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CARTESIAN CO-ORDINATE CONTROL OF A COMPLETE ARM PROSTHESIS

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Summary

It has been established by other workers that an arm prosthesis is easier to control if the signals required from the user are simply related to the desired position of the terminal device, rather than to individual joint angles or velocities. To facilitate this the position of the terminal device can be related to that of a reference point, usually on the shoulder, by a suitable set of co-ordinates. This method has been used on several arms with a limited number of degrees of freedom. A spherical polar co-ordinate system is usually adopted, largely because it can be implemented mechanically in simple systems. However if extra degrees of freedom are added the complexity of the problem increases irrespective of the co-ordinate system adopted, and only computer controlled systems have used co-ordinate position control with arms capable of the full range of human movements. These computer systems usually adopt cartesian co-ordinates.

The report describes a cartesian co-ordinate position control system for a seven degree of freedom arm which does not employ a computer. The system is capable of very precise positioning and of speeds compatible with natural movements. Also described is an electrically powered prototype arm, designed to test the control scheme. This prototype, although not an attempt at a clinical prosthesis, uses spiroid gearing which, although somewhat novel, may have potential for clinical application. The arm is capable of movements of sufficient speed and force to permit normal arm functions to be performed in a natural manner.

Introduction

The loss of an arm due to amputation or to congenital deformity implies the absence of not only the physical structure of the arm, but also of part of the nervous system relating to the arm.

Replacing the physical structure of a lost arm without attempting to replace the lost lower levels of the nervous system forces the user to consciously control the motion of each joint using visual feedback. The control loops thus formed are very slow and the concentration required to supervise even a single joint motion makes great demands on the user. Co-ordinating the actions of an arm capable of seven or eight independent movements would be beyond human capability, and such a system would inevitably result in movements of different joints being performed sequentially.

When positioning the human hand, one thinks only of its desired position, not of the necessary joint angles in the arm. Similarly a control system for an artificial arm will be easier to use if its input signals are related to the desired position of the terminal device, rather than joint angles, joint velocities or terminal device velocities. Several prosthetic arms have been constructed which use position control. Of particular significance is that due to Simpson(1). In this five degree of freedom pneumatically powered arm, the position of the terminal device is referred to the shoulder by a set of spherical polar co-ordinates, and the actuators act through a complex system of gears which enables motion in one co-ordinate to be produced independently by the action of a single actuator. Position control is incorporated in all degrees of freedom but perhension and thus the position of the terminal device is directly related to that of the control sites (the shoulders) enabling the user to "feel" the position of the terminal device. This effect has been referred to as "Extended Physiological Proprioception (E.P.P.).

Extending this method of control to include extra degrees of freedom adds considerable complexity, and only computer systems have used position control, usually with cartesian co-ordinates, on arms capable of the full range of human movements.

The aim of the work described in this report is to provide a system for applying co-ordinate position control to a seven or eight degree of freedom arm without employing a digital computer. The report will consider only those degrees of freedom associated with positioning the hand in space.

Choice of a Co-ordinate System

Most simple arms use spherical polar co-ordinates, because it is possible to arrange mechanically for motion in each co-ordinate to be controlled independently by a single actuator. However, as the complexity of the arm is increased, it becomes prohibitively difficult to implement any co-ordinate system mechanically and the degree of complexity of the necessary control scheme is similar for spherical polar or cartesian co-ordinates. The choice should therefore be made on the grounds of their suitability for the system in which they are to be used.

The spherical polar method describes the position of the hand in terms of R, θ and ϕ. Where R is the length of the line from the shoulder to the wrist, θ is the angle of elevation of this line and ϕ its angle of azimuth. If, as in the Simpson arm, movement in three co-ordinates are controlled by movement of the clavicles, it can be arranged that forward and backward motion of the clavicle on the same side as the arm being controlled results in extension and contraction of the limb (increased and decrease in R), and verticle motion of the clavicle rotates the arm about the shoulder in a verticle plane (varying θ). Therefore, when the hand is directly in front of the shoulder, moving the clavicle up and down raises and lowers the hand, and moving the clavicle forward and backward moves the hand forward and backward. This makes the hand very easy to control. However, when the hand is directly above the shoulder, motion of the clavicle forward will cause the hand to raise, and motion of the clavicle upward will cause the hand to move backwards. This is clearly not as easy to control. In the extreme case, when the hand is behind the shoulder, movement of the shoulder in either direction causes movement of the hand in the opposite sense.

The cartesian co-ordinate method describes the position of the hand in terms of X, Y and Z as defined in Figure 1. Using cartesian co-ordinates, a movement in a particular direction is always connected with an input charge in the same direction, regardless of the position of the terminal device. If, as before, the clavicles are used to input signals it can be arranged that the hand, in the saggital plane through the shoulder, follows the movement of the clavicle on a magnified scale. This should allow extremely simple control with the minimum of conscious effort, and, because of the close association between the position of the clavicle and the position of the terminal device, should produce considerable E.P.P. It therefore seems that cartesian co-ordinates are more suitable for systems with linear input signals.

It is proposed to investigate the possibility of using three dimensional movement of one clavicle to generate the co-ordinates X, Y and Z of the arm. This can be achieved by measuring the lateral displacement of the clavicle with respect to the base of the spine, or by measuring spinal curvature. Thus using a cartesian co-ordinate system the terminal device will follow the movement of the clavicle on a magnified scale in three dimensions. This should greatly improve the ease of use of the arm and the E.P.P., by using signals from only one side of the body to control the hand position. This method could be used for only one dominant arm.

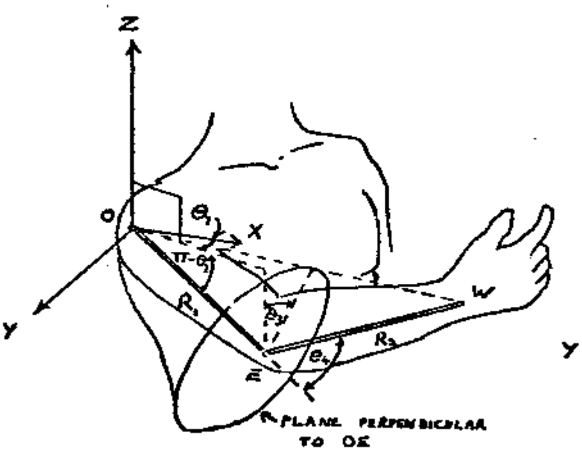

Figure 1

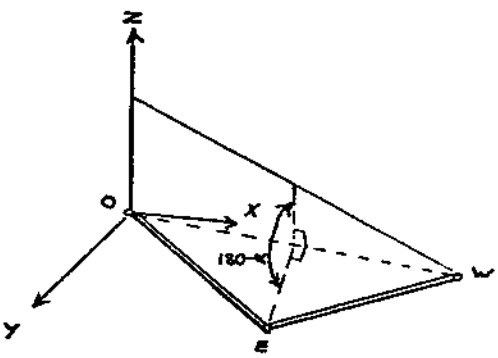

Figure 2

The Controller

The cartesian co-ordinates X, Y and Z and the system parameters R_1, R_2, θ_1, θ_2, θ_3 and θ_4 are defined in Figure 1. In order to obtain unique solutions to the system equations a fourth input variable is required, for this purpose the elbow angle α is defined in Figure 2.

Two methods of control were considered. The first involves calculating the necessary joint angles $\theta_1 - \theta_2$ from the co-ordinates X, Y, Z and α of the required position. Position servo-loops then drive the arm to conform with these calculated values. In practice calculation of these angles to give quadrature information is extremely complex, and this system was rejected. The second method is to calculate the actual position of the hand from the measured values of the joint angles and to compare this with the desired position of the hand to generate error signals. Position servo-loops are then used to minimise these error signals such that the hand is driven to the desired position. Difficulties arise because each degree of freedom is dependent on the position of several joints and because an error of a given polarity will be minimised by rotation of a particular joint in different directions when the arm is in different positions.

Figure 3 shows a simplied block diagram of the complete system.

$$\underset{\sim}{x} = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{vmatrix} = \begin{vmatrix} X \\ Y \\ Z \\ \alpha \end{vmatrix} \qquad \underset{\sim}{\theta} = \begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{vmatrix}$$

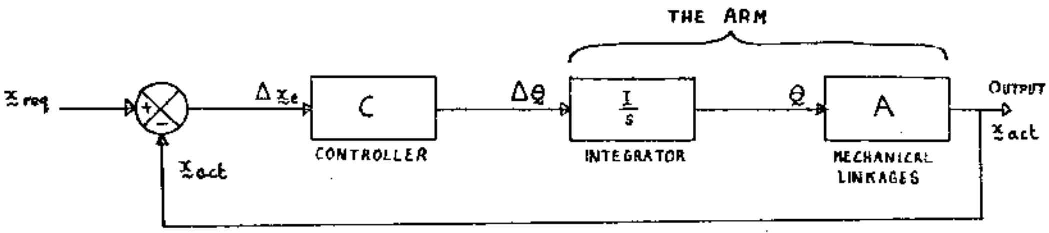

Figure 3

Simplified Block Diagram of the System

$\underset{\sim}{x}_{req}$ = required $\underset{\sim}{x}$ (input)
$\underset{\sim}{x}_{act}$ = actual $\underset{\sim}{x}$ (output)
$\Delta \underset{\sim}{x}_e$ = $\underset{\sim}{x}_{req} - \underset{\sim}{x}_{act}$
$\Delta \underset{\sim}{\theta}$ = change in $\underset{\sim}{\theta}$

The arm can be represented by an integrator and a block of transfer function A representing the relationship between the cartesian co-ordinates of the arm $\underset{\sim}{x}$ and the joint angles $\underset{\sim}{\theta}$.

The system equation for a small perturbation $\Delta \underset{\sim}{\theta}$ is

$$\Delta \underset{\sim}{x}_{act} = \frac{A}{s} \Delta \underset{\sim}{\theta}$$

where

$$A = \begin{vmatrix} \frac{\partial X_1}{\partial \theta_1} & \frac{\partial X_1}{\partial \theta_2} & \frac{\partial X_1}{\partial \theta_3} & \frac{\partial X_1}{\partial \theta_4} \\ \frac{\partial X_2}{\partial \theta_1} & \frac{\partial X_2}{\partial \theta_2} & \frac{\partial X_2}{\partial \theta_3} & \frac{\partial X_2}{\partial \theta_4} \\ \frac{\partial X_3}{\partial \theta_1} & \frac{\partial X_3}{\partial \theta_2} & \frac{\partial X_3}{\partial \theta_3} & \frac{\partial X_3}{\partial \theta_4} \\ \frac{\partial X_4}{\partial \theta_1} & \frac{\partial X_4}{\partial \theta_2} & \frac{\partial X_4}{\partial \theta_3} & \frac{\partial X_4}{\partial \theta_4} \end{vmatrix}$$

and the controller equation is

$$\Delta \underset{\sim}{\theta} = C \, \Delta \underset{\sim}{x}_e$$

C must be chosen such that $\Delta \underset{\sim}{\theta}$ is driven to make $\Delta \underset{\sim}{x}_e$ go to zero. The solution chosen is to make $C = A^T$.

In this case the forward transfer function of the system is $\frac{AA^T}{s}$, this has zero steady state error and since AA^T is non-negative definite, the system can be shown by Liapanov's theorem to be stable.

In practice the system is slightly more complicated than that shown in Figure 3, since the co-ordinates x cannot be measured directly from the arm. It is therefore necessary to calculate x from the joint angles $\underset{\sim}{\theta}$, and the system becomes that shown in Figure 4.

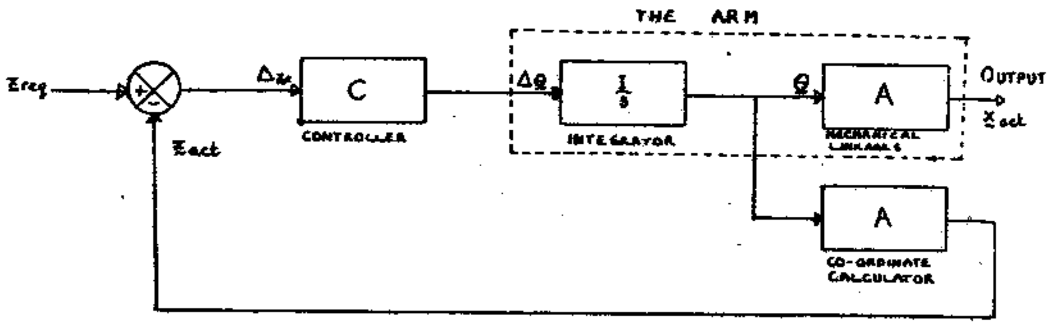

Figure 4

The Complete System

The equations used in the co-ordinate calculator are:

$$X = -R_1\mathrm{Cos}\theta_2\mathrm{Cos}\theta_1 + R_2\mathrm{Sin}\theta_4\ (\mathrm{Sin}\theta_3\mathrm{Sin}\theta_1 + \mathrm{Cos}\theta_3\mathrm{Sin}\theta_2\mathrm{Cos}\theta_1) - R_2\mathrm{Cos}\theta_4\mathrm{Cos}\theta_2\mathrm{Cos}\theta_1$$
$$Y = -R_1\mathrm{Cos}\theta_2\mathrm{Sin}\theta_1 + R_2\mathrm{Sin}\theta_4\ (\mathrm{Sin}\theta_3\mathrm{Cos}\theta_1 + \mathrm{Cos}\theta_3\mathrm{Sin}\theta_2\mathrm{Sin}\theta_1) - R_2\mathrm{Cos}\theta_4\mathrm{Cos}\theta_2\mathrm{Sin}\theta_1$$
$$Z = -R_1\mathrm{Sin}\theta_2 - R_2\mathrm{Cos}\theta_4\mathrm{Sin}\theta_2 - R_2\mathrm{Sin}\theta_4\mathrm{Cos}\theta_3\mathrm{Cos}\theta_2$$
$$\alpha = \frac{\pm Y\ \mathrm{Cos}\ \theta_2}{\left\{\frac{(X^2 + Y^2)(1 - \mathrm{Cos}\ \theta_4)}{2}\right\}^{\frac{1}{2}}}$$

The controller has a transfer function C defined by

$$C = \begin{vmatrix} \frac{\partial X_1}{\partial\theta_1} & \frac{\partial X_2}{\partial\theta_1} & \frac{\partial X_3}{\partial\theta_1} & \frac{\partial X_4}{\partial\theta_1} \\ \frac{\partial X_1}{\partial\theta_2} & \frac{\partial X_2}{\partial\theta_2} & \frac{\partial X_3}{\partial\theta_2} & \frac{\partial X_4}{\partial\theta_2} \\ \frac{\partial X_1}{\partial\theta_3} & \frac{\partial X_2}{\partial\theta_3} & \frac{\partial X_3}{\partial\theta_3} & \frac{\partial X_4}{\partial\theta_3} \\ \frac{\partial X_1}{\partial\theta_4} & \frac{\partial X_2}{\partial\theta_4} & \frac{\partial X_3}{\partial\theta_4} & \frac{\partial X_4}{\partial\theta_4} \end{vmatrix}$$

The terms of this matrix can be found by partial differentiation of the of the equations used in the co-ordinate calculations.

In order to investigate the performance of this system a simple computer simulation was devised using the equations derived for A and C. This showed the system to be stable for small and large step inputs.

Implementation of the Controller

Calculation of the sixteen partial derivatives in C would involve considerable hardware. To simplify the system the terms of C are generated by measuring the response of the outputs of the co-ordinate calculating system to small variations in each of its inputs. A small perturbation is added to each input in turn. Phase sensitive detectors are then used to measure the phase and magnitude of the resulting response on each output. These responses are directly related to the partial derivatives.

To produce the arm control signals $\Delta\underline{\theta}$ from the error signals $\Delta\underline{x}_e$ and the partial derivatives, the controller requires 16 analogue multipliers. In order to reduce the cost and complexity of the system, the possibility of assigning the partial derivatives values dependent only on the polarity of the calculated partial derivative and not on its magnitude, was considered. To increase the stability of the system, in the presence of inaccuracies in the calculated derivatives, a "dead space" can be added such that for $i = 1, 4$ and $j = 1, 4$

If $\left(\frac{\partial X_i}{\partial \theta_j}\right) < -h$ then $\left(\frac{\partial X_i}{\partial \theta_j}\right)' = -E$ where $\left(\frac{\partial X_i}{\partial \theta_j}\right)$ is the calculated derivative

$-h \leqslant \left(\frac{\partial X_i}{\partial \theta_j}\right) \leq +h$ then $\left(\frac{\partial X_i}{\partial \theta_j}\right)' = 0$ $\left(\frac{\partial X_i}{\partial \theta_j}\right)'$ is the value used in C

$+h < \left(\frac{\partial X_i}{\partial \theta_j}\right)$ then $\left(\frac{\partial X_i}{\partial \theta_j}\right) = +E$ E = a +ve constant

h = a threshold

This modification affords a considerable reduced in the complexity of the controller.

To investigate the effect this modification has on the stability of the system, a computer simulation was devised incorporating this condition. The resultant system was stable for large and small step inputs and the response was relatively insensitive to variations in the threshold, h. It was therefore decided to incorporate this modification into the system being developed. Figure 5 shows the resultant system.

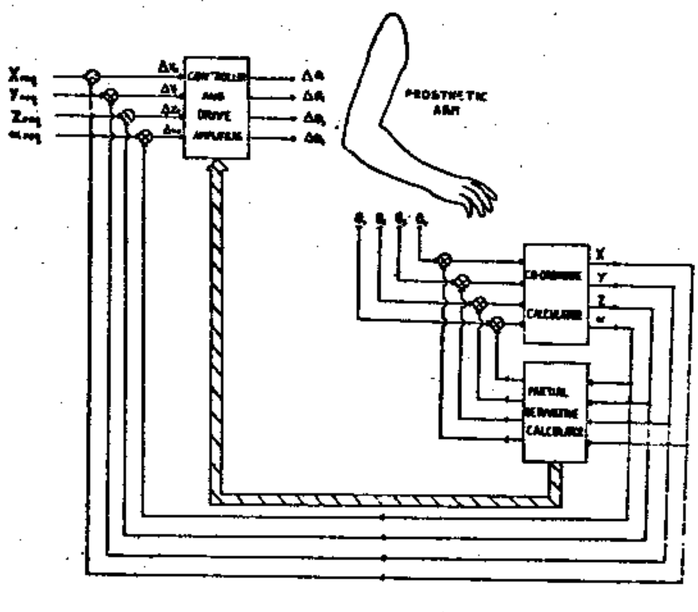

Figure 5

The Prototype Prosthesis

Although this project is not primarily concerned with the design of the mechanical structure of prostheses, it was thought necessary to produce a prototype with which to evaluate various control schemes. In the design of this prototype consideration was given to the range of movements of the arm and to its speed and torque capabilities, but no attempt was made to produce a clinically acceptable prosthesis.

In order to reduce the cost and complexity of the prototype it was decided that initially not all the degrees of freedom would be incorporated, but that the design would permit the addition of extra degrees of freedom later. It was thought desirable for the prosthesis to be of sufficient power and suitable construction for it to be used with the prosthetic hand which is at present under development within the control group at Southampton University (and which is described in a paper to be presented at this Symposium). For this reason the degrees of freedom included in the hand, namely, wrist flexion, wrist rotation and those connected with prehension, were omitted from the arm. The five degrees of freedom provided are shoulder flexion, shoulder rotation, elbow flexion, elbow rotation and wrist abduction/adduction.

D.c. permanent magnet motors were chosen for the actuators because of their high power to weight and torque to weight ratios, and because their linear torque/current and speed/voltage characteristics make them simple to control. In order to reduce the moments of the arm weight about the elbow and the shoulder it was necessary to move the actuators as far up the arm as possible. The possibility of positioning all the motors above the shoulder was rejected because of the difficulties involved in transmitting power down the arm to the lower joints. It was however decided to position the shoulder rotation motor above the shoulder joint since this could be connected directly.

In order that power is not consumed in supporting a stationary load the joints are required to lock when drive is removed from the motors. This can be achieved by applying a mechanical brake to some part of the gear train or joint, or by using a self-locking gear network. Initially two possible arrangements were considered, a spur gear train with a mechanical lock and a self-locking gear train using a worm and wheel.

Spur gearing is the move efficient of the two methods. However, to take the large torques required at the joints the gears would need very large teeth and therefore the reduction ratios that could be obtained from any pair of gears would be low (probably 2 or 3 : 1 for the early stages). The gear train would therefore be very long, each additional gear adding backlash and considerable weight. The aligning of the gears would require great precision to prevent excessive backlash and loss of efficiency. It is likely that the assembly would be large, heavy and have an overall efficiency of less than 50%.

Worm and wheel gearing has the advantage that a high reduction ratio can be obtained with few components thus reducing the number of gears required, the size, and the backlash. However, because of the operation of the system it is necessary to use different materials for the worm and wheel, and invariably in commercially available worm and wheel pairs the wheel is made of phosphor-bronze. This material is not as strong as steel and consequently the wheel has to be very large to take the necessary torque. Poor efficiency is also a disadvantage of this method, since if a system is designed to lock under all conditions it is likely that forward efficiency will be approximately 15%. Thus very large and consequently heavy motors are required.

It became apparent that neither method could be used in the proposed system and an alternative system was sought. This was found in the form of Spiroid gears(2). These gears are similar to high ratio hypoid gears and are made in England by the Davall Gear Company. The relatively low sliding velocities of these gears enable the use of steel for the gear and the pinion and this together with a very large area of contact makes them able to withstand much higher torques than spur or worm and wheel gearing of comparable size and weight. Careful setting-up can produce a self-locking system with effectively zero backlash, a forward efficiency of up to 60%, and a size and weight considerable less than a spur or worm and wheel system of similar reduction ratio and torque capability. Spiroid gears are therefore used in the final stage of the gear train, a spur gear train being used to drive the pinion from a geared motor shaft.

The structure of the arm was kept as simple as possible, and was made of aluminium. The weight of the finished arm, from shoulder to hand, was approximately equal to that of its human counterpart. The performance of the prototype is shown in Table 1.

Table 1

Performance of the Prototype

Joint	Worst Case Torque kg. m.	Maximum Speed Radians/Sec.
Shoulder Flexion	2.4	2
Shoulder Rotation	2.3	2
Elbow Flexion	1.7	3
Elbow Rotation	1.7	3
Wrist Abduction/Adduction	0.6	3

The worst case torque indicated is the torque which the arm can apply to an external object when opposed by the maximum possible moment of the arm about that particular joint. The speed/torque performance of the prototype is sufficient to allow it to perform most "normal" arm functions in a natural manner, even when supporting a prosthetic hand.

The Complete System

At the time of writing this paper the system is unfinished. The mechanical prototype has been completed and a limited number of tests have been carried out on this, with encouraging results. The necessary motor drive amplifiers have been designed and constructed, these are of the "chopper" type for maximum efficiency. Work on the co-ordinate calculator and controller stages are nearing completion and it is hoped that initial testing of the complete system will begin shortly, the results of which will be described at the Symposium.

Conclusion

Work to date suggested that an electrically powered multi-degree of freedom prosthetic arm is a realisable possibility from mechanical performance, weight and movement co-ordination considerations. The electronic controller described offers considerable advantages over existing mechanical and computer controlled systems.

References

1. Simpson, D.C. : 'The design of a complete arm prosthesis', Biomedical Engineering, February, 1973.

2. Nelson, W.D. : 'Spiroid Gearing', Machine Design, February 16th, 1961.

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