheme of simplest system realizing the optimal control of working medium in a piston cylinder.

1. Statical Analysis of Pneumatic Double-Acting Cylinder

1.1 Analysis Purpose and Assumptions Made

The analysis is aiming at determination of mathematical relations qualifying consumption of working medium according to the programme of activities performed and design parameters of a cylinder, as well as at indication of possibilities to minimalize that consumption.

A pictorial diagram of cylinder is shown in Fig.1.

The cylinder consisting of sleeve 3 and piston 2 has two working chambers: a small-diameter one 1' /including a piston rod/ and a large - diameter one 1".

To simplify the analysis, following assumptions has been made:

a/ movements of the piston are of the quasistatic character,
b/ a working medium is perfect gas,
c/ the gas temperature in both chambers of the cylinder is equal and constant,
d/ the chambers are perfectly leakproof.

The above assumptions well correspond with the facts in case of slow movements and not too high pressures, which take place in drives of protheses.

1.2 Notations and Output Relations

In the analysis, following notations has been assumed:

L - piston travel /maximum displacement/
$0 \leq X \leq L$ - piston displacement
F - external force applicating to the piston
S - chamber cross-section
M - mass of gas included in chamber
V - overall volume of chamber
V_s - clearance volume of chamber
p - absolute pressure in chamber
p_a - atmospheric pressure
ϱ_a - gas density at atmospheric pressure
R - gas constant

Symbols noting quantities related to the chamber 1' /the small - diameter one/ are indexed with ' while those related to the chamber 1" /the large - diameter one/ are indexed with "

The assumptions made in paragraph 1.1 allow in easy way to determine a dependence between gas masses M' and M'' included in the chambers of the cylinder in the static equilibrium state

and a piston displacement X and external force F. This dependence is determined by following relations:
- equation of forces applicating to the piston

$$(1) \quad p'S' - p''S'' + p_a (S'' - S') = F ;$$

- equation of the gas state

$$(2') \quad p'V' = M'RT ;$$

$$(2'') \quad p''V'' = M''RT ;$$

- relations between chamber volumes and piston displacement

$$(3') \quad V' = V'_s + S' \cdot X ;$$

$$(3'') \quad V'' = V''_s + S'' (L - X) ;$$

- limitations relating to pressures in chambers

$$(4') \quad p' \geqslant p_a ;$$

$$(4'') \quad p'' \geqslant p_a .$$

1.3 Static characteristics of cylinder

Introducing non-dimensional quantities

$$(5') \quad m' = \frac{M'}{\rho a S'' L} ;$$

$$(5'') \quad m'' = \frac{M''}{\rho a S'' L} ;$$

$$(6') \quad l' = \frac{V'_s}{S'' L} ;$$

$$(6'') \quad l'' = \frac{V''_s}{S'' L} ;$$

$$(7) \quad 0 \leqslant x = \frac{X}{L} \leqslant 1 ;$$

$$(8) \quad f = \frac{F}{p_a S''} ;$$

$$(9) \qquad d = \frac{S'' - S'}{S''} \ ;$$

we obtain from the equations (1) - (3") the dependence between gas masses in the chambers and displacement and load of the piston has a convenient form for further analysis

$$(10) \quad m' = m'' \frac{l' + x}{1 + l'' - x} + (f - d)(l' + x) \ .$$

At the given position and load of the piston, i.e. at the defined values of variables x and f, the state of static equilibrium is possible only when values m' and m" corresponding with gas masses included in the cylinder chambers satisfy the equation (10) and conditions determined by the inequalities (4') and (4"). There is here infinitely many permissible solutions. An optimal solution i.e. minimum values m' and m" are obtained for $f \geqslant 0$ when $p'' = p_a$, and for $f \leqslant 0$ when $p' = p_a$.

Hence for $f \geqslant 0$:

$$(11') \qquad m'_{min} = (1 - d + f)(l' + x) \ ;$$

$$(11") \qquad m''_{min} = 1 + l'' - x \ ;$$

and for $f \leqslant 0$:

$$(12') \qquad m'_{min} = (1 - d)(l' + x) \ ;$$

$$(12") \qquad m''_{min} = (1 - f)(1 + l'' - x) .$$

For established values x and f permissible solutions of the equation (10) create on a m', m" plane a ray coming out from a point with coordinates determined by either the relations (11') and (11") or (12') and (12") according to the value f.

In Fig. 2 there are shown several examples of such rays corresponding with a cylinder with geometric parameters $l' = l'' = 0,2$ and $d = 0,1$ for several different values x and f.

For the given value x, rays corresponding with different values f are shifted in parallel along the axis m' for $f > 0$ or along the axis m" for $f < 0$ in relation to the ray corresponding with the value $f = 0$.

The above presented characteristics corresponding with different states of static equilibrium of the piston constitute a basis to determine the gas consumption by a cylinder when performing quasistatic movements.

Moreover, it should be also taken into consideration a dependence m' (m',x) corresponding with balance of pressures in both chambers of the cylinder, i.e. $p' = p''$.

The dependence is expressed with an equation:

$$m' = m'' \frac{l' + x}{1 + l'' - x} \quad (1 - d) \tag{13}$$

From the equations (10) and (13) it results that for $f > 0$ pressure in the chamber 1' is always higher than in the chamber 1".

2. Optimal Utilization of Working Medium

2.1 Theoretical Grounds and Algorithm

The above considerations show that position x of the piston and its load f do not simultaneously determine the quantity of gas which must be led to the double acting cylinder to achieve the state of static equilibrium. This means that according to the way of control, we can realize the given activity utilizing different quantities of compressed gas.

Therefore, an optimal algorithm of control ensuring the minimum utilization of working medium was searched. As an optimalization criterion, there has been assumed mass of compressed gas flowing through a cylinder during execution of activity measured at the cylinder output.

Considerations were limited to the cylinder activities executed in the quasistatic way consisting in passing through succeeding states of static equilibrium determined by the variable sequence x_i and f_i. To realize the defined activities programme, the gas quantity in the cylinder chambers should be so changed that on the plane with co-ordinates m', m" to cross rays corresponding with succeeding states of equilibrium. In general case, a passage on a ray lying higher in diagram, i.e. corresponding with higher values f or higher values x, requires that either mass m' be increased, i.e. chamber 1' filled, or mass m" be decreased, i.e. chamber 1" emptied. On the other hand a passage on a ray lying lower in diagram requires that either chamber 1' be emptied or chamber 1" filled.

The attention should be paid to the following points:

1. In some cases desired changes in the values m' and m" can be obtained when ensuring free flow of gas from one chamber to the other.
2. The rays m' (m") corresponding with the states of static equilibrium are divergent ones, which means that as moving away from the origin of coordinates a passage from one ray to another requires larger and larger changes in the quantity of gas in the chambers.

   From the above remarks it results a general principle of controlling a double-acting cylinder ensuring optimal utilization of working medium.

It consists of three points as follows:

A. If pressure in the chamber being emptied is higher than in that being filled, then gas should be first drained from the chamber being emptied to the chamber being filled.

B. If pressure in the chamber being emptied is not higher than in that being filled, then gas should be vented from the chamber being emptied to atmosphere.

C. Only when pressure in the chamber being emptied is equal to the atmospheric pressure that gas should be led from the supply source to the chamber being filled.

The control consistent with the presented principle has been called the economical control. Technical realization of a system which would ensure the economical control at any loads and directions of motion is difficult enough. This is why in further considerations we will limit ourselves to activities in which an external force has always the same sign $f > 0$. Such activities take place in prostheses quite often and are usually realized by single-acting cylinders owing to the ease in their control as well as the fact that they use less gas than double-acting cylinders in the classical control.

The example programme of such activities is shown in Fig. 3a as diagram $f(x)$.

It corresponds with following activities:

0-1 - detachment of the load $Q = 1.4\ p_a S^*$ from ground and overcoming the friction force $T = 0,2\ p_a S^{*a}$, hence $F_1 = Q+T$,

1-2 - Rising of the load to the height corresponding with the piston deplacement $X = 0,8L$, $F_2 = F_1$,

2-3 - pressing of the load to stop requiring the force $F_3 = 2,6 p_a S^*$,

3-4-3-4-3 - detachment of the load from the stop requiring decrease of the force to the value of $F_4 = Q-T$ and repeated two-fold pressing with force F_3,

3-4-5 - detachment of the load from the stop and lowering it to the initial position without fixing it to the ground $F_5 = F_4 = Q - T$.

In Fig. 3b the above programme is shown on the plane with coordinates m', m''.

It consists in succeding passages through the rays noted with numbers 0-1-2-3-4-3-4-3-4-5.

In case of single-acting operated cylinder, the programme is realized according to the trajectory $A_0-A_1-A_2-A_3-A_4-A_3-A_4-A_3-A_4-A_5$ and it requires using of mass equal to $2\Delta m^*_{34} + \Delta m^*_{35}$ whereas in the economical control of double-acting cylinder, the programme is realized according to the trajectory $A_0-A_1-A_2-A_3-B_4-A_6-A_3-B_4-A_6-A_3-B_5$ and uses the mass of gas measured on the output of chamber 1' equal to $2\Delta m_{34} + \Delta m_{35}$ i.e. by 38 percent less than in preceding case.

During activities 0-1, 1-2 and 2-3 the programmes were in both cases realized according to the same trajectory $A_0-A_1-A_2-A_3$. This is regularity to be found for $f > 0$ in any activities requiring the a chamber 1' to befilled and the chamber 1" to be emptied.

Effects of the economical control appear only when performing activities requiring the chamber 1' to be emptied, and the chamber 1" to be filled. When performing reversed activities, gas is not drained from the chamber 1', therefore its consumption equals to zero.

2.2 Comparison of Working Medium Utilization in Economical and Traditional Control

Advantages from application of a double-acting cylinder with the economical control compared to a single-acting cylinder when performing activities consisting in a passage from the state of equilibrium in the point with coordinates x_j, f_j to the state of equilibrium in the point x_k, f_k /where $x_j \geqslant x_k$ and $f_j \geqslant f_k \geqslant 0$/, can be determined from equation resulting from the relations /10/ - /12"/.

$$(14)\quad \eta = \frac{l'(f_j x_j - f_k x_k) + x_j x_k (f_j - d) + l'(f_j - f_k)(l' + x_k) - x_k^2 (f_k - d) - l'd(x_j - x_k)}{(1 + l' + l'')\left[(1-d)(x_j - x_k) + f_j(l' + x_j) - f_k(l' + x_k)\right]} ;$$

where

$$\eta = \frac{\Delta m^*_{jk} - \Delta m_{jk}}{\Delta m^*_{jk}} ,$$

Δm_{jk} - mass of gas used by a single-acting cylinder,

Δm_{jk} - mass of gas used by a double-acting cylinder with economical control.

At simplifying assumptions $l' = l'' = d = 0$ the equation (14) assumes a form:

$$(15)\quad \eta = \frac{x_k}{1 + \frac{x_j - x_k}{x_j f_j - x_k f_k}} ;$$

and for $x_j = x_k = x$ we obtain

$$(16)\quad \eta = x$$

and for $f_j = f_k = f$

$$(17)\quad \eta = \frac{x_k}{1 + \frac{1}{f}}$$

It results from the equations (14) - (17) that the economical control gives big advantages at the high values of variables x and f.

Such conditions first of all occure in prostheses during catching of objects, when a cylinder in the latter part of working movement $(x \cong 1)$ must applicate with a large force. From the equation (16) it results that when performing activities requiring a change of the grasp force at an established position for $x = 1$, the economical control may ensure nearly 100 percent of working medium saving. The equation (16) is,

however, approximated and real savings are smaller.
On a diagram shown in Fig.4 there are noted with points the values of coefficient η characterizing effectivity of the economical control experimentally determined for a cylinder with geometric parameters $l' = l'' = 0{,}07$ and $d = 0{,}1$ when performing activities consisting in lowering of a load from the position $x_1 = 1$ to the position x_k and repeated rising to the initial position. During the experiment, the load was remaining constant $f = 3{,}7$ and only values x_k were changed.

Scatters of the values η obtained can be explained by occurence of large friction forces between the piston and the cylinder. For comparison, the dependence between η and x_k analytically determined on a basis of the equation (14) is shown on the diagram with a continuous line.

2.3 Example System Realizing Economical Control

The first model of system ensuring the economical control was made on a basis of typical elements of pneumatic automatics. It was intended for control of a cylinder of which load had always the same sense $f > 0$. The system scheme is shown in Fig. 5a. It consists of two manual comtrolled cut-off valves V_1 and V_2 and one pneumatically controlled cut-off valve V_3. In the state of equilibrium all valves are closed and pressures in the cylinder chambers satisfy condition $p' \gg p'' \gg p_a$. An opening of the valve V_1 causes joining of both cylinder chambers and flow of gas from the chamber 1' to 1". It is followed by movement of the piston to the left under application of the external force F.
It should be stressed that for a big difference between active surfaces of the piston S'' and S', i.e. for $d \gg 0$, it is possible movement of the piston to the left under the force $F < 0$.

On the other hand an opening of the valve V_2, when the valve V_1 closed, causes venting of gas from the chamber 1" to the atmosphere and movement of the piston to the right. When pressure in the chamber 1" drops to the value $p''_{min} = p_a + \Delta p$ then it follows an opening of the valve V_3 and feeding of gas from the supply source to the chamber 1', which ensures continuation of piston movement to the right. The value Δp may be adjust in the system. It results from considerations conducted in the paragraphs 1.3 and 2.1 that minimum gas consumption occurs for $\Delta p = 0$.
However, researches have proved that for $\Delta p = 0$ during the piston movement to the right, oscillations occur. For the system investigated, the best results were achieved when $\Delta p = (20 \div 40)\,kNm^{-2}$
The above presented system of the economical control of a double-acting cylinder is very simple and includes only one element /valve V_3/ more than the simplest system of a single-acting cylinder control shown for comparison in Fig.5b.

First researches proved that when performing movements in the range of $0,6<x<1$, it ensures ca 20 percent lower consumption of working medium compared to the traditional system shown in Fig.5b.

The effects obtained are distinct yet lesser to expected in theoretical considerations.

The differences are explained with the fact that the valve V_3 is actuated by a small bellows actuator which slightly increases a clearance volume of the chamber 1", as well as that in the theoretical analysis the absolute installation tightness has been assumed.

Acknowledgement

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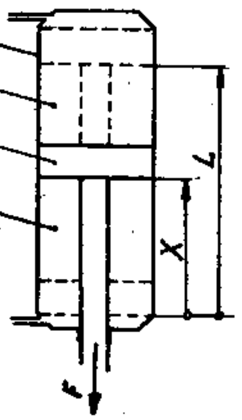


Fig.1 Scheme of a double acting cylinder

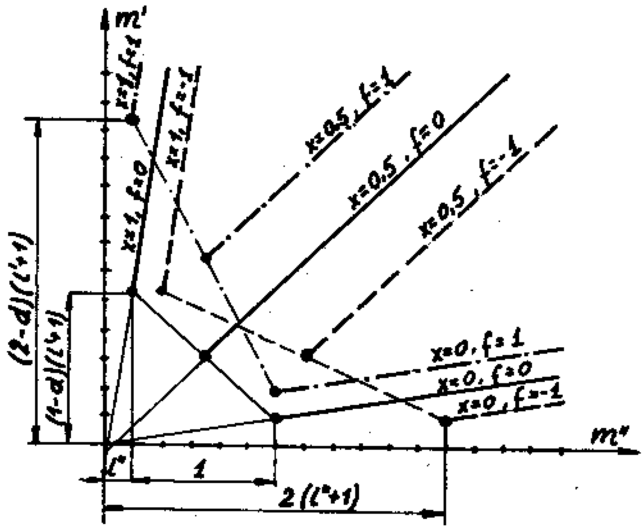


Fig.2 Static characteristics of a cylinder

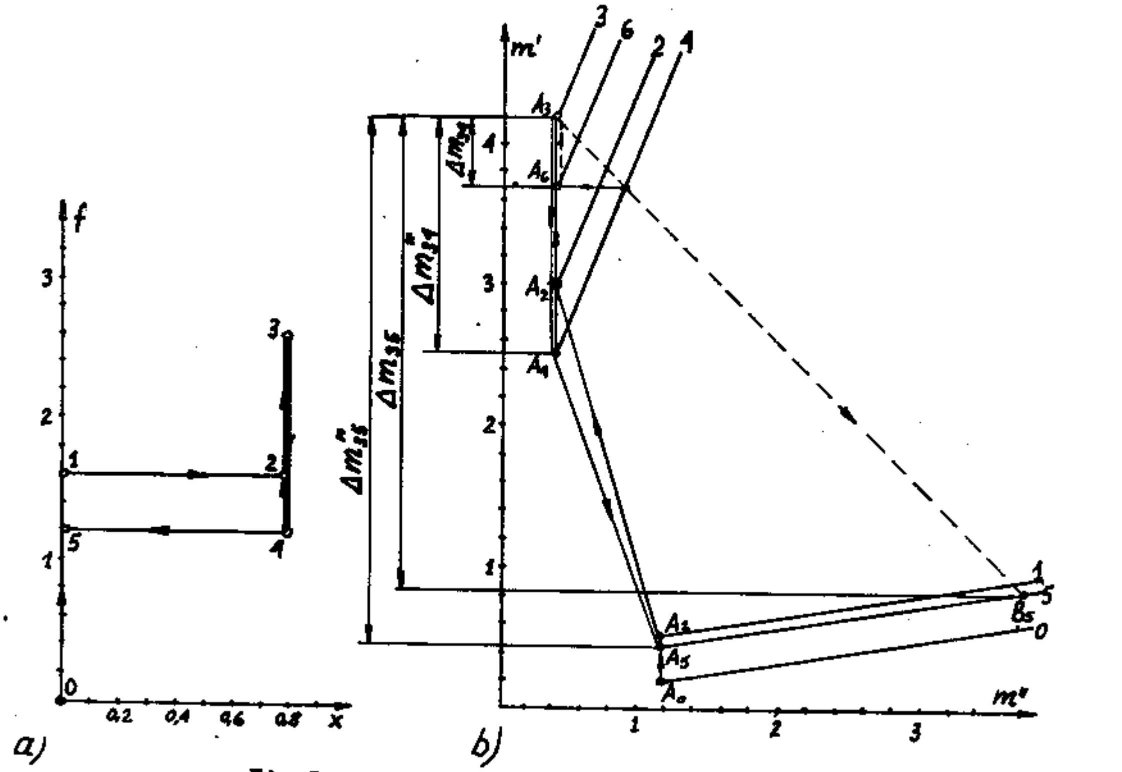


Fig.3 Example programme of activities performed by cylinder

a/ in coordinates x, f

b/ in coordinates m', m"

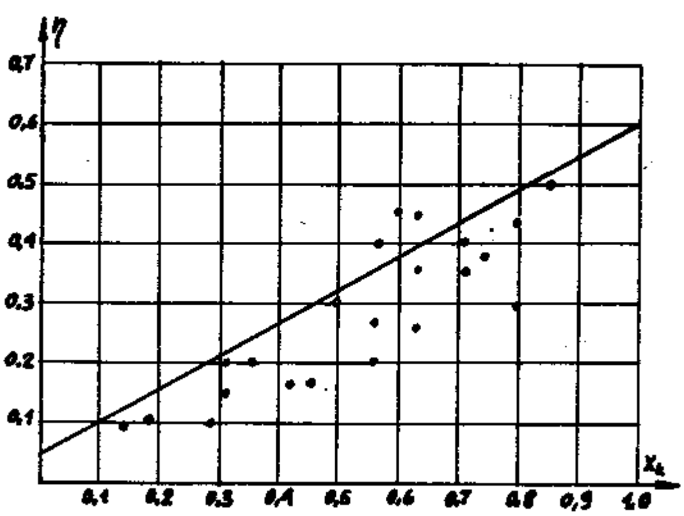


Fig.4 Comparison of theoretical and experimental values of coefficient η

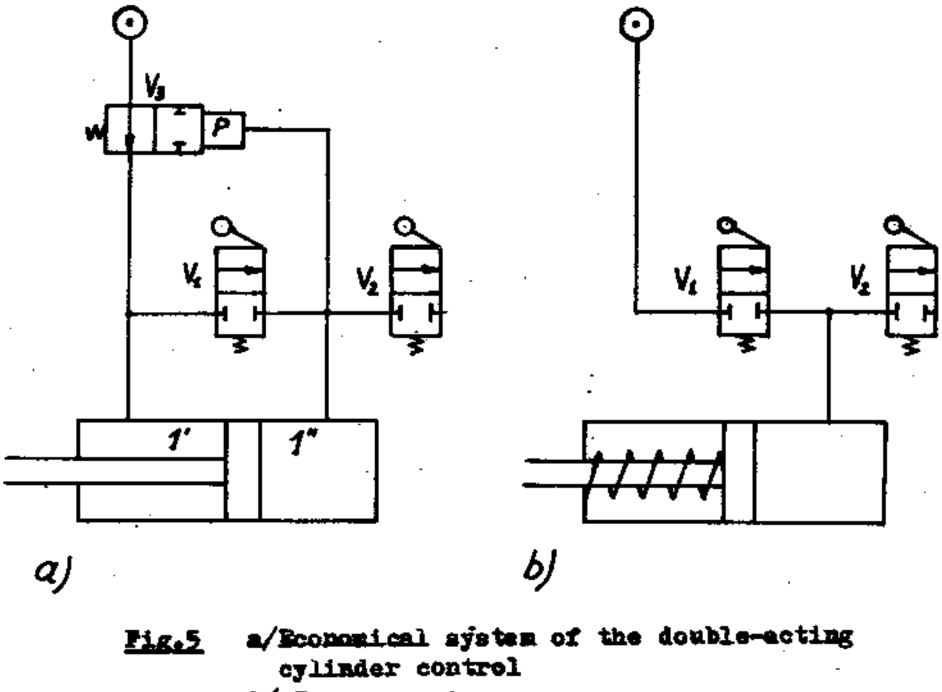
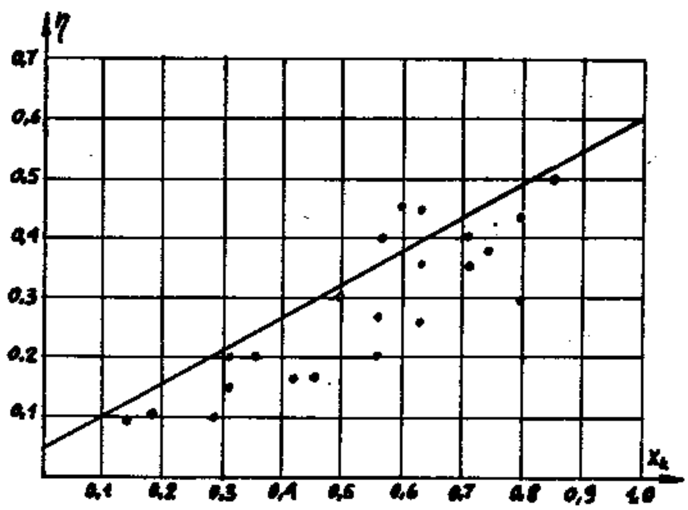


Fig.5 a/Economical system of the double-acting cylinder control
b/ Traditional system of the single-acting cylinder control



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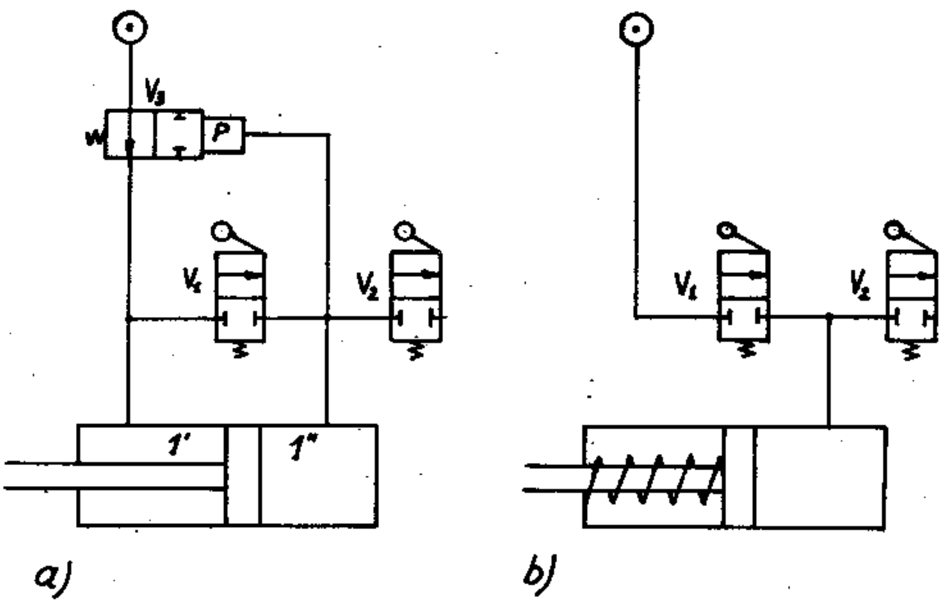

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b/ Traditional system of the single-acting cylinder control