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A METHOD FOR THE DYNAMIC CONTROL OF REDUNDANT MANIPULATORS

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ABSTRACT

The work presented in this paper deals with the study of the dynamic model and with the control system synthesis of adaptive advanced manipulators under complex and time varying environmental conditions. One of the problems to be solved in this case is the automatic coordination of the movements of the various degrees of freedom in the redundant mechanical system.

A method of dynamic coordination is presented in this paper which can be applied to articulated mechanical systems, which have a tree-like structure. Each branch may exhibit translational and rotational relative motions. The proposed method can be divided into three main stages :

(1) in the space of the meaningful variables (for instance those defining the manipulating device's orientation and position) a potential function is constructed in such a way that is has a unique minimum for the desired values of these variables,

(2) the generalized forces obtained from the potential function are applied to the joints,

(3) dissipative forces are added which are computed in order to insure the optimal stabilization of the mechanical system.

The practical application of this method in synthesizing the lower level control system requires the knowledge of the mechanism's dynamical model. The equations of motion, based upon the Lagrangian formalism, are presented. The different methods developed in the literature for obtaining such equations are summarized and compared.

Finally the theoretical results exposed in this paper are applied to the control of an experimental mechanism : the "RHOTHETAPHI", a telescopic gimballed manipulator.

1. INTRODUCTION

Some manipulators have a large number of degrees-of-freedom, which insure great flexibility and thus the ability to avoid obstacles. Here the problem of the automatic control, of such a redundant device is considered from a given initial position to a final position corresponding to the task completion. In

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general, this problem leads to arbitrariness since there is an infinite number of final configurations which enable the given goal to be attained. It may therefore be of interest to select, among all these configurations, the one which is "optimal" with respect to a secondary criterion. This will be the case, for example, if all the joint variables are required to be sufficiently "far" from the limiting and locking values.

The type of manipulator under consideration is an electro-mechanical system with many couplings. The mechanism is assumed to have a tree-like structure, i.e. without loops ; it is formed by the interconnexion of an arbitrary number of links, with revolute and prismatic joints. Each degree of freedom is actuated by a motor. The system inputs are the motor torques. The outputs are the positions and the velocities of the joint variables. The input-output general description of a manipulator as a mechanical system will be obtained in deriving the dynamical state equations which describe the motions. Thus the first problem arising in the dynamic control of manipulators is to obtain a dynamical model. Then, the second problem is to develop a control system which ensures a good coordination, or synergy, in a dynamic sense.

2. DYNAMICAL MODELS OF ARTICULATED MECHANICAL SYSTEMS

There are three main groups of methods, presented in the literature, suitable for obtaining the dynamical equations :

(1) the methods using the NEWTON-EULER equations,

(2) the methods using the LAGRANGE equations,

(3) a recent method using the D'ALEMBERT principle of virtual work.

2.1. *Methods based on the NEWTON-EULER equations*

These methods were first developed for the study of the dynamic behaviour of deformable spacecraft.

The particular model established in 1962 by H.J. FLETCHER et al /1/ was generalized later by W. HOOKER and G. MARGULIES /2/ ; the proposed model introduces the concept of "augmented bodies", which simplifies the equations. Furthermore, the matrix formalism used by R.E. ROBERSON and J. WITTENBURG /3/, following the HOOKER-MARGULIES vector equations, is suitable for simulations on digital computers.

Nevertheless these methods are limited to relative rotations and it is difficult to eliminate the constraint torques /4,5/.

Recently, J. WITTENBURG /6/ added the possibility of taking into account relative translational motions between adjacent links. This method can be compared to the work of G.V.KORONEV /7/ who had previously introduced the notions of kinematic and dynamic levels, together with the "principle of compatibility". Unfortunately, these methods, which are based upon tensor calculus, do not lead to an explicit formulation of the scalar equations of motion. Such a drawback does not appear in the method proposed by E.P. POPOV et al /8/.

M. VUKOBRATOVIC and J. STEPANENKO /9/ have developed the algorithms, for the dynamic simulations of anthropomorphic systems, by using the NEWTON-EULER equations and D'ALEMBERT'S principle (cf. § 2.3.). They introduce the notion of a connexion matrix which allows the study of closed Kinematic loops.

The method of F.W. OSSENBERG-FRANZES /10/ who deals with the most general case of articulated systems, is interesting principally from the theoretical point of view.

2.2. *Methods based on the LAGRANGE equations*

2.2.1. *J.J.UICKER'S method /11/*

This method is an extension of J. DENAVIT and R.S. HARTENBERG'S formalism /12/ ; its originality is to be found in the use of 4x4 matrices ; and also in the fact that, the mechanism under study may have an arbitrary number of closed loops. The authors have also presented an interesting way to obtain the kinetic energy and the potential energy, that leads to the LAGRANGE'S equations in a form suitable for simulation studies. R.A. LEWIS /13/ and A.K. BEJCZY /14/ have applied this method to the design and study of a 6-d.o.f. manipulator.

UICKER'S method appears to be the most general but it suffers from a great redundancy in the computations and it ignores the interesting notion of "augmented bodies".

2.2.2. *A method suitable for articulated systems with revolute joints*

For the special case of systems with revolute joints and with a topological tree-like structure, we have developed a method which is different from the above described one and which brings out the "augmented bodies" and the barycenters of such bodies /15-17/. This method has been applied to a 4-d.o.f. arm /18/.

The range of application of this method has recently been enlarged by also considering translatory motions /19/ , the corresponding results will be summarized in paragraph (§ 3) of this paper.

2.3. *A method using D'ALEMBERT'S principle of virtual work*

This method will not be developed here. It has been recently proposed by J.C. SAMIN /20/. According to this author, the main advantages are :

(1) the method does not need the explicit elimination of the constraint torques,

(2) the method does not need to compute the Lagrangian function and its derivatives.

3. *THE PROPOSED DYNAMICAL MODEL*

This model is obtained by using Lagrangian formalism. GIBB'S notation will be used throughout.

Let us consider an articulated mechanical structure (fig.1) consisting of (n+1) rigid bodies $\mathcal{C}_0, \mathcal{C}_1, .., \mathcal{C}_n$. Let $\mathcal{P} = \{1,2,...n\}$. A Galilean reference frame is enbedded in $\mathcal{C}_0$. The bodies are numbered naturally starting from $\mathcal{C}_0$.

The topology of the system is uniquely defined by the set of coefficients ε_{ij} :

$$\varepsilon_{ij} = \begin{cases} 1 & \text{if either } \mathcal{C}_i \text{ is a link of the chain } \mathcal{C}_0 \mathcal{C}_j \text{ or } i=j \\ 0 & \text{else} \end{cases}$$

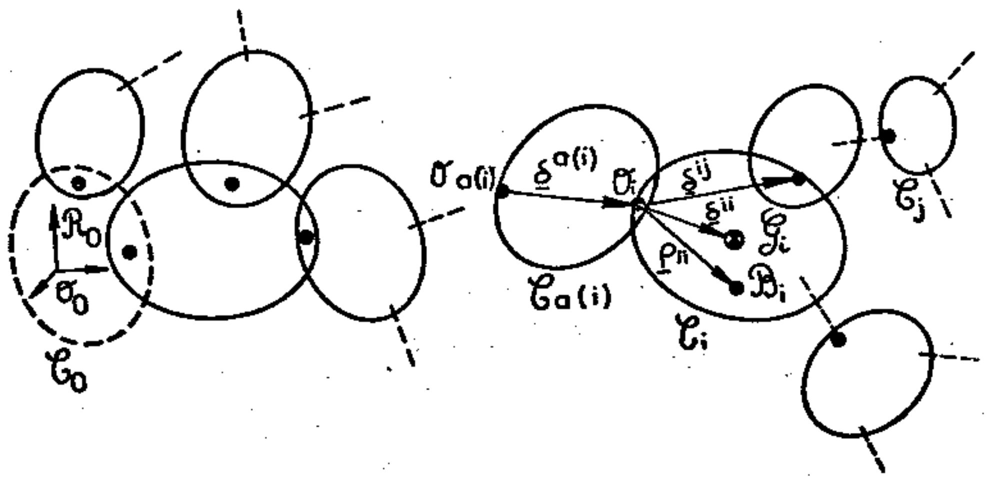

Fig.1. **The general structure under consideration.**

The relative motions can be (fig.2).

a) **either a rotation defined by the relative displacement** γ_i

b) **or a translation defined by the displacement** $\ell\tau_i$, **where** ℓ **is a normalization coefficient.**

The nature of the relative motions have been characterized by the set of coefficients s_i, **i=1,2,...,n, such that** s_i=0 **in case a) and** s_i=1 **in case b).**

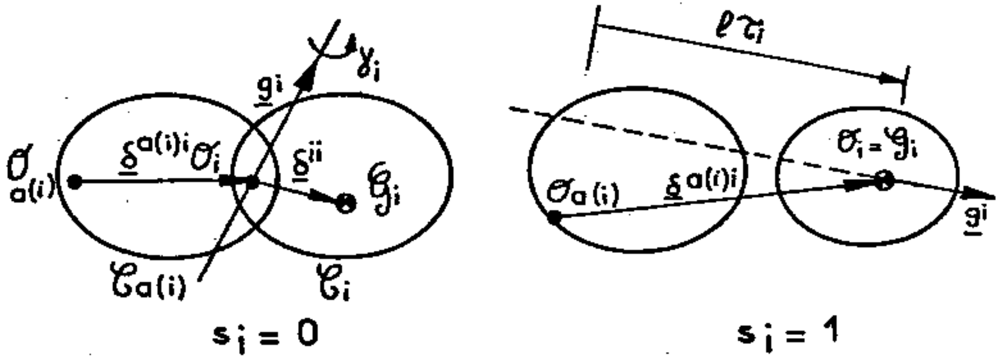

Fig.2. Definition of the relative motion.

An additional facility for establishing the dynamical model is introduced by considering the so-called "augmented bodies" : let m_i be the mass of $\mathcal{C}_i$ (with m_0=0), and let us write $m=\sum m_i$ and $\mu_j=\sum_k \varepsilon_{jk} m_k$. The "augmented body" $\mathcal{C}_i^*$ is formed by $\mathcal{C}_i$ to which the following point masses have been added :

- μ_j at points $\mathcal{O}_i$ such that i=a(j) (antecedent of j)
- $(m-\mu_i)$ at point $\mathcal{O}_i$.

It is useful to introduce, in the computations the barycenter $\mathfrak{B}_i$, of the augmented body, defined by $\underline{\rho}^{ii} \triangleq \underline{\sigma_i \mathfrak{B}_i}$ (fig.1).

The above notations, together with additional symbols which take into account generalized tensors of inertia $\underline{\underline{I}}^{ij}$ (cf./15/), allow us to derive the equations of motion in a straightforward manner. The corresponding algorithms are given in the work /19/.

Let $q_i = (1-s_i)\gamma_i + s_i \tau_i$ be the i-th generalized coordinate ; the kinetic energy T can be put in the form

$$(1) \quad 2T = \sum_{k,\ell} a^{k\ell} \dot{q}_k \dot{q}_\ell$$

where each coefficient $a^{k\ell}$ can be computed as a function of the generalized coordinates and of the structural parameters. So we can write

$$(2) \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_\ell}\right) - \frac{\partial T}{\partial q_\ell} = \sum_{i \in \mathcal{P}} a^{i\ell} \ddot{q}_i + \sum_{j,k \in \mathcal{P}} b^{\ell,jk} \dot{q}_j \dot{q}_k = Q^\ell \ , \ \forall \ell \in \mathcal{P}$$

where $b^{\ell,jk}$ is the CHRISTOFELL symbol.

In order to put the dynamic equations in the classical form we introduce the state vector X, the elements of which are

$$(3) \quad \begin{cases} x_\ell \triangleq q_\ell \\ x_{\ell+n} \triangleq \dot{q}_\ell \quad \text{with} \quad \ell \in \mathcal{P}. \end{cases}$$

Let A be the (n x n) symmetric matrix formed with the a^{jk}'s, and let us denote by α^{jk} the elements of its inverse A^{-1}. If we put $S^\ell \triangleq -\sum_{j,k \in \mathcal{P}} b^{\ell,jk} \dot{q}_j \dot{q}_k$ (the centrifugal and coupling forces), the LAGRANGE equations take the following form :

$$(4) \quad \dot{x}_i = \begin{cases} x_{n+i} \ , & i = 1, 2, \ldots, n. \\ \sum_{j \in \mathcal{P}} \alpha^{ji} (Q^j + S^j) \ , & i = n+1, \ldots, 2n. \end{cases}$$

The method summarized in this paragraph has been tested for obtaining the equations of various mechanical systems. It appeared to be shorter and simpler than the method used in /14/, even in the simple case of a gimballed telescopic structure. These advantadges seems to be due principally to the introduction of the concept of "augmented bodies".

4. *A METHOD FOR SYNTHESIZING THE DYNAMIC CONTROL LAW*

Several well known methods for synthesizing the control system of manipulators have been proposed. They are mainly based upon the use of the kinematic relationships between the orientation and the position of the terminal device and the generalized coordinates. Using these methods, however the dynamics of the mechanical system is ignored and this may lead to unexpected movements during fast transients of large magnitude. In other works /14,21/ the dynamic effects are taken into account but there is no redundancy in the system : the number of inputs of the dynamic level equals the number of degrees of freedom. The method proposed below is a theoretical approach to the problem of controlling redundant systems, i.e. systems which may have several different configurations able to execute the given task, or hit the goal. In our case, we shall not specify a priori which configuration must be used ; the coordination of the several degrees of freedom will be done automatically by choosing to derive part of the generalized force Q^ℓ in the LAGRANGE equation from a potential function /22/.

The proposed method will be illustrated by solving an end-point problem and will be applied to a simple practical experiment.

Let $Y = [y_1, \ldots y_j, \ldots y_m]^T$ be the vector whose final value Y^* must be the given goal Y^f ; we have $y_i = f_i(x_1, x_2, \ldots, x_n)$.

Let $Z = [z_1, \ldots z_k, \ldots z_p]$ be the vector whose final value Z^* must be as "near" as possible to Z^f; we have $z_k = h_k(x_1, x_2, \ldots, x_n)$.

The system is redundant with respect to the goal if $m < n$.

In order to obtain the part of the generalized force Q^ℓ deriving from a potential function, the following functions are introduced :

a) A function $-U_p$ for gravity compensation

b) A function V, smooth and having a finite number of minima, defined on the (m x p) Euclidian Space

(5) $$V = V(y_1, \ldots, y_m ; z_1, \ldots, z_p).$$

Let V have the property

(6) $$V = V_{min} \iff \begin{cases} y_j = y_j^* = y_j^f & , j = 1, 2, \ldots, m. \\ f(z_1, z_2, \ldots, z_p) = 0 \end{cases}$$

Example : Let us consider a system with 3 d.o.f. (x_1, x_2, x_3) and let

(7) $$\begin{cases} y_1 = f_1(x_1, x_2, x_3) \\ z_1 = h_1(x_2) \\ z_2 = h_2(x_3) \\ z_3 = h_3(x_2, x_3) \end{cases}$$

We can choose

(8) $$V = \alpha_1(y_1 - y_1^f)^2 + \beta_1(z_1 - z_1^f)^2 + \beta_2(z_2 - z_2^f)^2 + \beta_3(z_3 - z_3^f)^2$$

To make the state $[x_1^*, x_2^*, \ldots x_n^*, 0, 0, \ldots 0]$ asymptotically stable, it is necessary to introduce damping forces Q_d^ℓ:

(9) $Q^\ell = Q_p^\ell + Q_d^\ell$, where

(10) $$\begin{cases} Q_p^\ell = -\dfrac{\partial U(x_1, \ldots, x_n)}{\partial x_\ell} \\ U(x_1, \ldots, x_n) = V[y_1(x_1, \ldots, x_n), \ldots, z_p(x_1, \ldots, x_n)]. \end{cases}$$

The dissipative forces can be chosen in such a way that the transient motion is "optimal" with respect to a given criterion /22/.

5. *APPLICATION TO THE DYNAMIC CONTROL OF AN EXPERIMENTAL SYSTEM*

The experimental system is a gimballed telescopic arm with three degrees of freedom, as shown in fig.3.

The problem considered here for illustrative purposes consists of bringing the end-point of the manipulator anywhere in a given vertical plane defined by $y = y_1^f$ with respect to a fixed reference frame (fig.4).

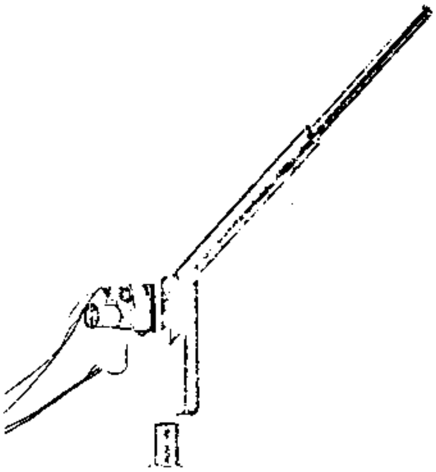

Fig.3. The experimental system.

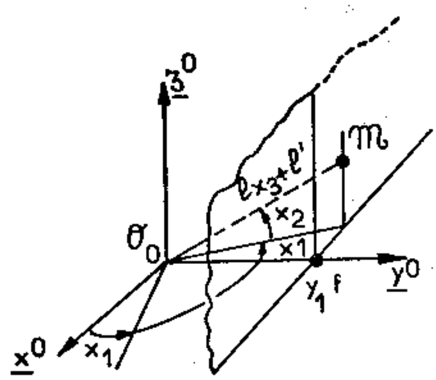

Fig.4. Geometry of the system.

The requirements on the final state are

a) $x_2 = \pi/4$

b) the telescopic portion half-deployed.

One can choose

$$(11)\quad \begin{cases} y_1 = (\ell x_3 + \ell')\cos x_1 \cos x_2 \\ z_1 = x_2 \\ z_2 = x_3 \\ V = \alpha_1 (y_1 - y_1^f)^2 + \beta_1 (z_1 - z_1^f)^2 + \beta_2 (z_2 - z_2^f)^2 . \end{cases}$$

When using the method proposed in § 3, one obtains the coefficients of the LAGRANGE equations :

$$(12)\quad \begin{cases} a^{11} = a_1^{11} + a_2^{11}\cos x_2 + a_3^{11} x_3^2 \cos^2 x_2 & b^{1,12} = -(a_2^{11} + a_3^{11} x_3^2)\sin x_2 \cos x_2 \\ a^{22} = a_1^{22} + a_2^{22} x_3^2 & b^{1,13} = a_3^{11} x_3 \cos^2 x_2 \\ a^{33} = a^{33} & b^{2,23} = a_2^{22} x_3 \end{cases}$$

From this, the following expressions can be computed :

$$(13)\quad \begin{aligned} S_1 &= -2 b^{1,12} x_4 x_5 - 2 b^{1,13} x_4 x_6 \\ S_2 &= -2 b^{2,11} x_4^2 - 2 b^{2,23} x_5 x_6 \\ S_3 &= b^{1,13} x_4^2 + b^{2,23} x_5^2 \\ Q_p^\ell &= -\left[\frac{\partial V}{\partial y_1}\frac{\partial y_1}{\partial x_\ell} + \frac{\partial V}{\partial z_1}\frac{\partial z_1}{\partial x_\ell} + \frac{\partial V}{\partial z_2}\frac{\partial z_2}{\partial x_\ell}\right] \end{aligned}$$

which allows us to write the LAGRANGE equations explicitly (4).

The following numerical values have been found experimentally :
$\ell = 1$ m ; $\ell' = 0.29$ m ; $a_1^{11} = 0.068$, $a_2^{11} = 0.636$, $a_3^{11} = 0.772$, $a_1^{22} = 0.650$, $a_2^{22} = a^{33} = 0.772$ (all units in m^2.kg).

An example of simulation of the transient mode is shown in fig.5. In that case the experimental values are :

a) coefficients of the potential function : $\alpha_1 = \beta_1 = 10$, $\beta_2 = 50$.

b) initial conditions : $x_1 = -\pi/2, x_2 = 0., x_3 = 0.5, x_4 = x_5 = x_6 = 0$

c) goal and constraints : $y_1^f = 0.5, z_1^f = \pi/4$, $z_2^f = 0.634$

The motion is shown to be asymptotically stable and to satisfy the terminal conditions.

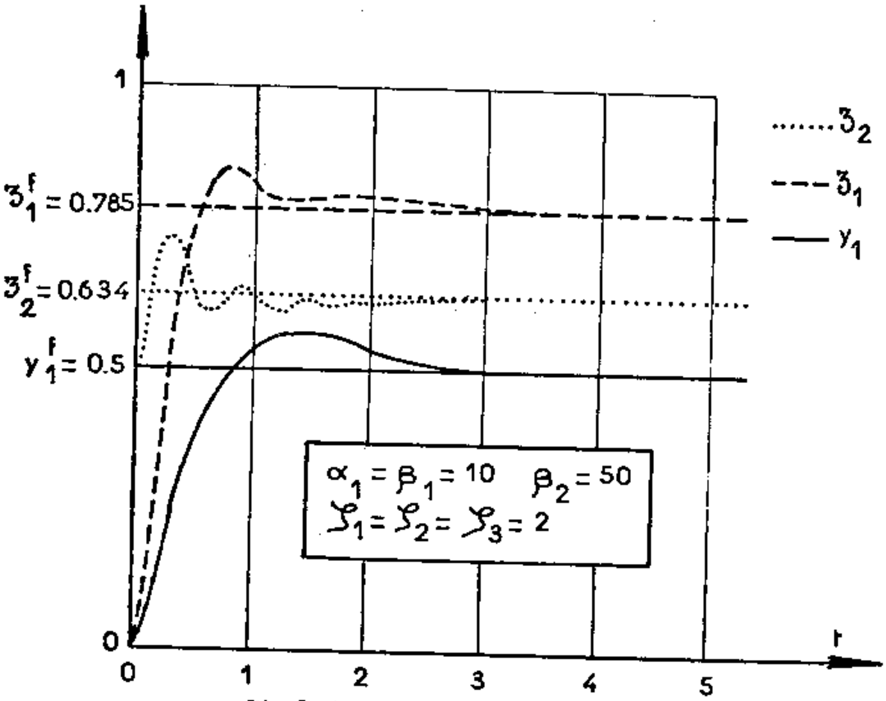

Fig.5. Example of simulation results.

6. CONCLUSION

The work presented in this paper deals with the dynamical model and with the control of redundant mechanical systems. A method for establishing the dynamical model of such mechanisms has been presented, which leads to a straightforward formulation of the LAGRANGE equations : further work is necessary in this field in order to extend the method to the case of closed kinematic chains. The proposed Lagrangian formalism has then been used to develop a method for stabilizing a redundant manipulator in a given manifold ; the results have shown that the transient response may exhibit poor qualities from the kinematic point of view (overshoot) but that the dynamic couplings between the various degrees of freedom can be suitably exploited in order to reach the goal.

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