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SOME PROBLEMS OF MATHEMATICAL QUANTITATIVE GAIT EVALUATION

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Abstract

In the paper some problems of quantitative gait evaluation are analyzed and new mathematical criteria are developed. Present, widely used evaluation methods are briefly discussed and their disadvantages poined out. The need for a scientific quantitative gait analysis is shown, and procedures of obtaining the necessary varibles for evaluation criteria are examined. Considering the great number of variables describing human locomotion and measurement difficulties, a minimal set of sufficient kinematic parameters is defined as well. Two evaluation methods convenient for computer implementation are defined.

Introduction

As the process of human locomotion still has not been rigorously defined, it is difficult to prescribe mathematically based criteria and methods of evaluation. In the field of gait rehabilitation various evaluation procedures (1,2,3,4,5) have been developed, which are quantitative, yet not scientifically enough based.

The lack of evaluation knowledge makes the selection and grading of available orthotic/prosthetic appliancies difficult. This represents a great problem particularly in the field of functional electrical stimulation, for on-line evaluation results are needed in order to adjust optimal stimulation parameters(6).This initiated present steps towards setting new evaluation methods.

The Aim of Evaluation

The evaluation of a process, in this case of human gait, implies the attribution of certain values to the variables describing this process. The method of evaluation can be qualitative or quantitative.

The qualitative method relies on the impressions of the expert,and his knowledge and experience with the process to be evaluated. Conclusions thus obtained frequently vary from expert to expert and are not precise.

The quantitative method, on the other hand, is based on measurable data of variables which are interpreted by means of adequate criteria, which are mathematically prooved,and whose choice depends on the aim pursued by the process. Such a procedure is objective and the conclusions are one-sense, independent on our temporal impression.

The main problem then consists in determining the relevant variables and in choosing the criteria for their complex interpretation. As a principle the choice of criteria is free. The selecting procedure and the assumptions on which it is based, depends on our knowledge of the evaluated process and our desire as to what the gait weighing criteria should point out. It is possible, however, to choose a wrong criterion, which is often the case when we have to do with the fields where the cause and consequence relations are unknown.In our case we are unable to write differential equations for the modeling of human gait in classical way, so we simply rely on our own experience and knowledge. We decide for regressive or correl=

tive techniques as the 'objective criterion'. The above stated possibility can easily happen. To eliminate it as far as possible it is necessary for the selected criteria to satisfy the followin conditions : To set the criterion for the evaluation, we have to find out such a model of gait, which is invariable in the class of normal gait population. Then the choice of stimulation sequences or parameters of the prosthetic system , which is expected for certain to be good, shows a higher numerical measure of the gait. And vice versa, for parameters or stimulation sequences chosen at random, we expect, as a principle, lowering of the index of the quality of gait (7). So, the most important part of the problem then consists in setting the model correctly,(i.e.the model must correspond to reality), or rather in selecting and deciding correctly on the variables chosen for the description of the gait. We must keep in mind the fact, however, that all the variables are not simply measurable, even though, due to their character, they should be considered in the model. But only those that are easily measurable should be selected then.

It is therefore necessary to find a compromise regarding the model, between our simple measuring technique and the selected variables, which must be statistically well defined. It is quite obvious that attention is necessary if we do not want our conclusions to become questionable because of the accepted compromise.

Variables Necessary for the Evaluation

Human gait can be described as a periodical movement of sophisticated mechanical system in space and in respect to ground.Performing these movements the system uses perfect drivers - the muscles, coordinated by perfect control mechanisms - the nerve system. To siplify the analysis of the previously mentioned system, motion will be represented as :

$$\dot{q}_i = f_i(q_i, u_j) \quad i=1, ..n, \; j=1, ...m \qquad (1)$$

where q_i are system coordinates, $\dot{q}_i$ respective velocities, and u_j the control moments. The solution of equation 1 is periodical, supposing stationary gait, i.e. $q_i(t) = q_i(t + nT)$ and must satisfy a certain hypothetical criterion :

$$J = \int_0^T F(q, u)dt \qquad (2)$$

under condition :

$$G = G(L, T_s, V, \gamma) \qquad (3)$$

where L is the step length, T_s the step duration, V the average velocity and γ the quotient of stance phase duration and stride time.

Using states $q_i \in Q$ and a set of control moments $u_i \in U$, and also a gait parameter G, the gait patterns can be defined as :

$$X = \{ U, Q, G \} \qquad (4)$$

with the element $x_i \in X$.

Keeping in mind the fact that during locomotion, especially under experimental conditions, some stohastic perturbation disturbs the system, it is desired to define the vector X statistically :

$$X = X(x_i, \sigma_i) \qquad (5)$$

where x_i is an average value and σ_i a standard deviation. In this way the eq.5 not only enables us to test different hypotheses over the defined pattern, but also gives us information about the quality of the system (stationarity).

Statistical Invariability of Variables

Taking into account the fact that the anatomic structure of normal human gait is statistically similar over the whole population, and the fact that the solution of eq.1 takes value in the sense of eq. 2 and 3, we can say that the normalized variables of the eq.5, which are describing the gait, are statistically invariable. The coeficient of variation was taken as a measure of invariability. On the basis of the above said, we can define the statistical pattern of gait as follows :

$$Y = Y\ (Y_i, \sigma_i) \quad i=1, \ldots n \qquad (6)$$

in the eq. 6 y_i represents the average value for a population of normal gait

$$Y_i = \frac{1}{N} \sum_{k=1}^{N} x_{ik}(t/T) \quad i=1, \ldots n \ (7)$$

and σ_i the corresponding standard deviation. The index i=1,...n denotes the number of variables, and k=1,...N the number of objects in normal population. The sign t/T represents temporal normalization of variables. The step length is normalized by being devide by the leg length, while ground reaction forces, e.g., must be devided by the whole body weight.

The Principle of Symmetry - the Evaluation Method Criterion

In their studies many authors have intuitively realized the importance of normal gait symmetry (8 ,4), but seldom have they attempted to explain why the symmetry is important and to proove its invariance, making the best use of it. We have prooved that symmetry is an invariable characteristic of normal gait and we then used this fact to set a complex evaluation method (7). It is easy to understand that minimum energy is consumed by the symmetrical gait, that its information contents is minimal, and that it provides maximal movement confort. Now then, although symmetry is above all the characteristic of normal gait, it can certainly be used also as a criterion of pathological gait evaluation where the rehabilitation goal is a cosmetic normal locomotion.

The above assertions are logical. Yet we should try to have them qualitatively derived as well. The eq.1, regarding the stance phase, could be written in relation to the support line τ_i; $i = L , R$ (fig 1) . The support line describes the motion of resultant groun reaction forces upon the foot. Its importance in walking systems is explained in literature (9).

Let us suppose that the solution of eq.1 must satisfy the criterion J expressed in eq.2 , and the condition G prescribed in eq. 3. Then there exists an optimal value of variables q,u at period T. But as the gait period $T = T_{sL} + T_{sR}$, eq. 2 can be broken up in two parts :

$$J = \int_{T_{sL}} F(q,u)\,dt + \int_{T_{sR}} F(q,u)\,dt \qquad (8)$$

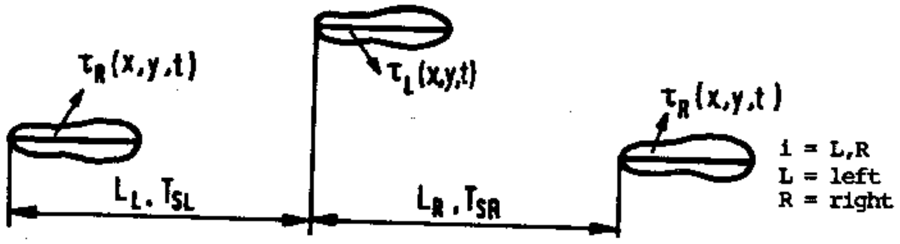

Fig. I

If we want the system to work optimally during the whole period T, at a certain average velocity, then both parts of equation 8 must be optimal or rather the variables must be of identic trajectory:

$$x_{iR}(t) = x_{iL}(t + \delta) \qquad (9)$$

The same condition is valid for the support line :

$$\tau_L(x,y,t) = \tau_R(x,y,t) \qquad (10)$$

The eq. 10 and the state of system as expressed in eq.9 can be estimated indirectly with the symmetry of the following set :

$$G^* = \{L_i,\ T_{si},\ \gamma_i\} \quad i=L,R \qquad (11)$$

which actually represents the minimal quantity of simply measurable parameters necessary for the evaluation.

Methods of Evaluation

A) the Method of Minimal Square Deviation of Measured Parameters with Respect to the Statistical Pattern of Normal Gait.

This method is based on comparing the data measured on the patient with a statistical sample of the normal gait pattern, and is a logical consequence of the existence of the 'invariable model' (eq. 6). Supposing that eq. 5 and 6 have a normal distribution, we can define the criterion of evaluation (3,7) :

$$\min_{j\in J} d_{jk} = \left[\sum_{i=1}^{n} z_{ij}^{2}\right]^{1/2} ; \quad k=Z,P$$

$$z_{ij} = \frac{y_i - x_{ijk}}{\sigma_i} \qquad (12)$$

where z_{ij} is a normalized variable expressed in standard units.

The index $j \in J$ presents the applied orthosis or the stimulation sequences, and k=Z,P denotes variables of the healthy (Z) and the injured side(P) respectively. The necessary variables and their everage value and standard deviations for normals are represented in Table I.

The eq.6 defines also the vertical and the horizontal ground reaction forces, which, for once, could not have been measured over multy-strides. Yet the absence of these variables does not essentially effect the correctness of evaluation. Its presence would, however, rise the method sensitivity. The results of evaluation according to this method are shown in Table 2.

The best results of evaluation and with them of the applied

Table 1

names of variables	value Y_j	j	dimensions
step length	0.72	0.06	(m)
step duration	0.52	0.04	(s)
stance phase / ~~stride~~duration	60	4.8	(%)
max. value of hip flexion in swing phase	35	3.78	(°)
max. value of knee flexion in swing phase	60.8	3.9	(°)
max. value of anlke dorsal flexion in sw.ph.	-15.5	3.5	(°)
time of maximal hip flexion	85	1	(%)
time of maximal knee flexion	70	1.7	(%)
time of maximal ankle dorsal flexion	62	1.8	(%)

stimulation sequences are those where the value of expression d_{jz} / d_{jP} is maximal.

Table 2
The Results of Hemiplegic Patient Gait Evaluation According to the Method A

patient	d_{Pj}	d_{Zj}	$\frac{dZ}{dP}$	$j \in J$ kind of stimulation
1	13.42	5.44	0.40	1. without stimulation
	17.23	6.40	0.37	2. stimulation of type I
	14.46	7.05	0.48	3. stimulation of type II
2	9.40	7.43	0.79	1. without stimulation
	9.02	7.98	0.88	2. stimulation of type I
	11.27	8.27	0.73	3. stimulation of type II
3	21.02	14.00	0.66	1. without stimulation
	18.52	13.49	0.73	4. stimulation of type III

In Figure 2 also the vector Z_{ij} components of patient No. 3 are shown.Aalyzing them, we come to the conclusion that there is a given influence of chosen parameters of stimulation on gait variables on the basis of which it is possible to do the wanted corrections and determine better stimulation parameters.

B) The Method of Absolute Symmetry Deviation Between Left and Right Side Gait Parameters.
This method is based on the assumption that the gait is most confortable if the condition of symmetry of the right and left side of body is satisfied. The symmetry S is defined as a set of partial symmetries S_i for variables x_i as $S = \{S_i\}$ and the partial symmetry is defined as :

$$S_i = \frac{1}{m} \sum_{k=1}^{m} \frac{X_{iL}}{\bar{X}_{iR}} \quad (k) \qquad i=1. \dots r \tag{13}$$

where m is the number of steps. For the ideal gait $S_i = 1$, for normal gait $S_i \doteq 1$, and for pathological gait $S_i \neq 1$. The integral symmetry is then :

$$S_j = 1 - \Delta S_j \tag{14}$$

where ΔS_j is the average value of absolute deviation from the ideal symmetry :

$$\min_{j \in J} \Delta S_j = \frac{1}{n} \sum_{i=1}^{n} \text{abs } (1-S_{ij}) \; W_{ij} \; ; \; S_{ij} > 0 \qquad (15)$$

The subscript i denotes the measured variables and $j \in J$ the applied orthotic system. The maximum value of integral symmetry determines the optimal orthosis. In the eq. 15 W_{ij} represents the weighing factor defined as $W_{ij} = 1 + \sigma_{ij} / S_{ij}$, where σ_{ij} is the standard deviation of symmetry S_{ij}.

In the paper (11) this method and the variables to be evaluated are explained in detail. For the evaluation presented in the eq. 15, the minimum set of variables was used (as defined in eq. 11) : the right and the left leg step length, the step duration, the ratio stance phase and the stride duration. At the first sight it may appear that the average velocity was ignored. Actually it was implicitely taken into account by the symmetry of the step length and step duration. In this way we can evaluate with certainty also the energetic optimum (minimum) of the system. In addition, the symmetry of the stance phase duration of the right and the left leg represents the symmetry of average value of body speed in stance phase. The results of evaluation are shown in Table 3.

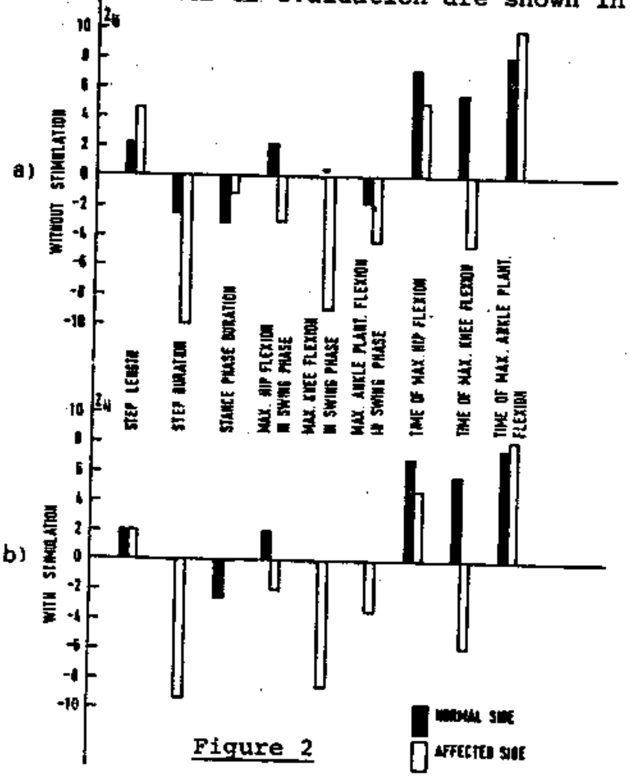

Figure 2

Interpretation of Gait Evaluation on Patient No. 3
a) without Stimulation b) with Three-channel Stimulation (10)

Table 3
The Results of Hemiplegic Patient Gait Evaluation According to the Method B

patient	integral symmetry S_j	step length X_{1j} (m)		standard dev. of step length σ_{1j}	number of steps n	kind of stimul. $j \in J$
1	0.75 $\underline{0.82}$ 0.71	0.58 $\underline{0.61}$ 0.58	$\alpha<0.05$ no sign.	0.096 0.030 0.070	39 34 40	1 2 3
2	0.81 0.82 $\underline{0.95}$	0.61 0.59 $\underline{0.64}$	no sign. $\alpha<0.05$	0.070 0.070 0.050	39 88 40	1 2 3
3	0.80 $\underline{0.89}$	0.37 $\underline{0.41}$	$\alpha<0.01$	0.060 0.040	61 61	1 4

Table 4
Partial Symmetry : S_1= step length symmetry, S_2=step duration sym. S_3=stance phase sym.

patient	partial symmetry						waiting factor		
	S_{1j}		S_{2j}		S_{3j}		w_{1j}	w_{2j}	w_{3j}
1	1.38 1.06 1.21	$\alpha<0.01$ $\alpha<0.01$	0.81 0.76 0.68	$\alpha<0.01$ $\alpha<0.01$	0.87 0.81 0.77	$\alpha<0.01$ $\alpha<0.01$	1.068 1.065 1.103	1.055 1.073 1.153	1.033 1.044 1.085
2	1.23 1.15 0.98	$\alpha<0.01$ $\alpha<0.01$	0.80 0.79 0.92	no sign. $\alpha<0.01$	0.92 0.87 0.94	$\alpha<0.01$ no sign.	1.080 1.094 1.134	1.070 1.062 1.070	1.096 1.126 1.070
3	0.84 0.94	$\alpha<0.01$	0.82 0.92	$\alpha<0.01$	1.16 0.85	$\alpha<0.01$	1.24 1.18	1.13 1.08	1.14 1.10

The best results are underlined. To enable the comparison of both methods, the patients are listed here in the same order as in Table 2. In Table 4 the partial symmetry and its waiting factor is shown. The stimulation sequence notation is omitted because it is the same as in Table 3.

Over the step length as shown in Table 3 and the partial symmetry as shown in Table 4, a one-tailed significant test was done in order to see whether the change of the mentioned parameters is the consequence of stimulation or is it random. The tables clearly show that where the integral symmetry is highest, the partial symmetry is also highly significant and approaches the ideal value of one. The step lenhtg, too, shows the best results.

At this point it is necessary to call attention to the fact that with the patient No 3, Table 4, partial symmetries S_{31} and S_{32} differ significantly (because of the method of interpretation in Table 4 itself), yet from the view-point of locomotion, they are equivalent, i.e. non functional. This example shows that it is possible to acheive the wanted result by correctly choosing the parameters of stimulator.

Conclusion

In the paper some problems of quantitative gait evaluation are analyzed and new mathematical criteria are developed. The present widely used evaluation methods are briefly discussed and their disadvantages are pointed out. The need for a scientific quantitative gait analysis is shown and the procedure of getting variables necessary for the evaluation criteria is examined. Considering a great number of variables describing human locomotion, measurement difficulties, only a minimal set of sufficient kinetic parameters has been defined as well as two evaluation methods convinient for computer implementation.

The first method is based on the minimal square deviation of the measured parameters with respect to the statistical pattern of normal gait. The second method is defined as the absolute symmetry deviation between left and right side gait parameters. In both methods variables are weighed according to their relative dispersion. Practical examples using the described methods of evaluation of hemiplegic patient gait assisted by functional electrical stimulation are shown and results obtained by the two methods are compared.

Although the results clearly show the applicability of the two methods in gait estimation as a whole, we still do not know exactly how each variable changed under the influence of different orthotic systems (analysis). So we cannot predict which parameters would lead to the wanted result. This cannot be expected by the help of variables as defined in Table 1 and eq.11. It is therefore absolutely necessary to expand the evaluation over the complete vector eq. 6 which describes the dynamic of gait as a whole. Here again we meet some difficulties, for some components of eq.6 vector are hardly satisfactorily measurable. It is necessary first to improve the measuring technique instrumentation - at least develop the attachable shoe force plate. We may hope that our efforts will enable measurements of ground reaction forces on a sufficient number of steps in near future already, and that with the available instrumentation (12) it will be possible to study gait from energy and power view-point. Under such conditions only, will it be possible to connect the cause, in this instance functional electrical stimulation, with the resulting gait.

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