APPLICATION OF MATHEMATICAL MODELS IN THE STUDY OF PATHOLOGICAL GAIT

M.Vukobratović*), D.Surla**)

Abstract

In the paper are presented in shortest lines the results from mathematical modeling of biped gait dynamics. Some new results of the simulation of various asymmetric gait types, as well gaits with different overlapping degree are presented, which form an entity with existing previous results (1, 2, 3, 4). The mentioned results yield a solid basis for the forming of a dynamic criterion for gait evaluation.

Introduction

Mathematical modeling of biped gait is primarily aimed at creating the basis for controlling artificial systems of anthropomorphic type for application in rehabilitation robotics. Mathematical modeling is also significant in imitating some posture and gait phenomena of living systems, notably systems of human beings, and, finally, in evaluating the achieved results of using the orthotic devices in cases of heavily damaged locomotor acts with handicapped persons.

The paper is not aimed at giving full insight into the up-to-date results in biped gait and posture mathematical modeling. Many researchers were successfully engaged with these problems during the past 12 years. They are mentioned here, though not all of them, but according to the authors´ opinion the most important results from the field of mathematical modeling of the biped gait and posture dynamics. The first published paper, treating biped dynamics, appeared at the start of 1969 (1). After that, other paper emerged, which widened the original model of biped gait at the basis of the same idea (2, 3, 4, 5, 6). In parallel with biped gait mathematical modeling attempts were originated, trying to implement these results, at least partially, into the synthesis of complex active orthotic devices for producing the basic locomotor act of paralyzed persons (7, 8, 9). Noted results in modeling the biped gait and posture, and in the synthesis of control algorithms were presented in papers (10, 11, 12, 13, 14), while the corresponding effors\t to develop new orthotic systems for lower extremities were reported in (15, 16, 17), confirming a continuity of efforts to find at least partial application for the mathematical models of biped locomotion.

The interesting and very important problem stays, concerning the forming of some criterion for the validation of biped gait at

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*) *Mihailo Pupin Institute, Beograd, Yugoslavia*

**) *University of Novi Sad, Novi Sad, Yugoslavia*

the basis of dynamic system properties, i.e. its mathematical model. Lack of such a criterion has been noted, which does not rely solely on the kinematical system properties and on the cosmesis of the realized gait. The synthesis of a criterion for gait validation, based of its dynamic data, presents of course a delicate task. This paper does not pretend to answer this question completely. It is intended to present the broad possibilities of the dynamic model of biped gait, or, the broadeness and voluminosity of the simulation results concerning symmetrical and asymmetrical gait, as well all the relevant derived dynamic values, based on which can be contemplated about some validation criterion, which would be dynamic by its nature. The paper is also intended to demonstrate the purposefulness of using the biped gait mathematical model as a means for the generalization and systematization of the results of numerous investigations, by means of very modern equipment for measuring and recording the kinematical (gait pattern, gait speed, etc.) and dynamical gait parameters (dynamic support reactions, driving torques of the mechanism joints, acceleration of characteristic body points, etc.). Such a statement is primarily based on the earlier shown fact (4), that some simulation results of the dynamic state synthesis of the anthropomorphic configuration with identical parameters (geometry, segment masses of the body, upper and lower extremities, tensors of inertia of the segments), as well with identical imposed kinematics to the lower extremities (gait type), are practically identical with the measured and recorded values on the human being. This conclusion concerns first of all the character and values of the dynamical reaction forces in the foot-support contact, as well the driving torques in all the active joints of the extremities and of the body. For reasons of mechanical analogy of the anthropomorphic system with the human being, whose skeletal activity is the result of the action of numerous muscle groups, greater coincidence concerning the dynamical behaviour of the human trunk (compensating motion) and its mechanical analogue does not hold.

Mathematical model of biped dynamics

In order to perform description of the biped motion dynamics, the known procedure based on the semi-inverse method is adopted (1, 2, 3). Here are repeated only some basic postulates, on which are based the forming of the mathematical model of biped dynamics.

It should be pointed out that the zero-moment point is of great practical significance. It is the conditional name for the instantaneous point at which the total reaction forces are acting, produced by man during gait. The trajectory of the zero-moment points can be recorded in a relatively simple way. Recordings have been made in order to obtain the most realistic portrait of gait dynamics.

Let τ be the zero-moment point (Figure 1). According to D'Alambert's principle, the conditions of dynamic equilibrium, called the dynamic connections, are

$$\sum_{i=1}^{n} (\vec{\rho}_{\tau i} \times (\vec{F}_i + \vec{G}_i) + \vec{M}_{Fi})\vec{e}_x = 0$$
$$\sum_{i=1}^{n} (\vec{\rho}_{\tau i} \times (\vec{F}_i + \vec{G}_i) + \vec{M}_{Fi})\vec{e}_y = 0 \qquad (1)$$

where: $\vec{\rho}_{\tau i}$ is the vector from point τ to the center of gravity of the i-th element; $\vec{F}_i$, $\vec{M}_{Fi}$ are the main vector and main moment of the inertial force of the i-th element; $\vec{G}_i$ is the weight of particular elements; and $\vec{e}_x$, $\vec{e}_y$ are unit vectors of orthogonal axes, X and Y through the point τ.

The equilibrium equation with respect to the acting point of the resulting friction force can be written as

$$\sum_{i=1}^{n} (\vec{M}_{Fi} + \vec{\rho}_i \times \vec{F}_i)\vec{e}_\zeta = 0 \tag{2}$$

where $\vec{\rho}_i$ is the vector from the zero-moment point to the penetration point of axis ζ on the contact sufrace between the foot and ground, and $\vec{e}_\zeta$ is the unit vector of axis, ζ.

Equations (1) and (2) represent the dynamic equilibrium conditions written in general form for an anthropomorphic system with fixed arms. If free arms were considered, this model would have to be extended by as many second-order differential equations, as the "arms" have degrees of freedom. By fixing the arms, only three equations of dynamic connections are possible.

Since the number of degrees of freedom of the anthropomorphic mechanism is considerably greater than three, the law of change in the remaining (n-3) coordinates should be given to provide periodic consecutive movement of the legs. Consequently, the artificial synergy synthesis is performed in the following simple way: for the (n-3) coordinates, the motion program is prescribed, and the remaining coordinates are found from the equations of dynamic connections (1), (2).

The inertial forces, $\vec{F}$, and moments of inertial forces, $\vec{M}_F$, in equations (1), (2), in the general case, can be written as a linear form of generalized accelerations and a square form of relative angular velocities

$$\begin{aligned} \vec{F} &= \sum_{i=1}^{n} \vec{a}_i \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \vec{b}_{ij} \dot{\phi}_i \dot{\phi}_j \\ \vec{M}_F &= \sum_{i=1}^{n} \vec{c}_i \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \vec{d}_{ij} \dot{\phi}_i \dot{\phi}_j \end{aligned} \tag{3}$$

where: $\vec{a}_i$, $\vec{b}_{ij}$, $\vec{c}_i$, $\vec{d}_{ij}$ are functions of generalized coordinates. If the prescribed ϕ coordinates are separated from the set of coordinates, ϕ_i, and the ϕ^* coordinates computed from dynamic connections, the general system (1), (2) can be written in a concise form as:

$$\sum_{i=1}^{n} \vec{c}_i \ddot{\phi}_i^* + \sum_{i=1}^{n} \sum_{j=1}^{n} \vec{d}_{ij} \dot{\phi}_i^* \dot{\phi}_j^* + \vec{g} = 0 \tag{4}$$

where: $\vec{c}_i, \vec{d}_{ij}$ are vector coefficients depending on $\ddot{\phi}$, $\dot{\phi}$, $\ddot{\phi}$, and $\vec{g}$ is a vector coefficient depending on $\ddot{\phi}$, $\ddot{\phi}^*$, $\ddot{\phi}$, ϕ^*. Eq. (4) with the imposed repeatability conditions (1, 2, 3):

$$\phi_i^*(0) = \phi_i^*(T/2) \tag{5}$$

where T is the step period, gives the compensating synergy (motion of upper part of the body) at the basis of prescribed synergy of the lower extremities (gait pattern). Since the adopted gait is symmetrical, the repeatability conditions for the coordinates ϕ^- can be written for the half-step.

Since the complete synergy is known, the next step is to define the driving torques for the purpose od performing the anthropomorphic mechanism. To define these torques, we "break" the chain along the axes of particular joints. Thus for the part of the system not connected with the ground, the total torque of each joint can be computed according to (10):

$$\vec{M}^k = \sum_{i=1}^{n} \vec{c}_i^k \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \vec{d}_{ij}^k \dot{\phi}_i \dot{\phi}_j + \vec{g}^k \qquad (6)$$

where: the index, k, shows the number of the joint where the kinematic chain of the mechanism has been "broken". Since the coordinates, $\phi(t)$, are known, equation (6) makes it possible to determine all the driving torques M_i, $i=1,\dots,n$.

It this way, the "unusual" problem of mechanics has been solved. Thus, the problem of artificial anthropomorphic gait synthesis has been solved, and the algorithmic level (nominal dynamics) of the system is generated.

At a basis of the semi-inverse method for the forming of mathematical gait models, sufficiently instructive models were developed based on the adopted mechanical configuration in Figure 2. In the course of development, the algorithm for construction of the mathematical gait model acquired all the properties of a computer method, which resulted in a program package of significant flexibility consisting of the modification of the initial information block about the gait type, the geometry of members of the anthropomorphic mechanism, its masses and tensor of inertia. By changing these data, it automatically arrived to new solutions of functional movements of the anthropomorphic mechanism. The program package contains the automatic processing of all relevant system parameters, as all components of dynamic reactions, driving torques in all active joints of the mechanism, the necessary power values of the individual actuators and frequency analysis of the trajectories and loads.

Some simulation results

Based on such a program package, some characteristic results for gait upon level ground are presented. The basic and derived results (in the form of diagrams) of simulation will be used for the formation of the functional dependence of the characteristic gait parameters, which can be used as the idea for forming the criterion for gait validity evaluation. The parameters of the mechanical configuration in Figure 2 are given in Table 1.

Simulation has been performed with various gait parameters. As basic gait parameters were adopted:

- S - step size (length)
- T - step period (duration), cadence
- displacement of ZMP (zero-moment point)
- overlapping % (double-support phase duration).

Both symmetrical and asymmetrical gaits were treated. In that way all basic cases were taken into consideration, which should be significant for real biped system behavior. With the parameters, adopted in that way, a much broader spectrum of energy characteristics of the anthropomorphic mechanism was obtained, and beside the kinematic performances, estimation of the realized gait and a large number of dynamic data introduced, which in majority of cases were left out of analyses of this type.

Due to limited space, only condensed examples of the complete simulation results will be given here. Therefore, energy analysis concerning only symmetrical gait types upon level ground will be presented. Also, symmetrical gait cases are given with single-support and double-support phases.

All analyses have been performed for prescribed synergy of the lower extremities (desired type of leg motion), illustrated in Figure 3. Also for the single-support type of gait, five cases of ZMP displacement were adopted (Figure 4), while ZMP displacement for the double-support phase will be shown later.

In Figures 5 to 7 are presented the corresponding values as functions of gait speed increase via the parameters T and S for various ZMP-laws.

The more the ZMP on the foot in aft (nearer to the heel), or if it persists longer in that position, the maximal values of the driving torque and mechanical work during full step diminish with actuators for compensation in the sagittal and augment for those in the frontal plane (Figure 5).

For smaller T and greater S, faster gait in both cases, the maximal values of the driving torque and mechanical work during full step, augment for all adopted ZMP-trajectories with actuators for compensation in the sagittal and diminish for those in the frontal plane (Figure 5).

Also total mechanical work of all actuators and the maximal values of the vertical component of the reaction force (Figure 6) augment for greater S or smaller T, faster gait in both cases.

In Figures 8 to 10 are presented the corresponding values as functions of gait speed increase via the parameters T and S for various durations of the double-support phases.

The influence of gait speed via parameter T is the following:

- maximal values of the driving torques augment with actuators for compensation in the frontal and sagittal plane (Figures 8);
- mechanical work during full step augments with actuators for the compensation in the sagittal and diminishes in the frontal plane (Figures 8);
- total mechanical work of all actuators augment (Figure 9);
- maximal values of the reaction force vertical component augment (Figure 10).

Maximal values of the driving torque and mechanical work values during full step augment with actuators for compensation in the frontal plane with longer duration of the double-support phase p. Maximal values of the driving torques differ negligibly for parameter p variation from 20 to 40% with actuators for compensation in the sagittal plane. For p=0, the same are smaller. Mechanical work during full step with actuators for compensation in the sagittal plane diminishes with parameter p increase.

In Figures 11 to 13 is illustrated the dependance of the corresponding values as functions of the double-support phase duration, in % of the given gait speed. These diagrams demonstrate various influence of the parameters T and S on the presented values.

Apart from the presented diagrams, others for investigating the behaviour of the maximal values of the driving torques during full step with knee and hip actuators were established, depending on the following parameters: step size S, step period T, ZMP-law and double-support duration p. By analysing these diagrams, the following conclusions were arrived at:

- influence of the adopted ZMP-trajectories onto the ankle actuator is such, that the maximal values of the driving torques and mechanical work values during full step are the smallest, when the ZMP is stationary during the half-period;
- maximal values of the driving torques and mechanical work values during full step diminish for the knee and hip actuator, when the ZMP on the foot sole is more aft (nearer to the heel), and also if it stays longer in that position;
- maximal values of the driving torques and mechanical work during full step augment with actuators in the ankle, knee and hip joints, in case of gait speed increase for both cases, i.e. smaller T and greater S;
- maximal values of the driving torques with actuators in the ankle, knee and hop joint differ negligibly for double-support phase duration of 20, 30 and 40%;
- for p=0, maximal values of the driving torques are smaller for all actuators, compared with the cases when p differs from zero;
- power during full step augments with actuators in the ankle, knee and hop joint during growth of parameter p.

In Figures 14 to 16 are presented the corresponding values as functions of the difference of the support phase duration for the left and right leg ΔT in the case of asymmetric gait.

Maximal values of the driving torques for compensation in the frontal and sagittal plane augment for greater ΔT. This is more expressed in the sagittal plane, in the frontal one it is very small (Figures 14).

Mechanical work during full step of the compensating actuators in the frontal and sagittal plane differ rather slightly for ΔT growth (Figures 25).

Total mechanical work of all actuators and the maximal values of the reaction force vertical component have the tendency to augment for greater ΔT (Figures 15, 16).

In Figures 17 to 19 are illustrated the corresponding values as functions of gait speed by augmenting the step size of the right leg for S=0.8 and T=1,5 sec with asymmetric gait cases.

For faster gait, the maximal values of the driving torques diminish with actuators for compensation in the frontal plane and augment for those in the sagittal plane (Figures 17).

Mechanical work during full step augments for faster gait with with actuators both for compensation in the sagittal and frontal planes (Figures 17).

Total mechanical work of all actuators and the maximal values of the reaction force vertical component augment for faster gait (Figures 18, 19).

In Figures 20 to 22 are presented the corresponding values as functions of the right leg mass reduction versus the left one, which stay constant, expressed in per cent.

When reducing the mass of one leg, for all adopted ZMP trajectories the maximal values of the driving torques with actuators for compensation in the frontal plane, the mechanical work during full step with actuators for compensation in the frontal and sagittal plane (Figures 20), total mechanical work of all actuators (Figure 21) and the maximal values of the reaction force vertical component (Figure 22), tend to diminish very weakly, while the maximal values of the actuators for compensation in the sagittal plane tend to augment weakly (Figure 33).

Apart from the presented diagrams for asymmetric gait cases, diagrams were formed for investigating the behaviour of the maximal value of the driving torques and mechanical work during full step with actuators in the ankle, knee and hip joint depending on the following parameters: various double-support phase duration of the right and left leg and different right and left leg masses. Analysing these diagrams to the following conclusions was arrived:

- difference of the maximal values of the driving torques for the left and right leg augments for greater ΔT for actuators in the ankle, knee and hip joint;
- difference of the power during full step of the left and right leg augments for greater T for the actuators of the ankle and hip joint, while this difference is much less expressed for the knee actuator;
- for faster gait the maximal values of the driving torques in the ankle, knee and hip actuator augment, while the corresponding values for the left and right leg differ slightly, and
- for all adopted ZMP-trajectories, the maximal values of the driving torques and the mechanical work during full step are greater with the knee and hip actuator for the leg with smaller mass (this applies to the ankle actuator, too, for the ZMP stationary during the half-period).

Conclusion

The paper was intended to prepare the simulation basis for the forming of the criterion for the evaluation of biped gait. Compared with the preceding results at the basis of dynamic models of various complexity and symmetrical gait, in this paper were presented cases of asymmetrical gait, which are of practical importance, especially in the evaluation of pathological gait. Notably interesting are the new simulation results of gait with different degrees of overlapping, which renders also the possibility of a more realistic gait evaluation. Beside the dynamical reaction forces in the foot - support surface contact, the energetic analysis, acceleration analysis of the compensating part, for a series of characteristic

ZMP - patterns, a rather broad basis was created for concluding about the functional connection between the basic dynamical performances of biped gait and the relevant kinematic-dynamical parameters, which yields also the potential possibility for quantification of the mentioned performances in the scope of a criterion for the validation of biped gait.

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T.1. Mass and intertia DATA

NR. of PART.	MASS ($kg\,sec^2 m^{-1}$)	LENGTH (m)	PROPER MOMENTS INTERTIA ($kg\,sec^2 m$)		
			Jx	Jy	Jz
1(10)	0.152	0.121	0.00006	0.00055	0.00046
2(9)	0.612	0.397	0.00815	0.00815	0.00058
3(8)	1.090	0.431	0.01705	0.01570	0.00272
4	1.222	0.182	0.02080	0.00990	0.01980
5	2.140	0.417	0.13400	0.12320	0.01950
6(7)	0.468	0.785	0.00447	0.00447	0.00036

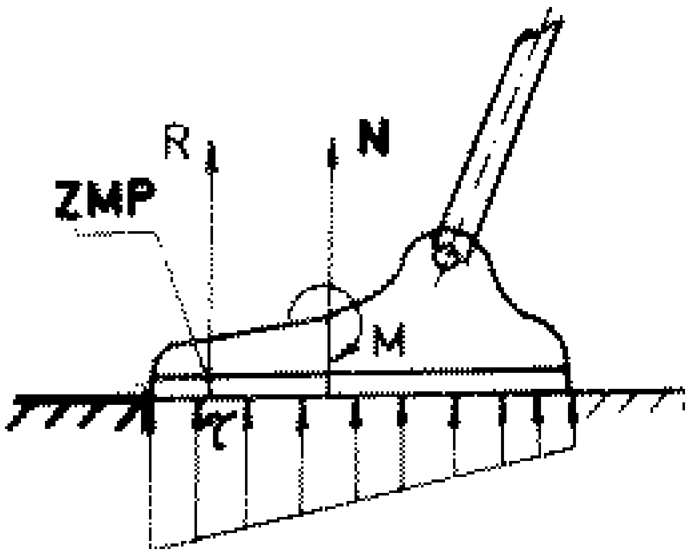


Fig. 1. Zero Moment Point (ZMP)

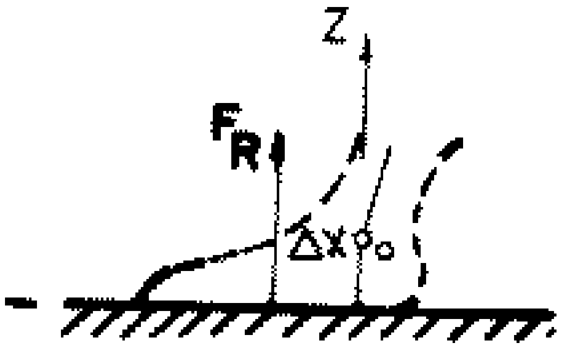


Fig. 4. ZMP displacement

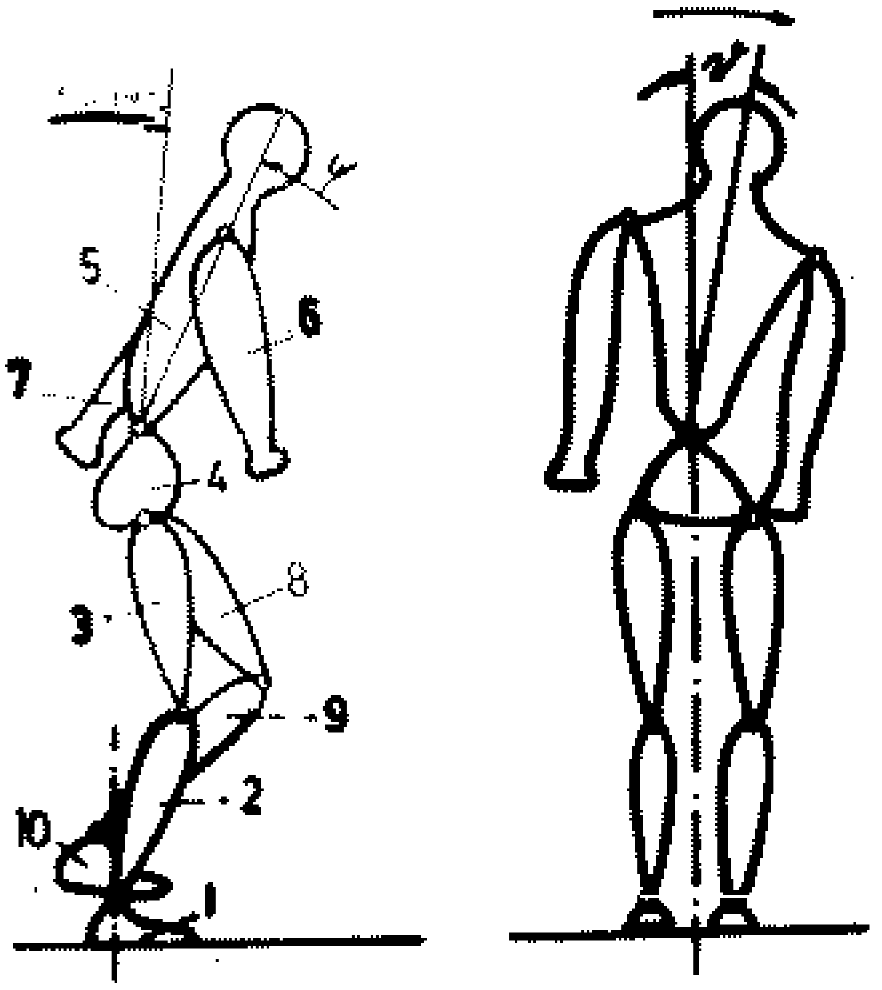
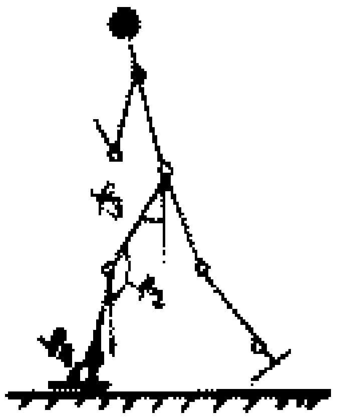


Fig.2. Adopted Mechanical Configuration



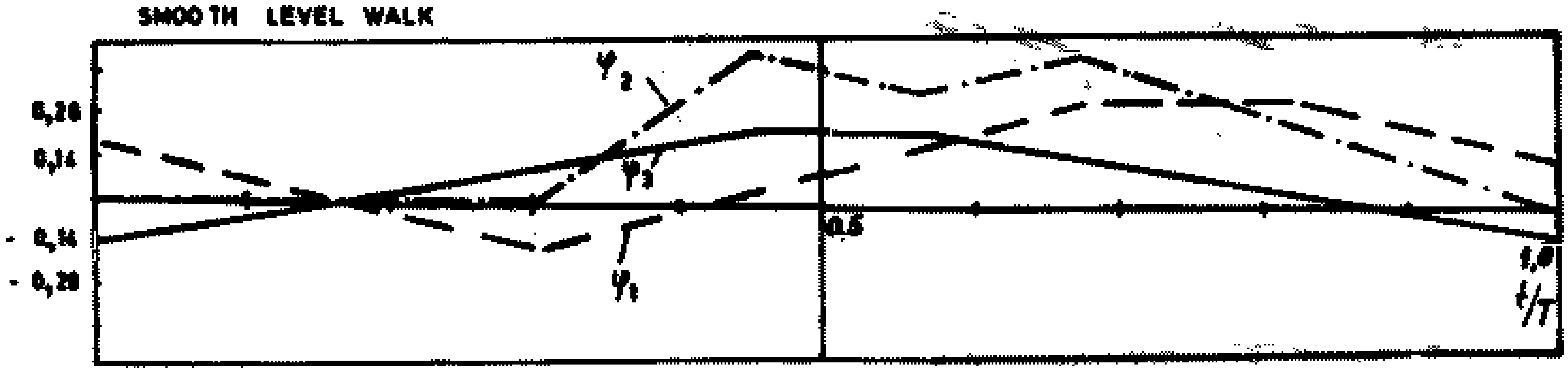


Fig. 3. Adopted Trajectories of Legs

Maximal values of driving torques and mechanical work in the function of gait speed for different stride and T=1.5sec (symmetrical gait, single support phase)

case	t(sec)	Δx(m)
I	0 ÷ T/2	0,0
II	0 ÷ 0,3	0,0
	0,3÷T/2	0,035
III	0 ÷ 0,5	0,0
	0,5÷T/2	0,035
IV	0 ÷ 0,2	-0,02
	0,2÷T/2	0,0
V	0 ÷ 0,2	-0,02
	0,2÷0,6	0,0
	0,6÷T/2	0,035

case I ————

II

III — · — · —

IV — ·· — ·· —

V — — · — — ·

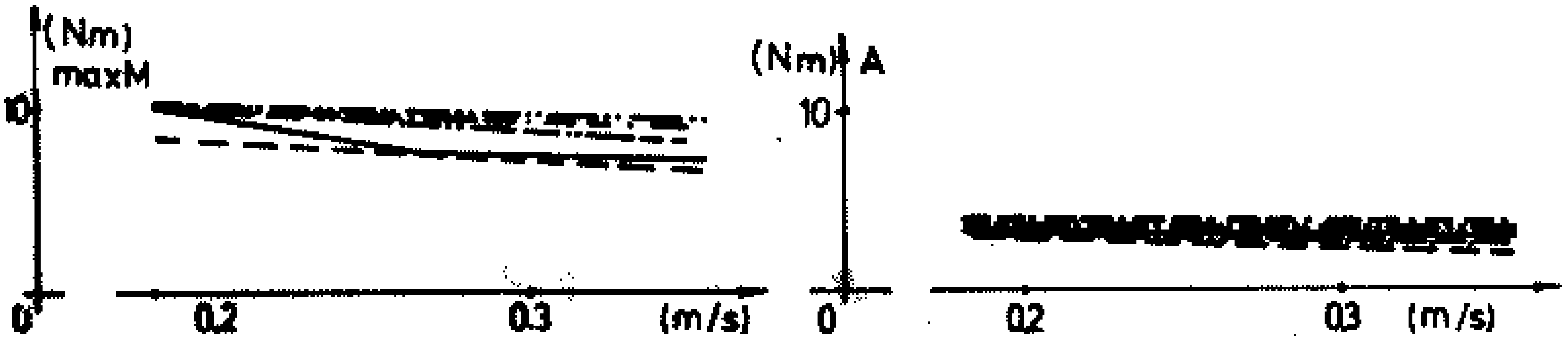


Fig. 5/1 Compensating actuator in frontal plane

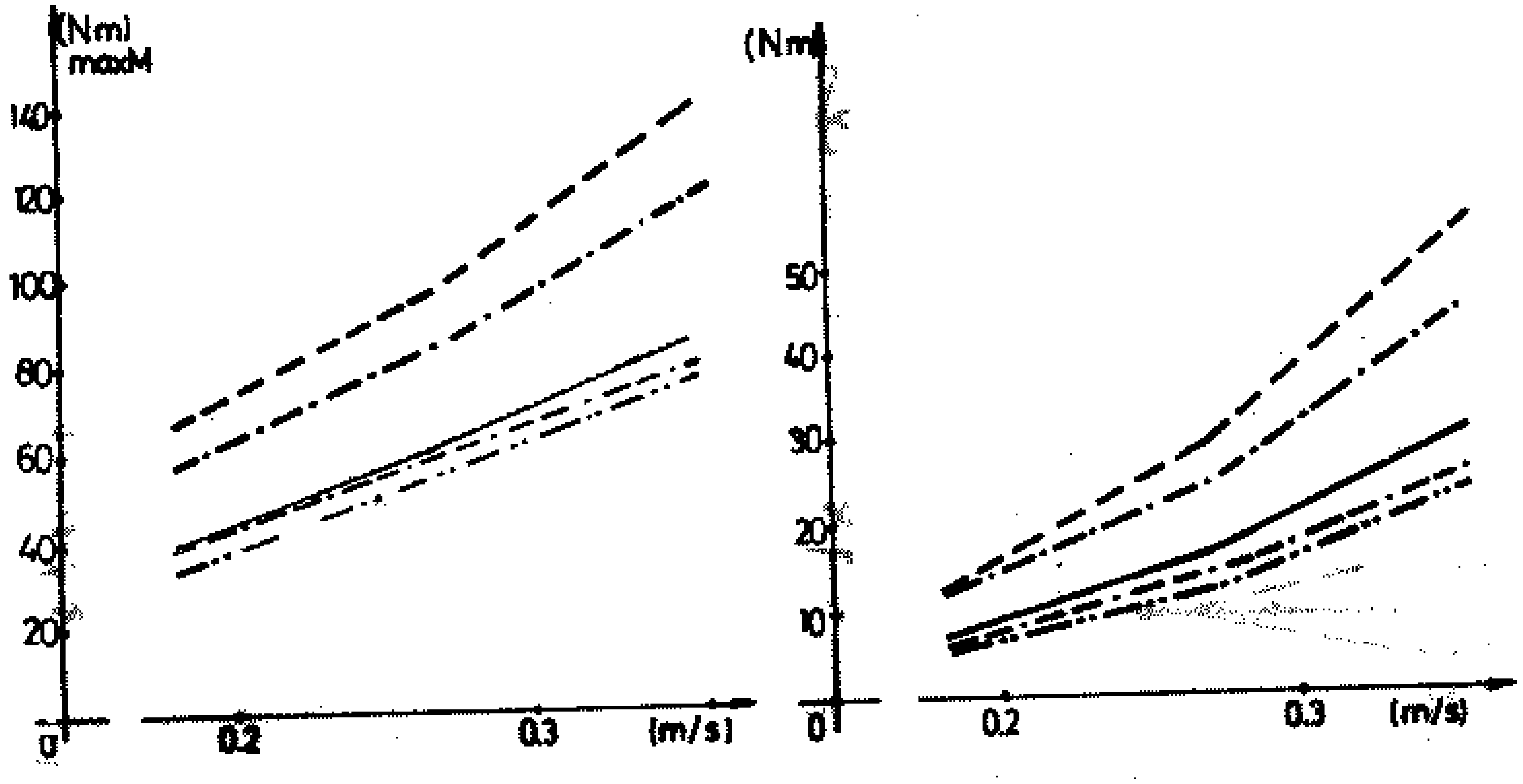


Fig.5/2 Compensating actuator in sagittal plane

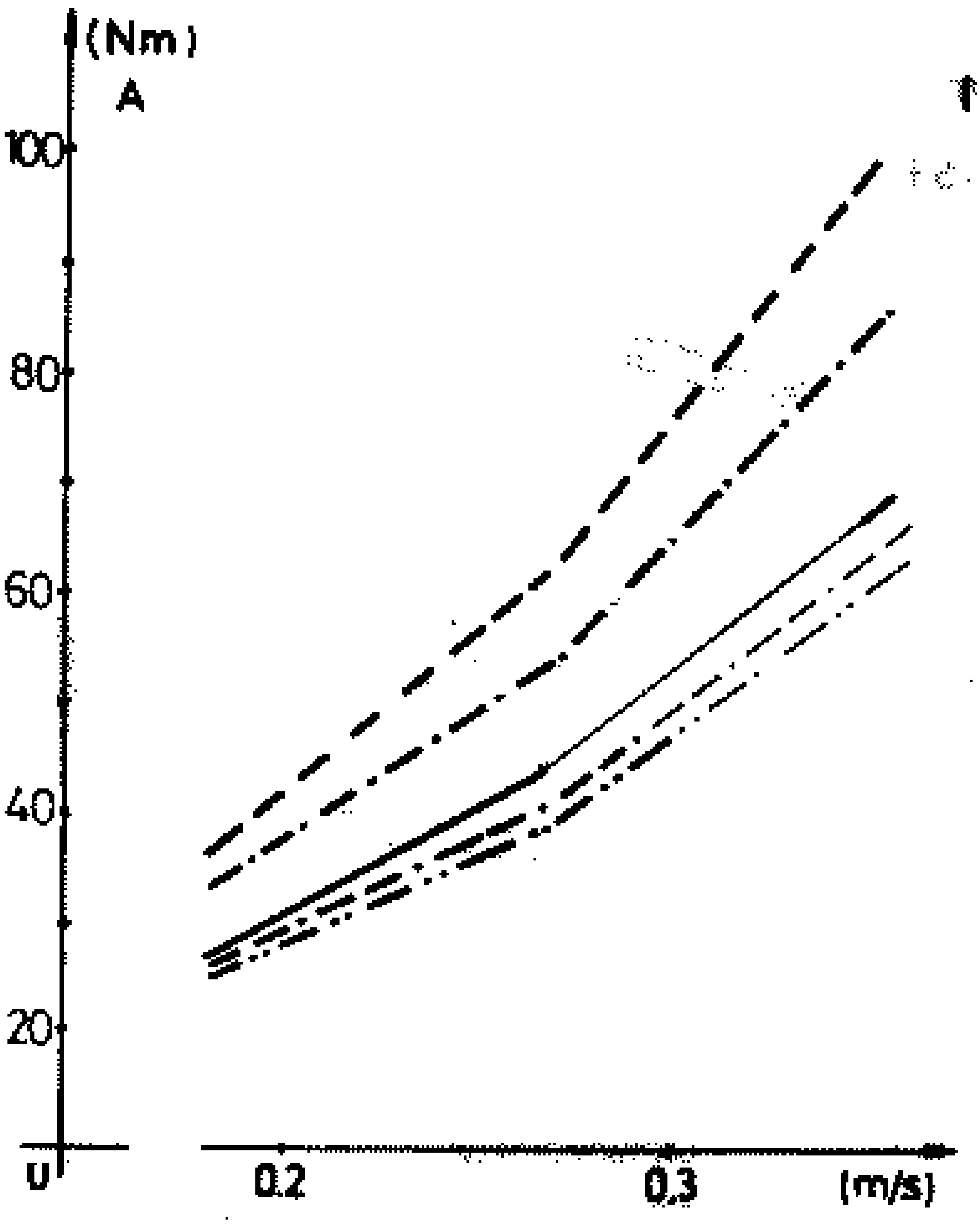


Fig. 6. Total mechanical work

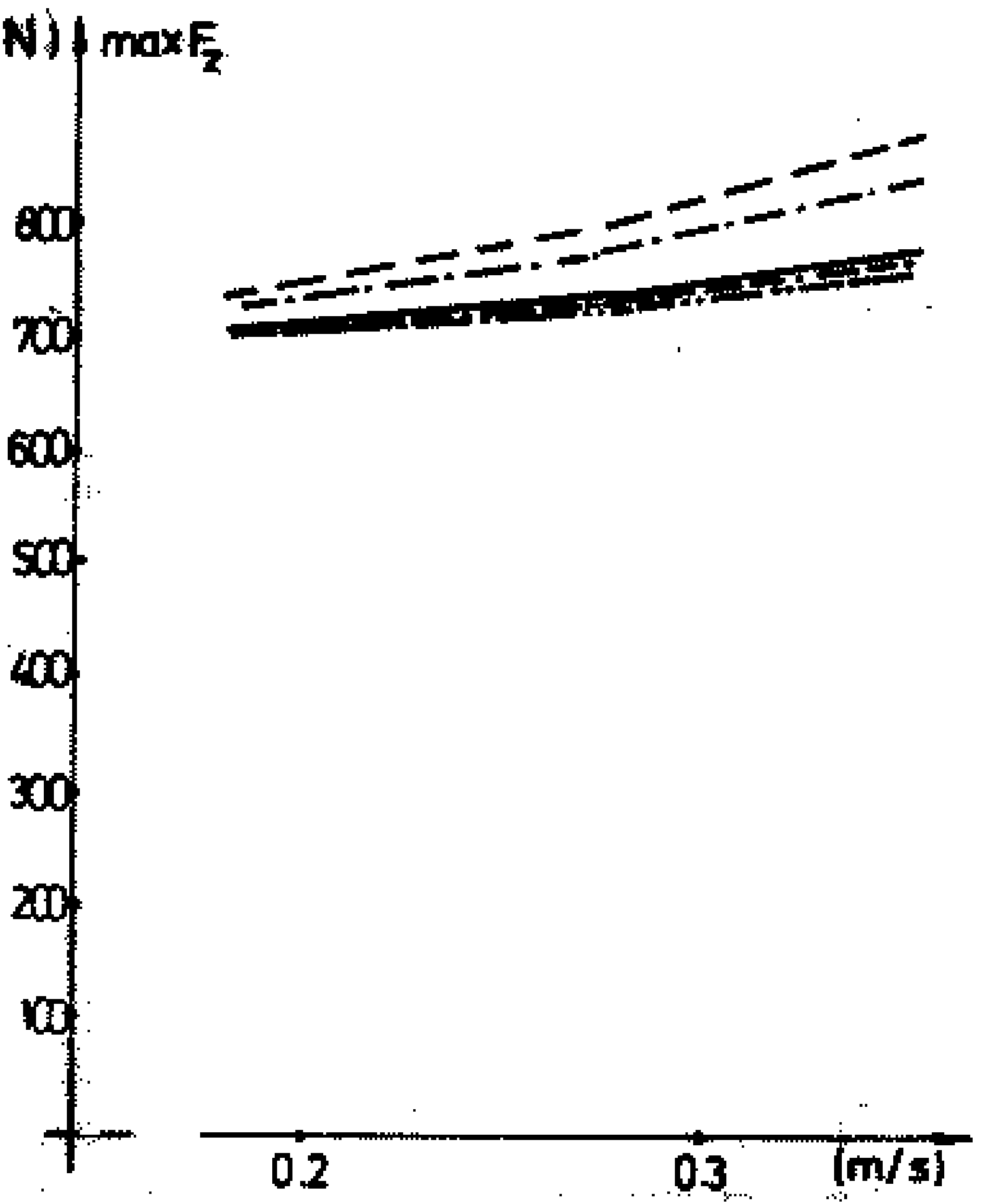


Fig. 7. Maximal values of vertical component of reaction force

Maximal values of driving torques and mechanical work in the function of gait speed for different cadence and S=0.4 (symmetrical gait, different duration of double support phase)

ZMP LAW		
Δx[m]	Δy[m]	t[sec]
0	d	$0 \div T/2$ for $p=0$
0,5S	d	$0 \div \tau$ ($\tau=Tp/400$)
0	0	$\tau \div (T/2 - \tau)$
-0,5S	d	$(T/2 - \tau) \div T/2$

p = 20% ————

p = 30% –·–·–·–·–

p = 40% – – – –

p = 0% - - - - -

T: step period sec; p: % of double support phase duration;

d: semi-distance of feet in frontal plane

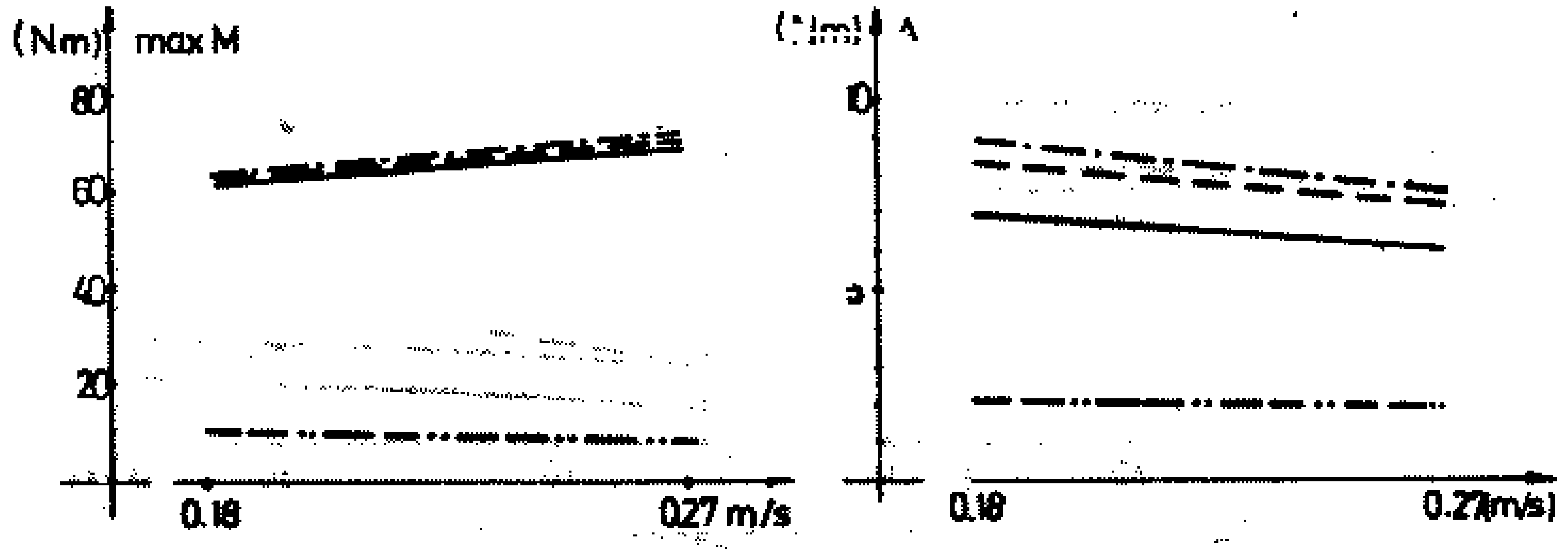


Fig.8/1 Compensating actuator in frontal plane

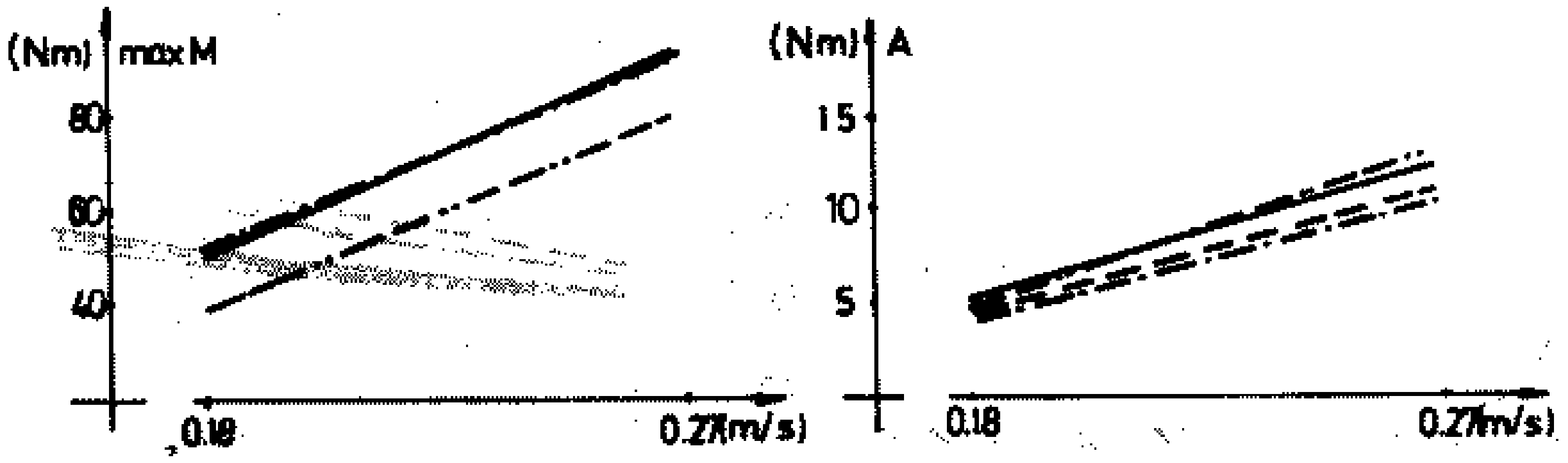


Fig. 8/2 Compensating actuator in sagittal plane

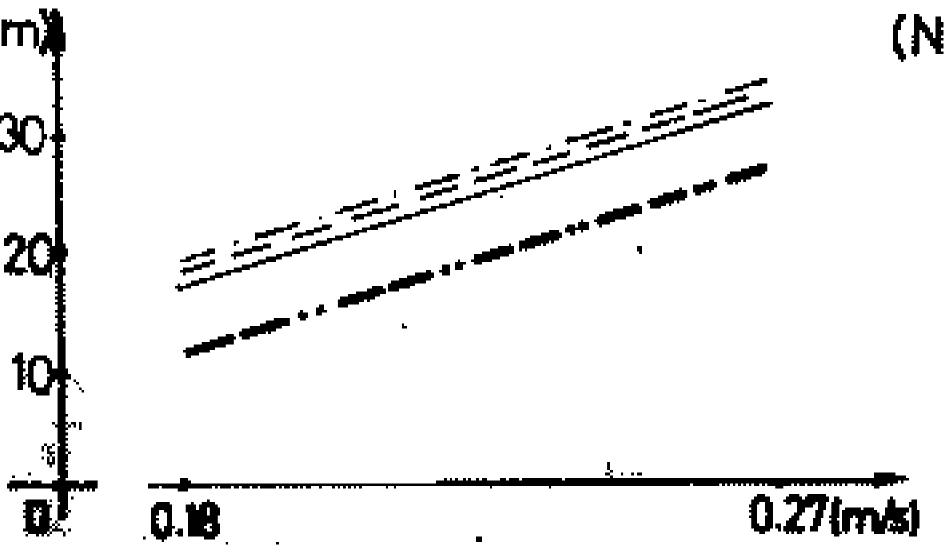


Fig. 9. Total mechanical work

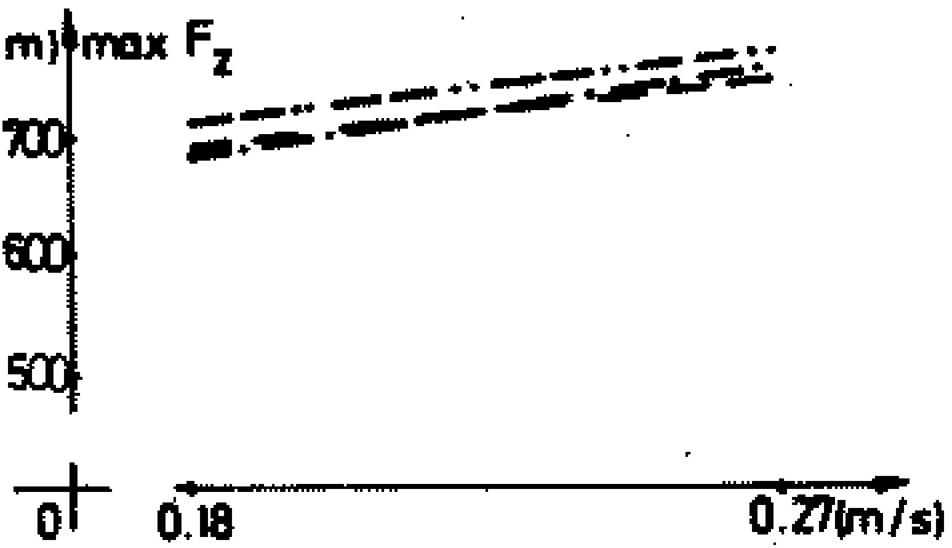
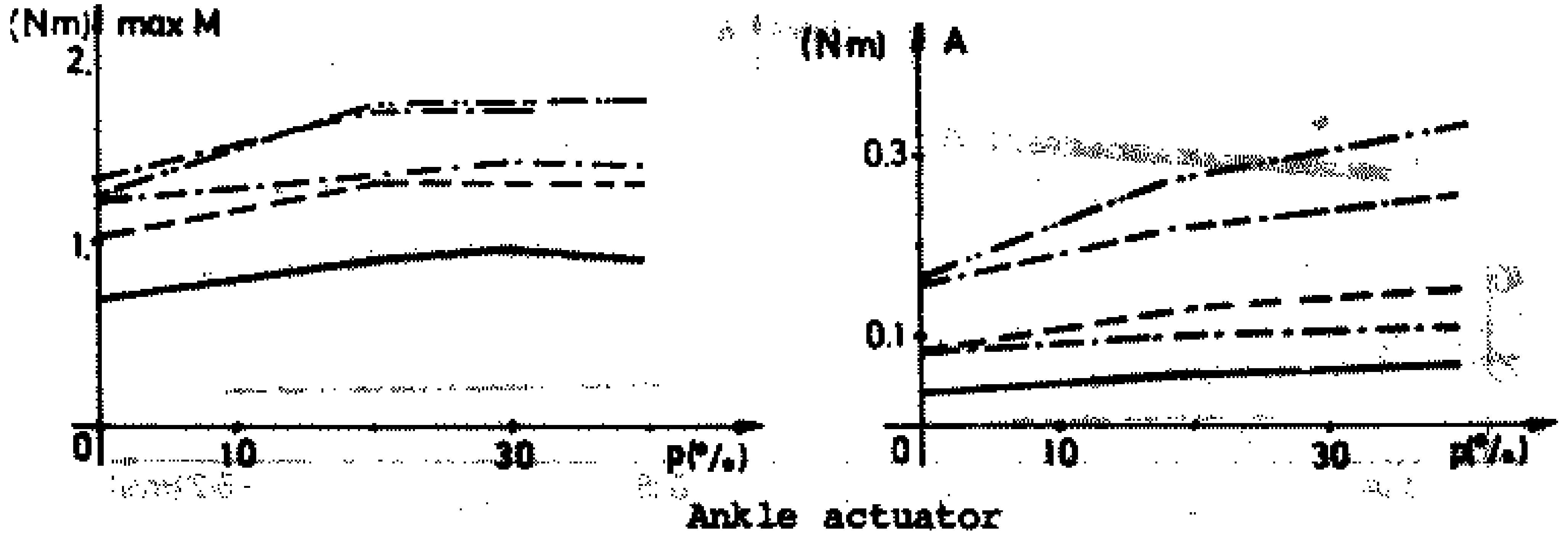


Fig. 10. Maximal values of vertical component of reaction force

Maximal values of driving torques and mechanical during full step in the function of double support phase duration, for different gait speed.

ZMP LAW		
Δx[m]	Δy[m]	t[sec]
0	d	$0 \div T/2$ for $p=0$
0,5S	d	$0 \div \tau$ ($\tau = Tp/400$)
0	0	$\tau \div (T/2-\tau)$
-0,5S	d	$(T/2-\tau) \div T/2$

case
T = 1.5 S = 0.4 ————————
T = 1.5 S = 0.6 — — — — — —
T = 1 S = 0.4 —·—·—·—·—
T = 2 S = 1 —··—··—··
T = 1.5 S = 0.8 ——·——·——



Ankle actuator

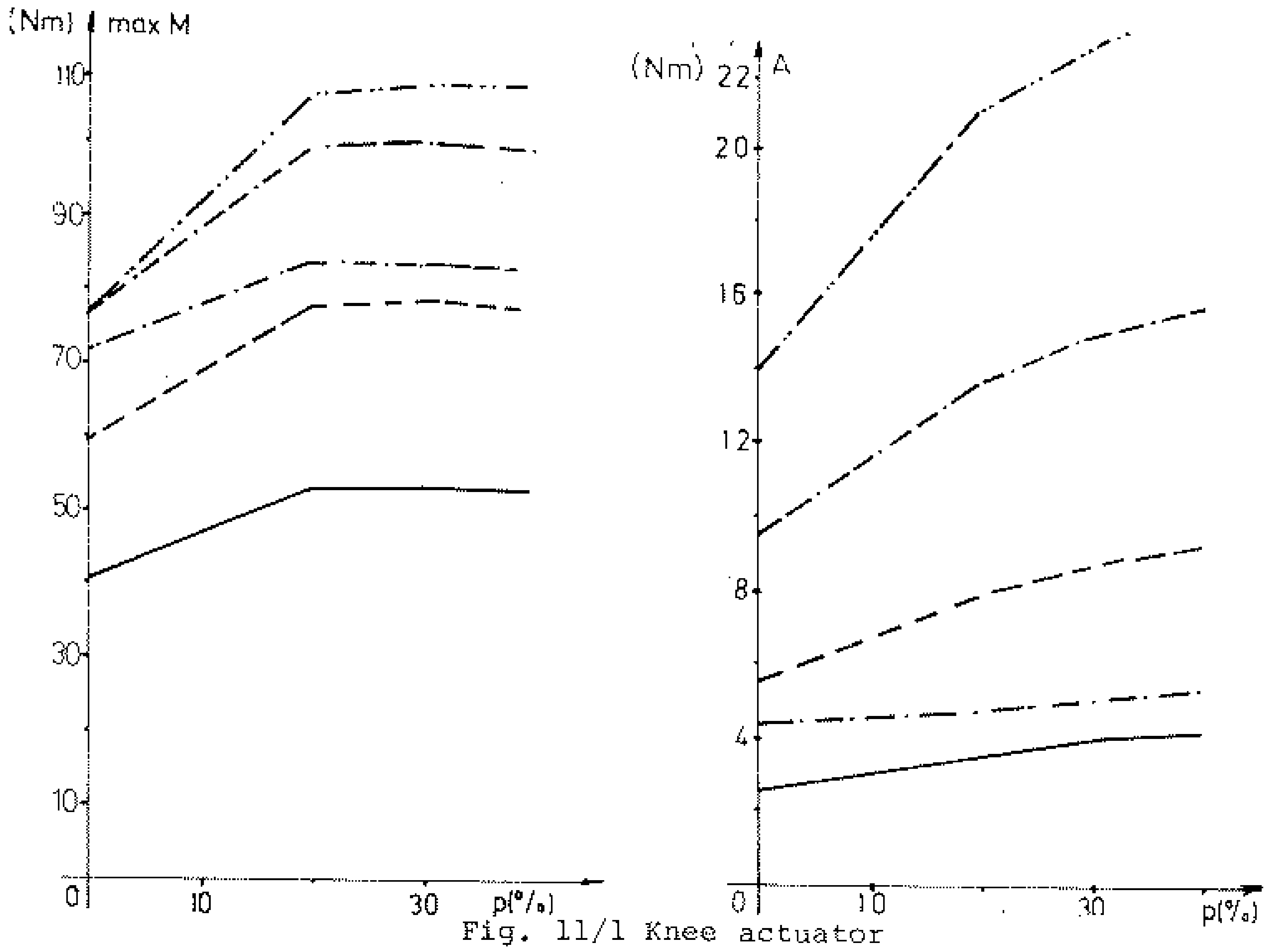


Fig. 11/1 Knee actuator

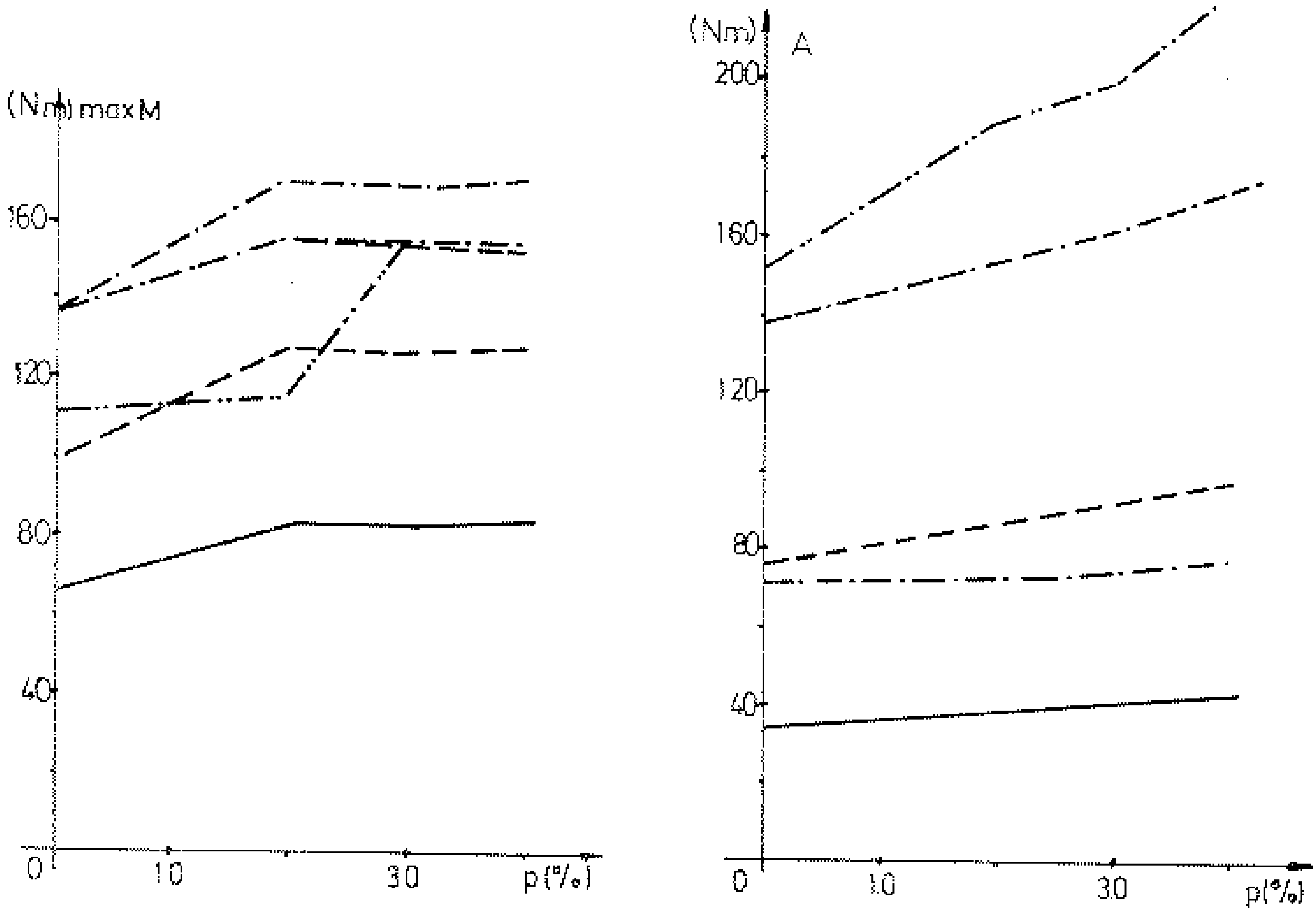


Fig. 11/2 Pelvic actuator

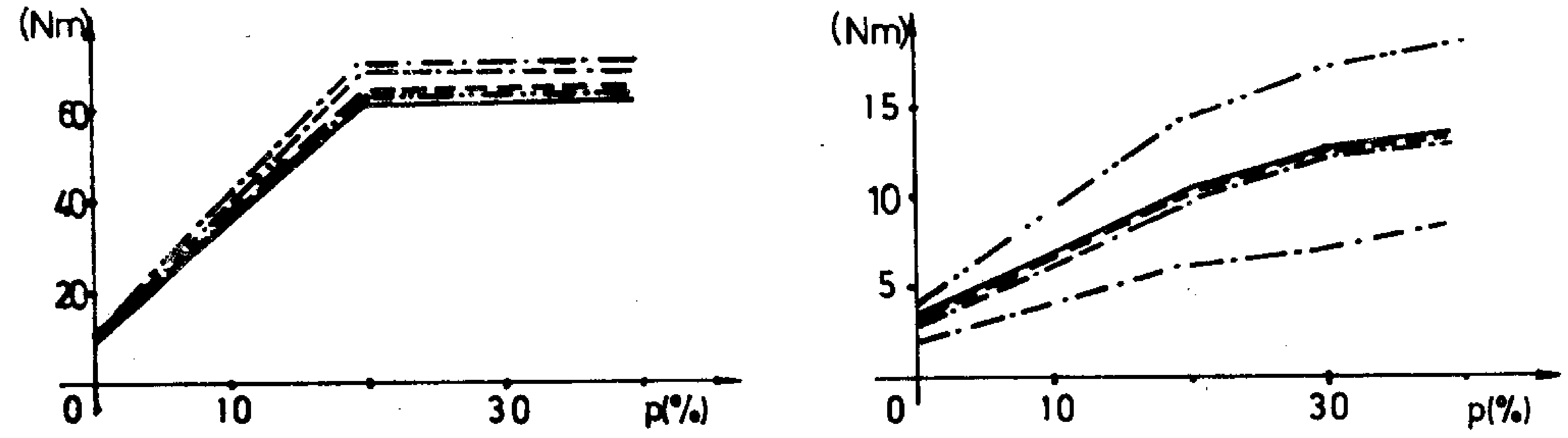


Fig. 11/3 Compensating actuator in frontal plane

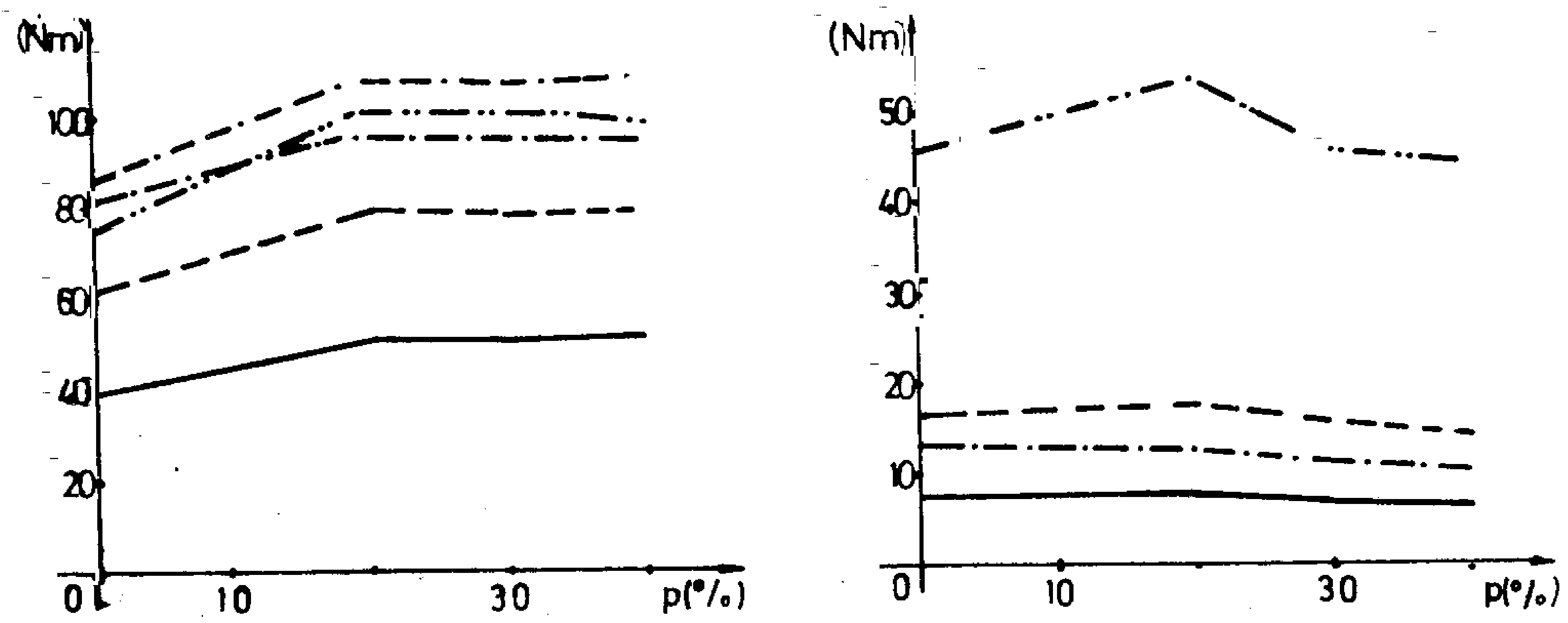


Fig. 11/4 Compensating actuator in sagittal plane

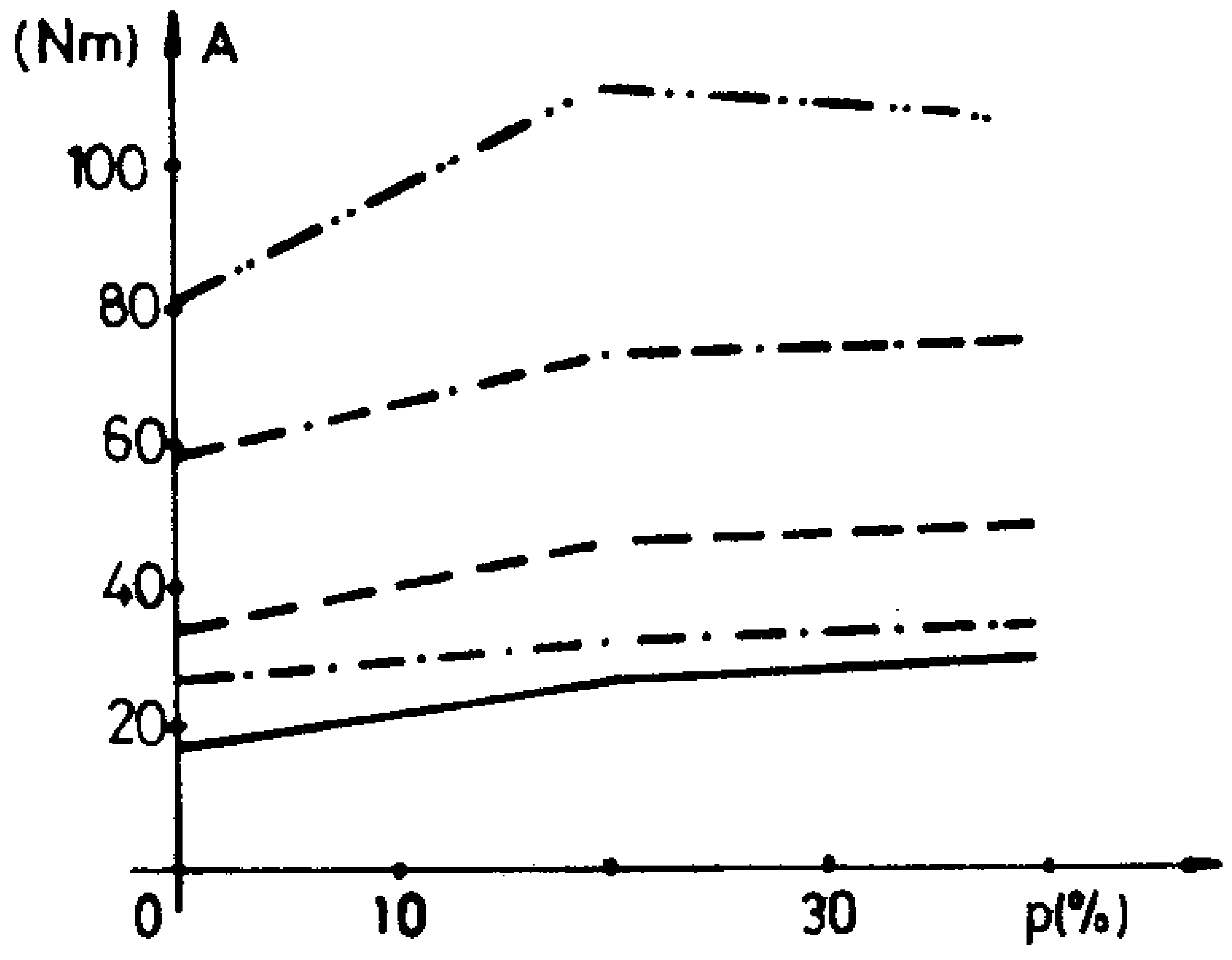


Fig. 12. Total mechanical work

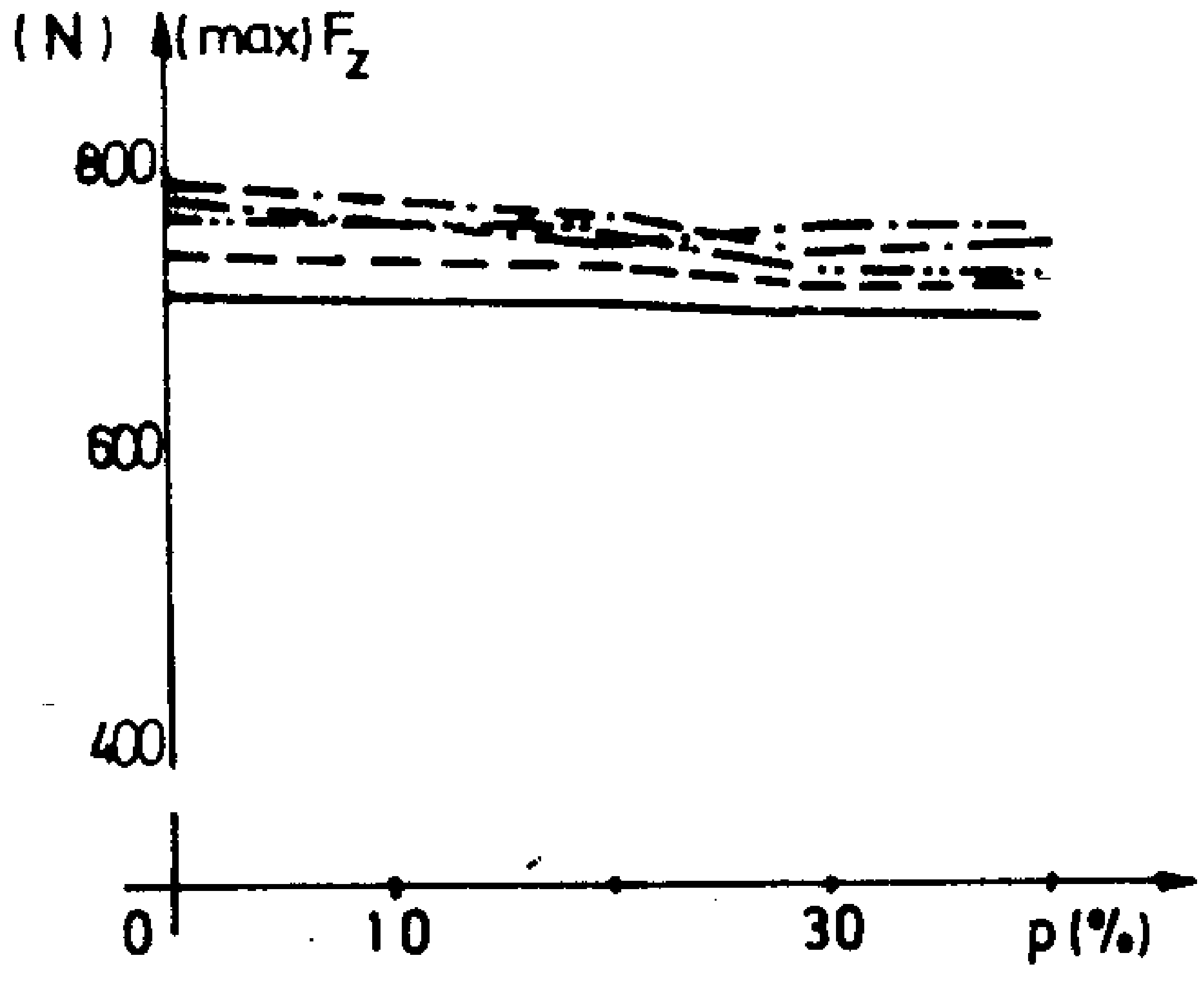


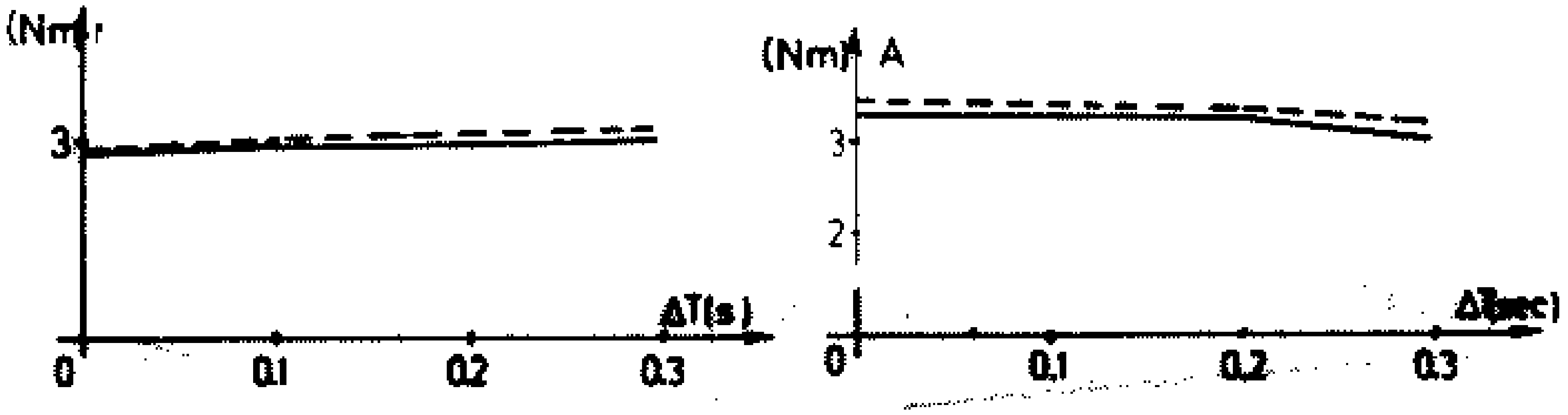
Fig. 13. Maximal values of vertical component of reaction force

Maximal values of driving torques and mechanical work during full step in the function of cadence difference between the left and right leg (ΔT) (asymmetrical gait, constant stride)

support phase of left leg			support phase of right leg			ΔT
t (sec)	S	ZMP law	t (sec)	S	ZMP law	
0-0,75	0,8	I	0,75-1,5	0,8	I	0,00
0-0,72	0,8	I	0,72-1,5	0,8	I	0,06
0-0,70	0,8		0,70-1,5	0,8	I	0,10
0-0,65	0,8	I	0,65-1,5	0,8	I	0,20
0-0,60	0,8	I	0,60-1,5	0,8	I	0,30
0-0,75	0,6	I	0,75-1,5	0,6	I	0,00
0-0,72	0,6	I	0,72-1,5	0,6	I	0,06
0-0,70	0,6	I	0,70-1,5	0,6	I	0,10
0-0,65	0,6	I	0,65-1,5	0,6	I	0,20
0-0,60	0,6	I	0,60-1,5	0,6	I	0,30

case: $T=1,5$ $S=0,8$ ———

$T=1,5$ $S=0,6$ – – – – –



Compensating actuator in frontal plane

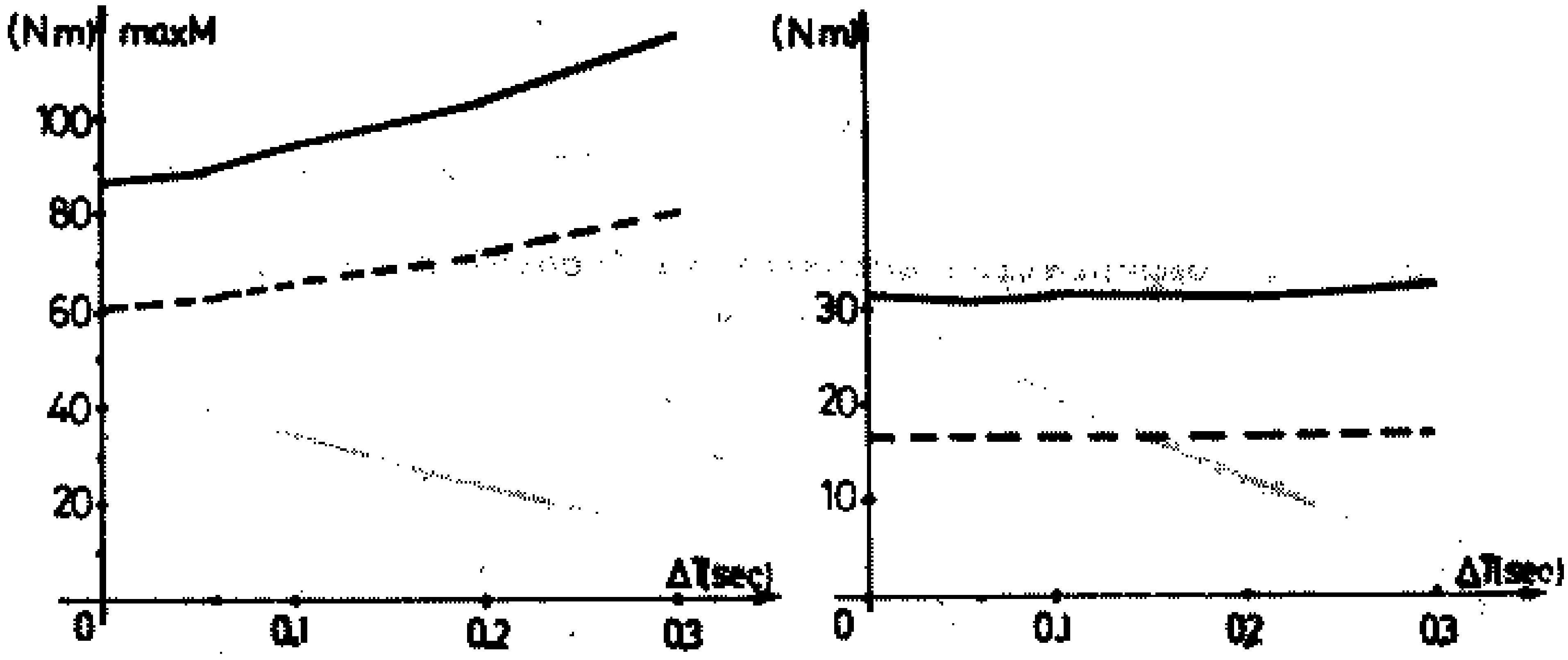


Fig. 14 Compensating actuator in sagittal plane

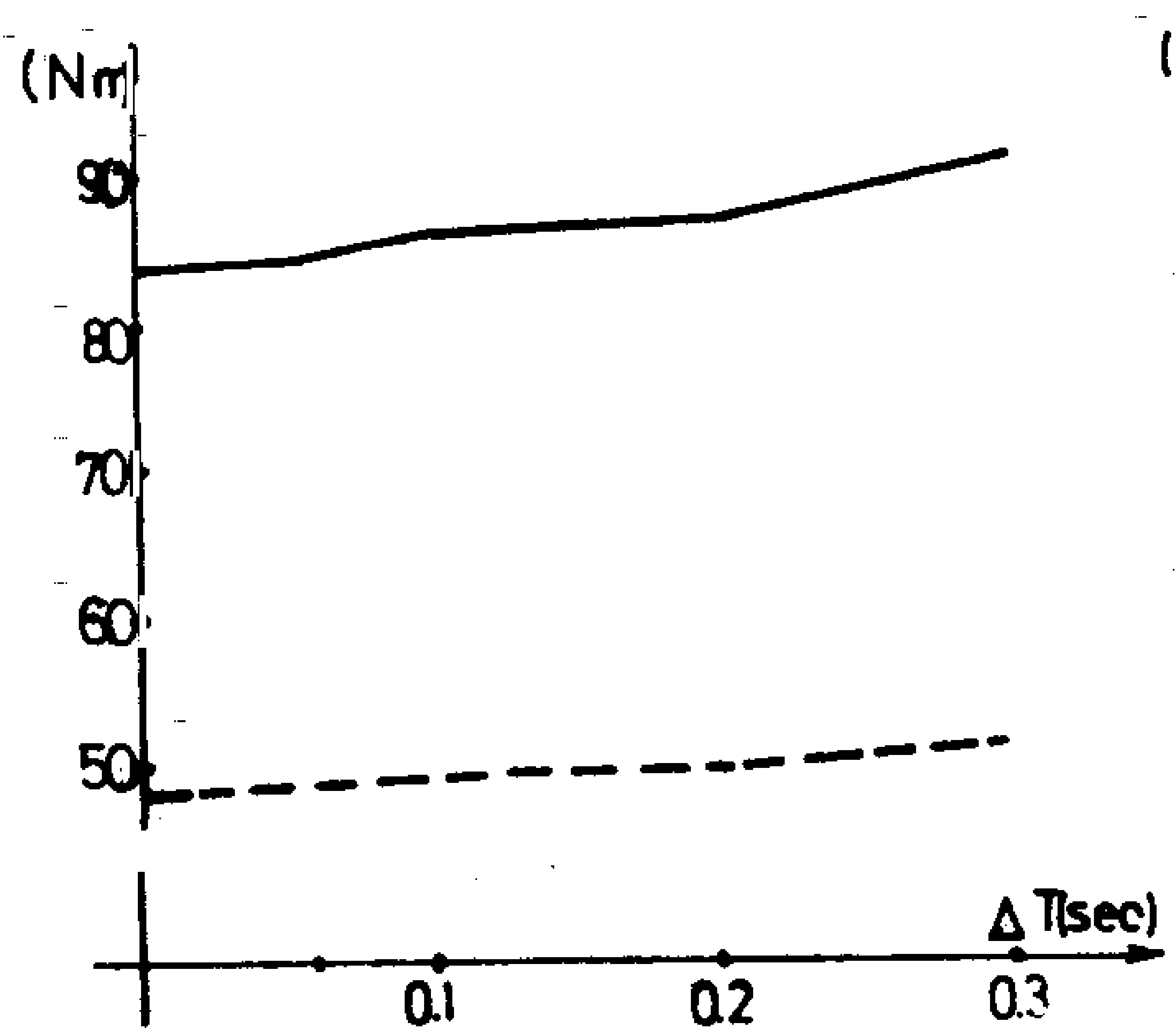


Fig. 15. Total mechanical work

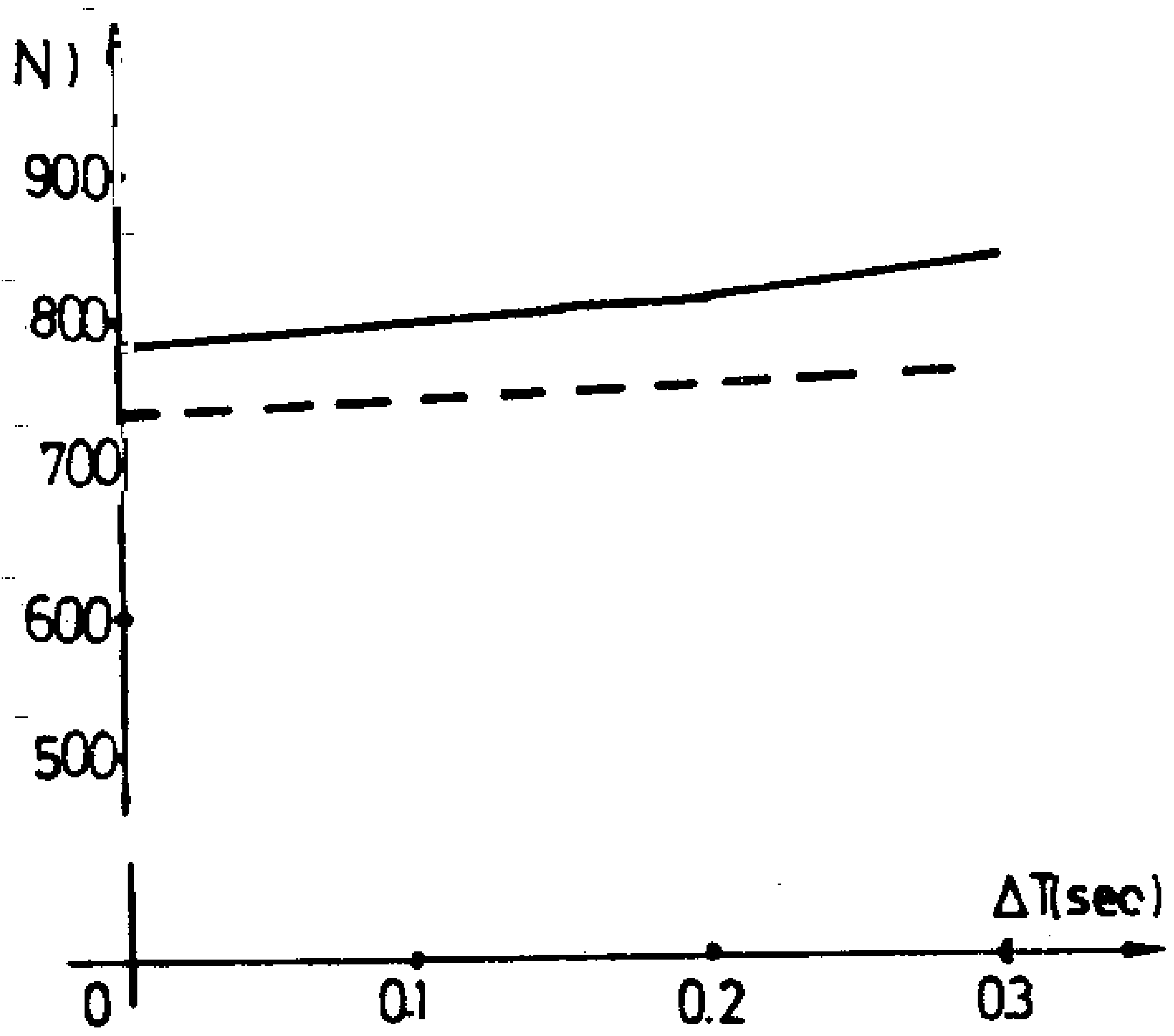


Fig. 16. Maximal values of vertical component of reaction force

Maximal values of driving torques und mechanical work during full step in the function of stride difference between the left and right leg (asymmetrical gait)

support phase of left leg			support phase of right leg			
t(sec)	S	ZMP law	t(sec)	S	ZMP law	ΔS
0-0,75	0,8	I	0,75-1,5	0,80	I	0,00
0-0,75	0,8	I	0,75-1,5	0,85	I	0,05
0-0,75	0,8	I	0,75-1,5	0,90	I	0,10
0-0,75	0,8	I	0,75-1,5	1,00	I	0,20

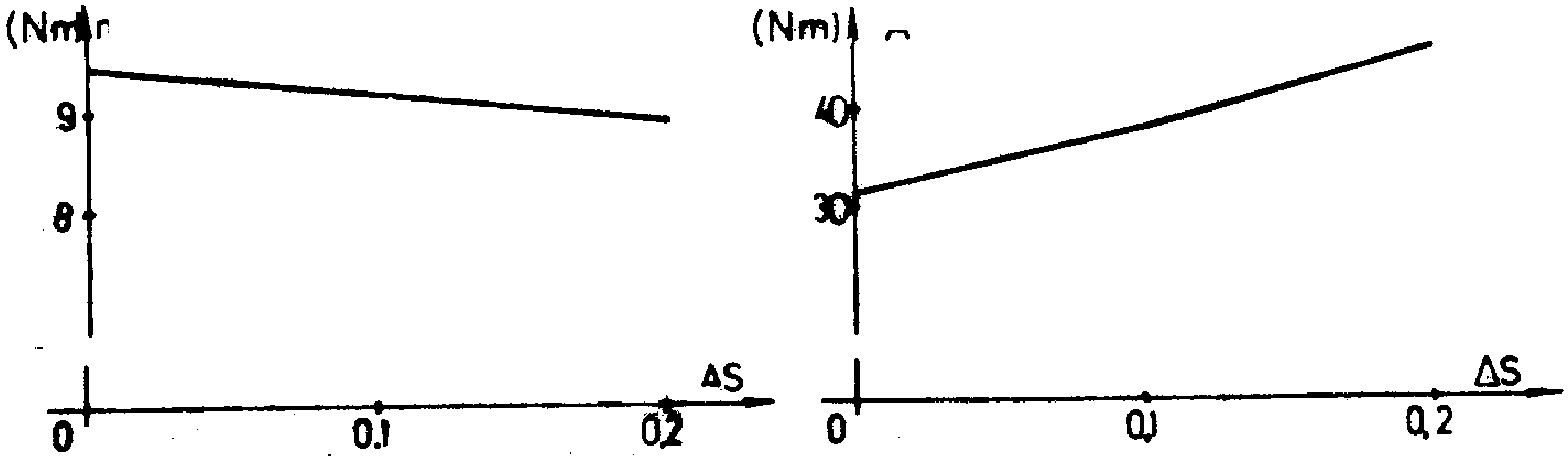


Fig. 17/1 Compensating actuator in frontal plane

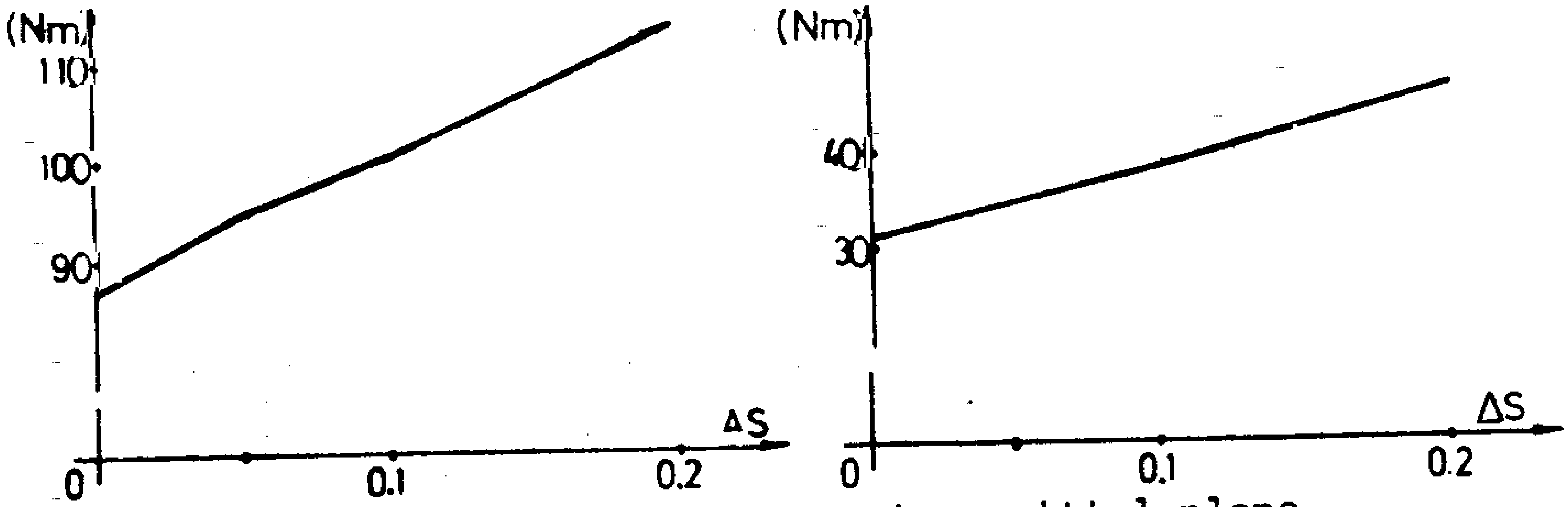


Fig. 17/2 Compensating actuator in sagittal plane

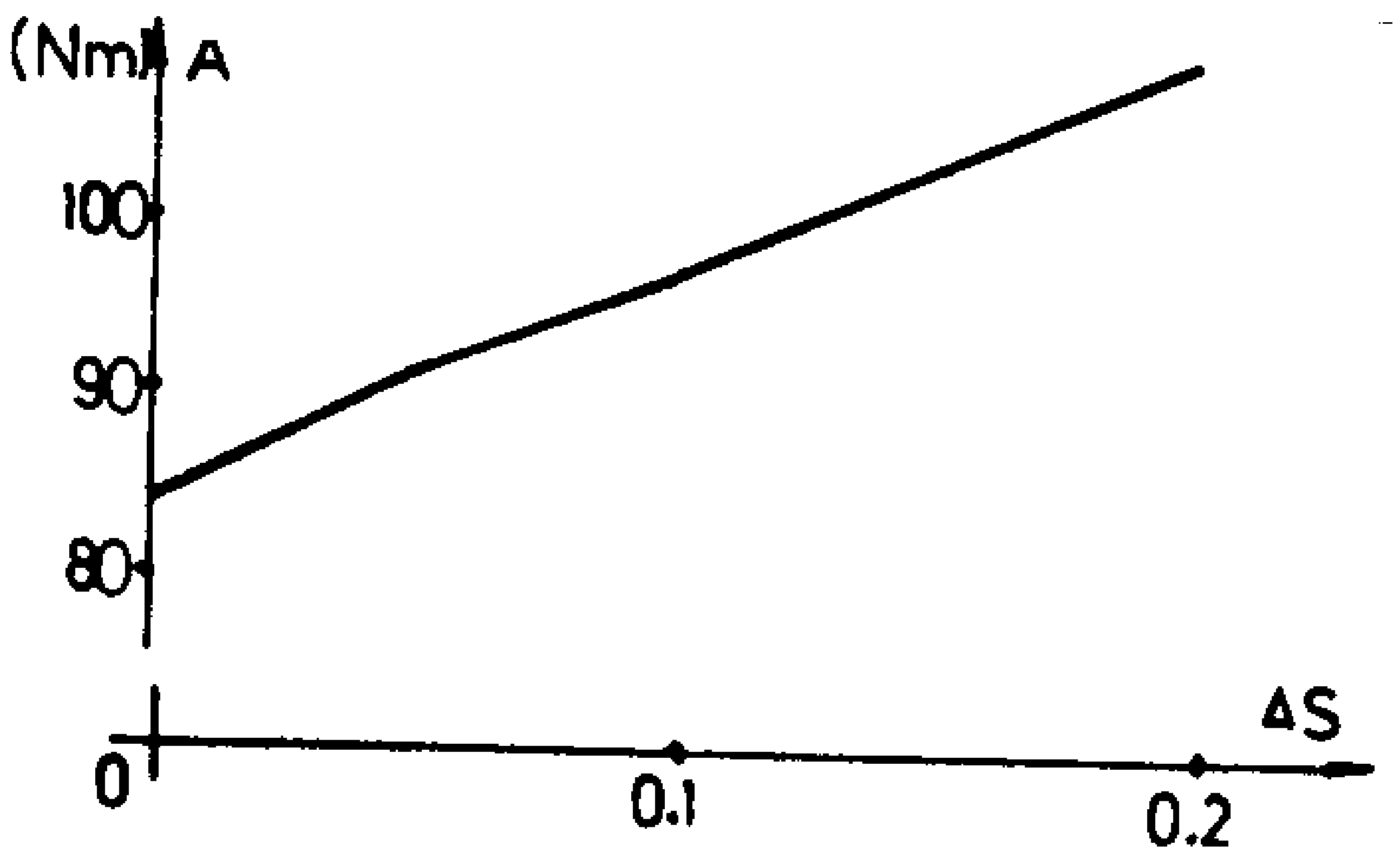


Fig. 18. Total mechanical work

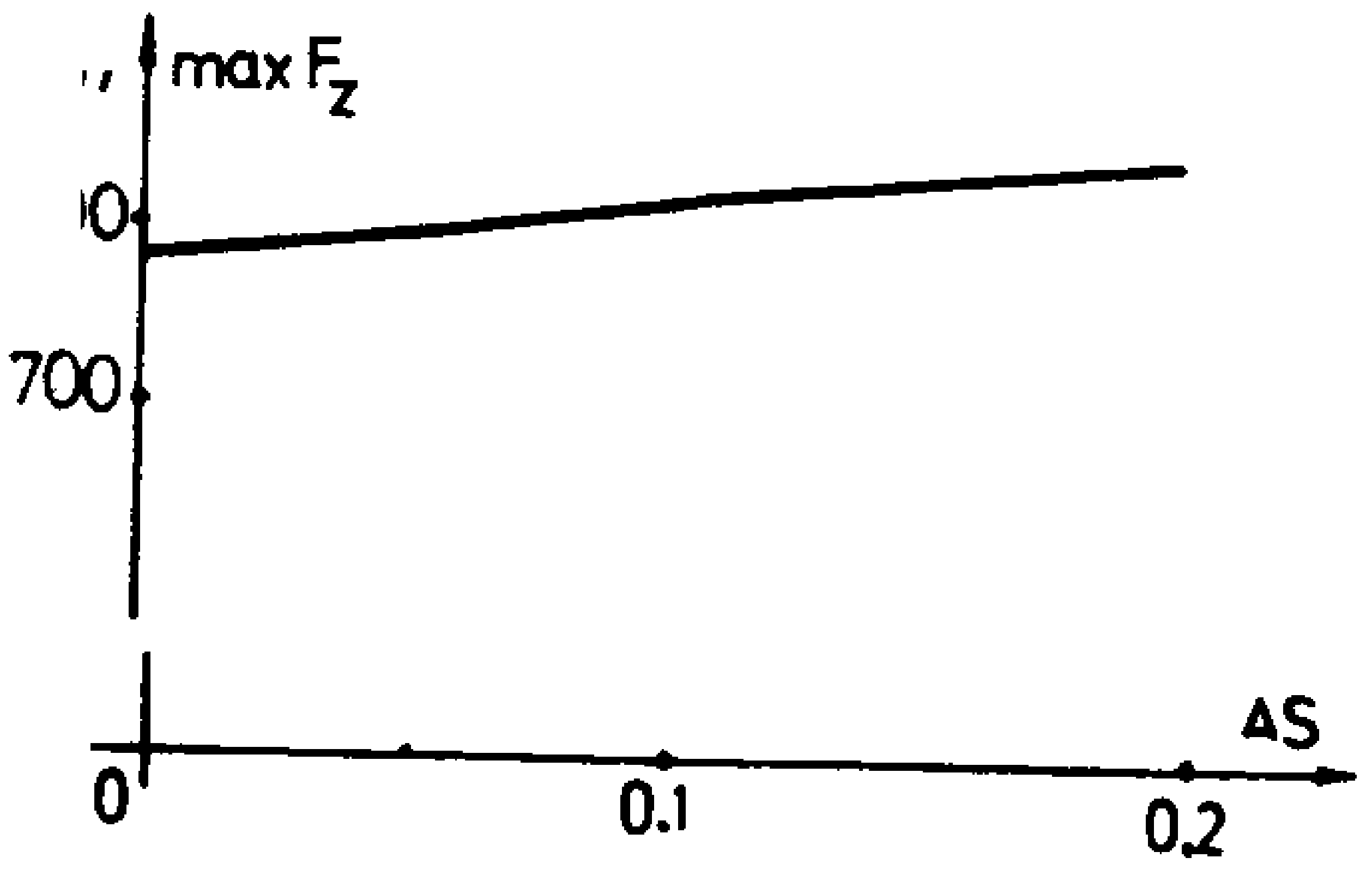


Fig. 19. Maximal values of vertical component of reaction force

Maximal values of driving torques and mechanical work during full step in the function of mass decreasing between left and right leg

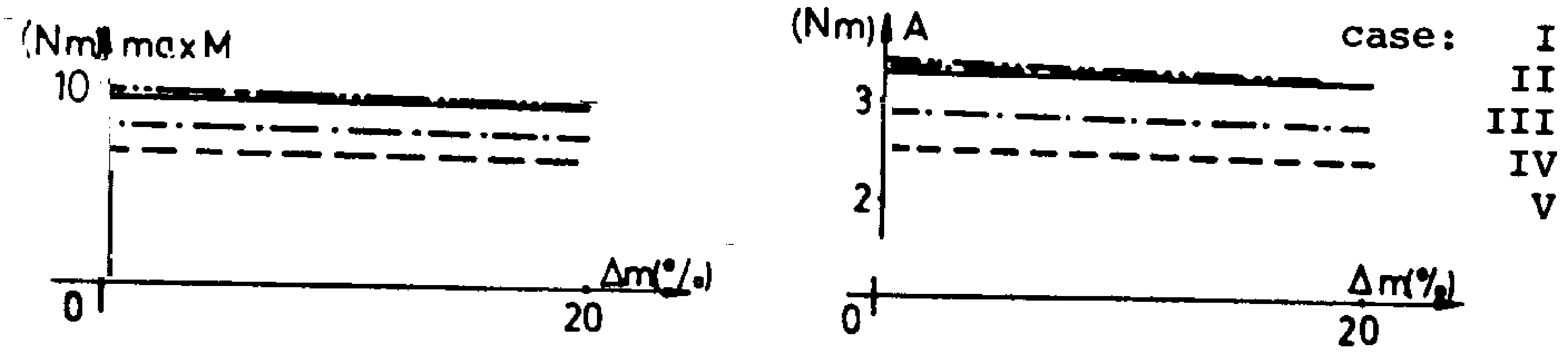


Fig. 20/1 Compensating actuator in frontal plane

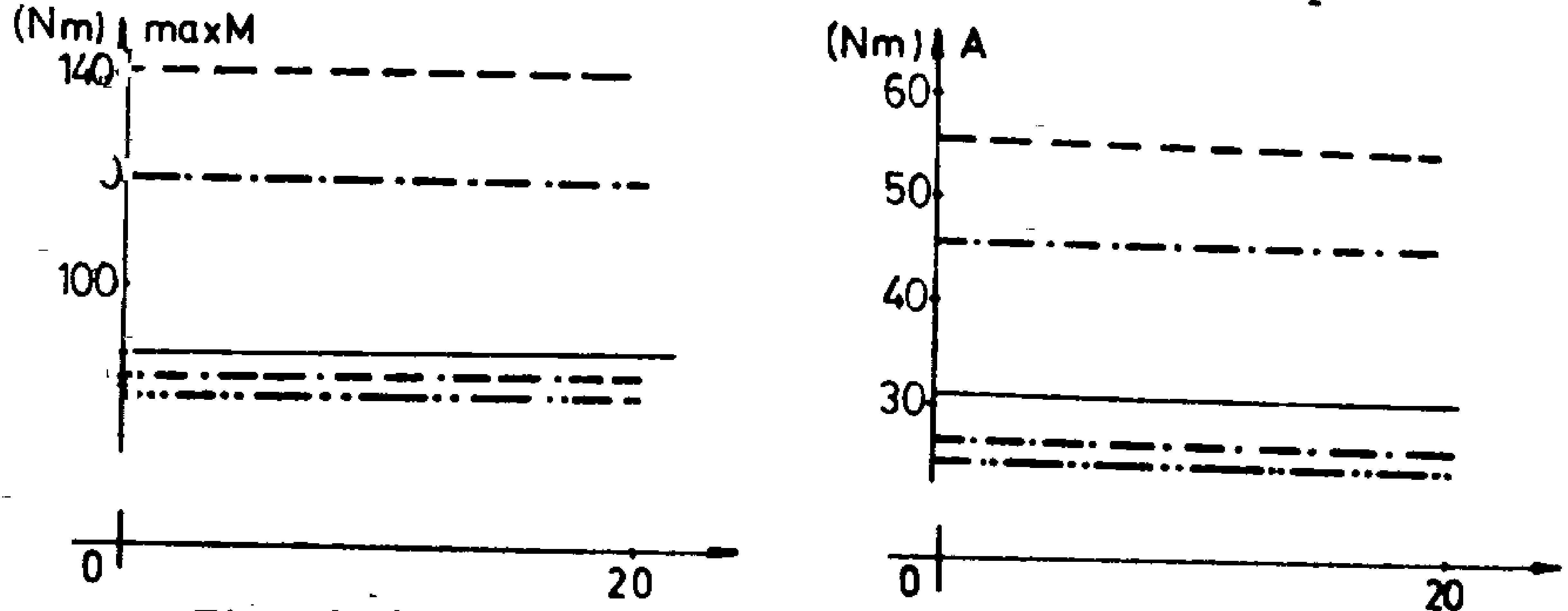


Fig. 20/2 Compensating actuator in sagittal plane

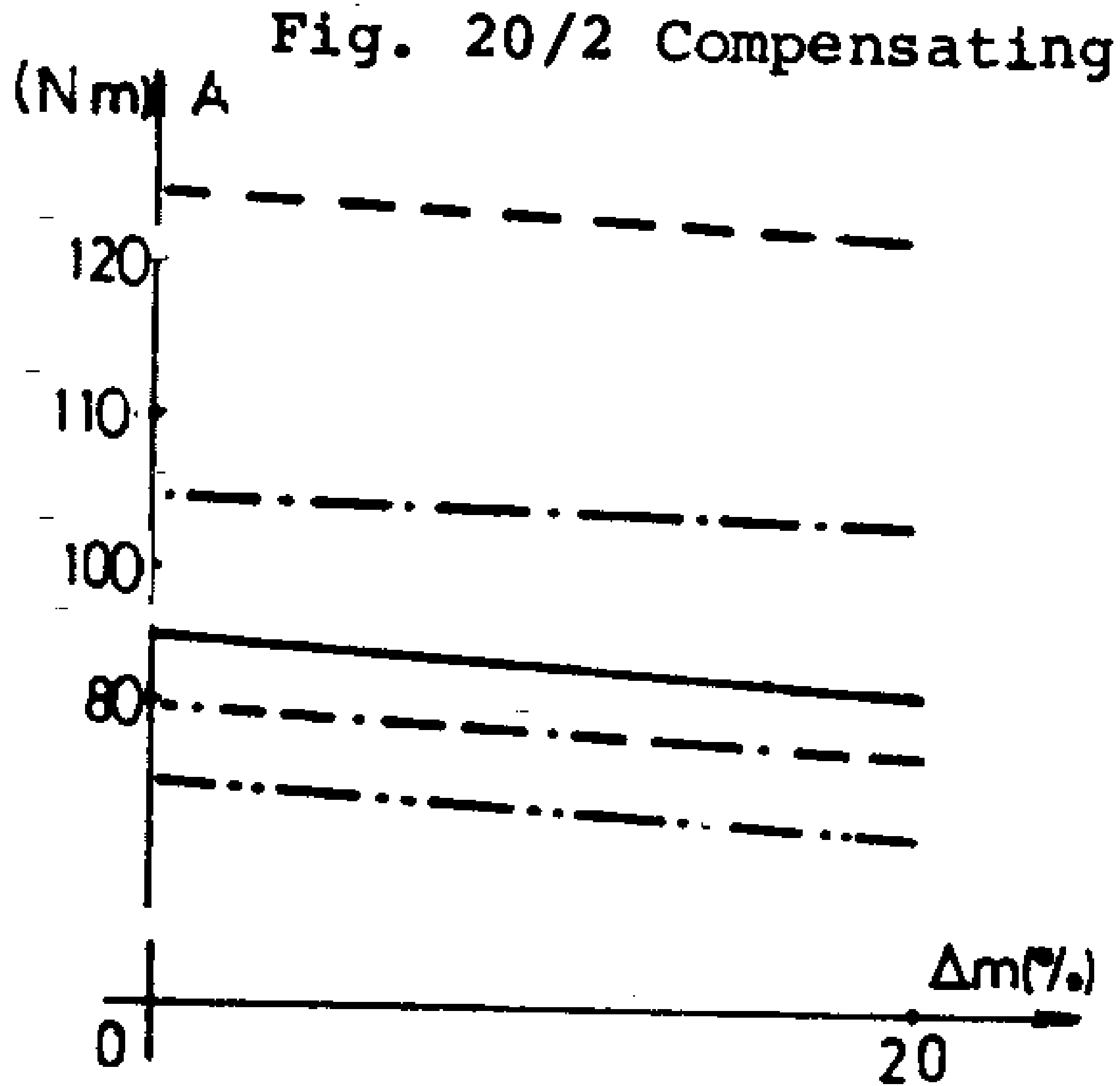


Fig. 21. Total mechanical work

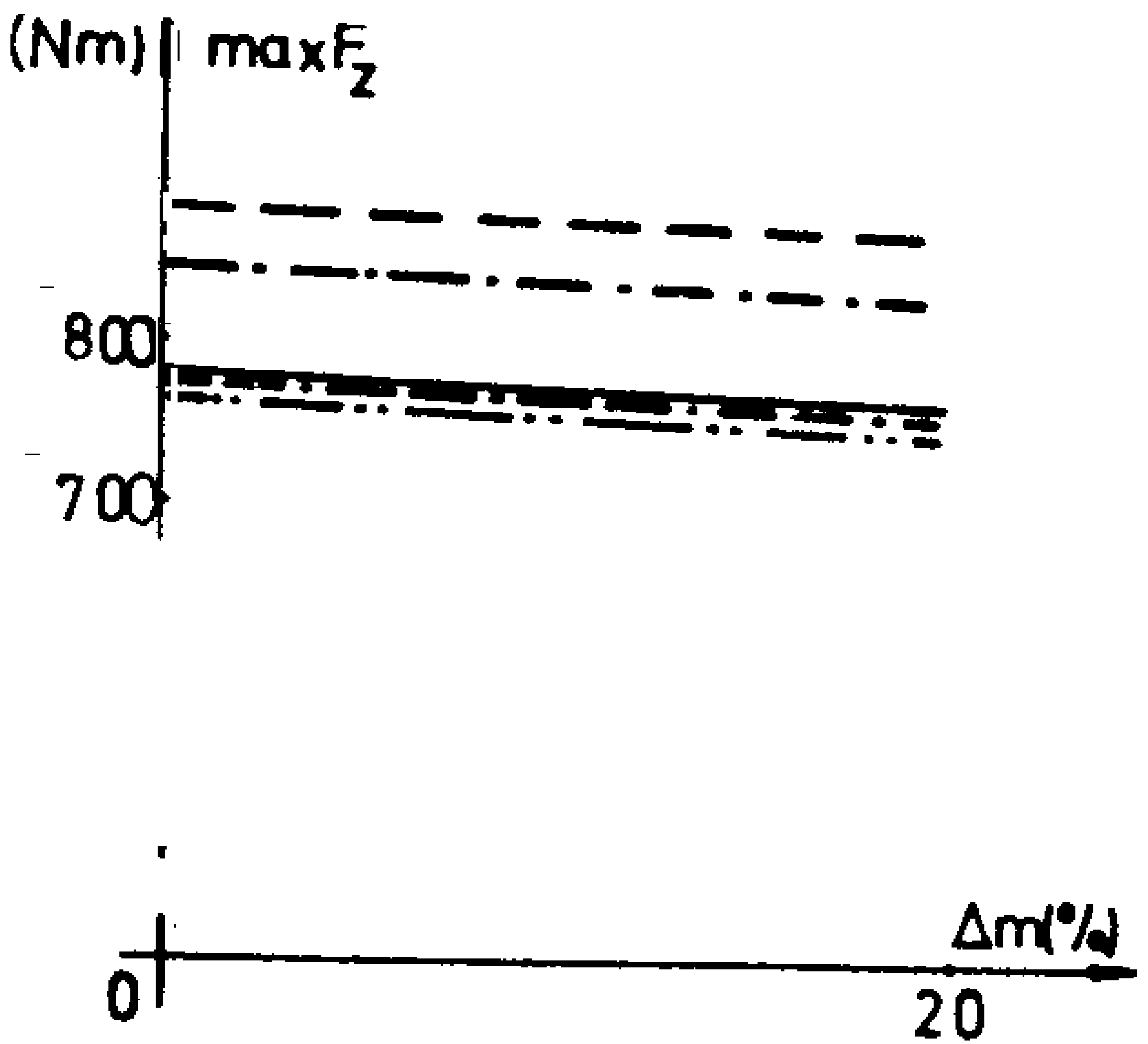


Fig. 22. Maximal values of vertical component of reaction force