AN APPROACH TO THE SYNTHESIS OF LOWER-EXTREMITY CONTROL

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Introduction

The desire of men to build animal-like locomotion automata, able to move on legs, is at least as old as the technical civilization. Althought a quite numerous family of wheeled vehicles was perfected in the meantime, the advantages of a legged vehicle in a natural environment are still obvious.

Aplication of legged automata is seen not only in off-road locomotion but in prosthetics, ground work machinery and, nowadays, in extraterrestrial exploration as well. The failure to achieve up to now significant results in building legged automata cannot be explained by difficulties either in mechanical construction or in finding suitable power units. The true cause for failure has been the inability to synthetize an adequate control system.

Investigators of the locomotive mechanism in animals claim that the included information processing is discouragingly massive. At the same time they offer little hope that the exact nature of the mechanism will be revealed in the near future. However, the results achieved in control and information processing encourage research on the synthesis of bionic locomotion systems.

Control of Bionic Locomotion Automata—General Considerations

The locomotive mechanism of legged animals shows some extraordinary qualities: learning, high degree of adaptation to terrain and internal changes, and ability of optimal motion under the given circumstances.

A bionic locomotion automaton should have all three of these qualities to some extent.

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Even the simplest legged mechanical construction exposed to control and external forces is a large-scale system in the cybernitic sense. The only way to cope with the problem of control is to decompose the system. An adequate control structure should correspond to the system kinematics and should be necessarily hierarchically organized in accordance with the principle of minimal information flow between control levels[1]. Not treating the problem of goal decomposition and general control structure, we give a possible list of control levels, as follows:

- — control of speed and direction;
- — automaton stability in motion;
- — synchronization of movements of extremities;
- — control of extremity movements.

In the following we shall consider the control of extremity movements only. This control level we define as exerting the following properties:

- — adaptivity to terrain within a step;
- — adaptivity to parameter changes of the automaton within reasonable limits;
- — near optimal control property with respect to a given criterion;
- — simplicity enabling implementation.

An Approach to the Synthesis of Extremity Control

We consider that a mechanical extremity is like an extremity in animals. It is built of joined inert levers. The joints are actuated by means of control torques. In order to obtain the mathematical model one has to state a second order differential equation for each degree of freedom of such mechanical systems.

Let us consider a movement of the extremity the initial and final positions of which are assumed to be known. The kinematic space of the extremity is constrained by terrain. The control torques are constrained as well. The main control task is to force the extremity to perform the movement in a given time, the constrains not being violated. The next task is to perform the movement with a minimal energy consumption.

It is worthwhile to point out that the problem stated above is too complex to be treated by the optimal control theory, even in a simple case of an extremity having two degrees of freedom and being composed of two levers. Instead a more practical approach to the synthesis is adopted.

A control system is envisioned that will enable the extremity to adapt to its own parameter changes, external forces and terrain. According to the proposed approach, the control system has to comprise two main subsystems:

- — adaptive generator of trajectories;
- — tracking control system.

The extremity has to be equipped with sensory elements to signal a contact with ground and an eventual obstacle. The control system comprises an efficiency evaluator as well (Fig. 1).

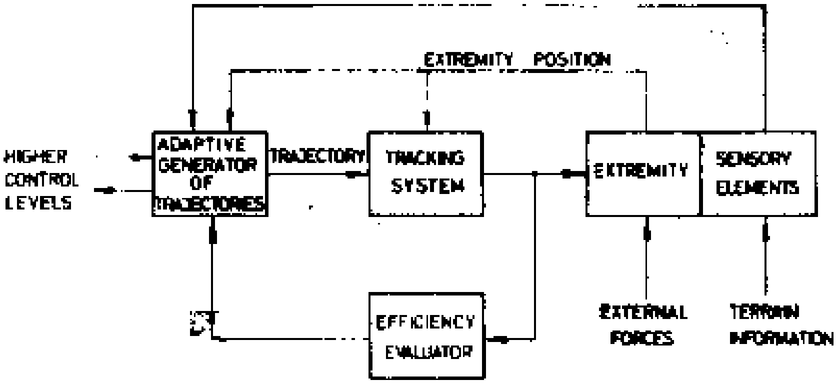

Figure 1.

If necessary the generator switches from the current trajectory to a new one according to information obtained from the sensory elements.

The tracking system is conceived to be based on classical PD regulators. It gives the adaptivity to system parameter changes and external forces.

A practical approach to the synthesis of a near optimal trajectory is to decompose it into the synthesis of a space trajectory and a speed function along the trajectory. The space trajectory has to fulfill the boundary conditions, to lie in the kinematic space of the extremity constrained by the terrain configuration. In order to get a near optimal space trajectory, one can search trough feasible solutions and try to find one simple enough to be easily implemented. An adequate speed function has to be chosen among those fulfilling boundary and integral conditions. Tracking such speed function need not produce the violation of the constraints on contral torques. Once the space trajectory is determined, search for a near optimal speed function is a much easier task.

Control of Extremity Movement in Swing Phase

Lower-extremity cycles in normal movement through swing and supporting phases.

The synthesis proposed will be applied to the control of extremity movement in swing phase.

Let us consider a two-lever extremity (Fig. 2), having two degrees of freedom, described by the following equations:

$$
\begin{aligned}
&(J_1 + M_1 l_1^2 + M_2 L_1^2)\ddot{\varphi}_1 + M_2 L_1 l_2 \ddot{\varphi}_2 \cos(\varphi_1 - \varphi_2 - \gamma_2) + \\
&+ M_2 L_1 l_2 \dot{\varphi}_2^2 \sin(\varphi_1 - \varphi_2 - \gamma_2) + M_1 g l_1 \sin(\varphi_1 + \gamma_1) + \\
&+ M_2 g L_1 \sin\varphi_1 = m_1 - m_2
\end{aligned}
$$

$$
\begin{aligned}
M_2 L_1 l_2 \ddot{\varphi}_1 \cos(\varphi_2 + \gamma_2 - \varphi_1) + (J_2 + M_2 l_2^2)\ddot{\varphi}_2 + M_2 L_1 l_2 \dot{\varphi}_1^2 \sin(\varphi_2 + \gamma_2 - \varphi_1) + \\
+ M_2 g l_2 \sin(\varphi_2 + \gamma_2) = m_2
\end{aligned}
$$

It is assumed that the ground under the extremity is a horizontal plane. The initial and final position of the extremity are shown in Figure 2. The boundary conditions of the movement are:

$$
\begin{aligned}
&\varphi_1(0) = \varphi_2(0) = -\varphi_0 \qquad && \varphi_1(T) = \varphi_2(T) = \varphi_0 \\
&\dot{\varphi}_1(0) = \dot{\varphi}_2(0) = 0 && \dot{\varphi}_1(T) = \dot{\varphi}_2(T) = 0
\end{aligned}
$$

where T is the given time period of movement.

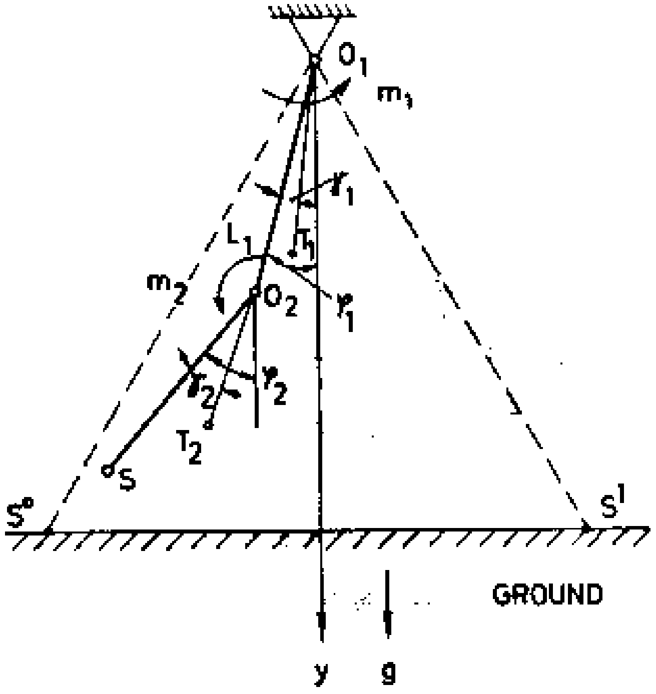

Figure 2.

O_i — joint axis, T_i — center of gravity of a lever, L_i — lever length, γ_i — angle $< L_i\ l_i >$, l_i — joint to center of gravity distance, φ_i — angle $< L_i\ 0_y >$, M_i — lever mass, J_i — inertial moment of lever proper, m_i — control torque

According to the general considerations, the block diagram of the system is given in Figure 3.

The extremity is represented with two interacting subsystems (S_1, S_2) each representing one lever.

The system with parameters

$$M_1 = M_2 = M, \quad L_1 = L_2 = 2l, \quad l_1 = l_2 = l \quad \text{and} \quad \varphi_1 = \varphi_2 = 0$$

was simulated on an analog computer.

The trajectory of the end point S uniquely determines the space trajectory of the extremity.

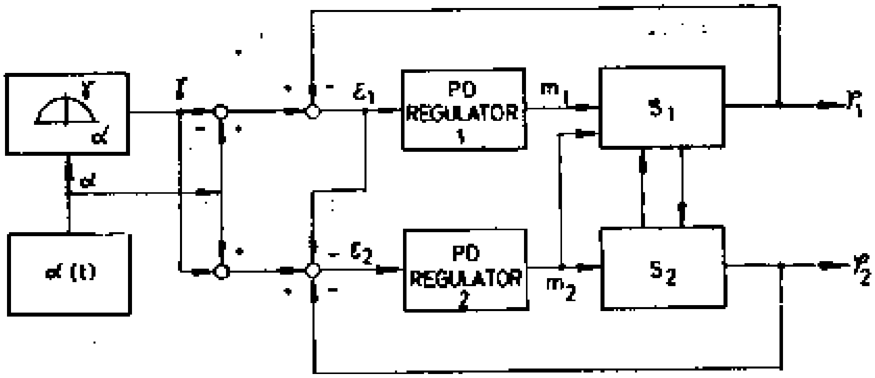

Figure 3.

A near optimal space trajectory very suitable for implementation appears to be the straight line S^0S^1. The equation of that trajectory expressed in polar coordinates 2ρ, α is:

$$\rho = h \frac{\cos \alpha_0}{\cos \alpha} \qquad |\alpha| \leq \alpha_0 = \varphi_0$$

Very simple expressions for φ_1, and φ_2 in function of coordinates ρ, α are obtained as an auxiliary parameter γ is introduced:

$$\varphi_1 = \alpha - \gamma$$

$$\varphi_2 = \alpha + \gamma$$

where $\gamma = \arccos \dfrac{\cos \alpha_0}{\cos \alpha} \qquad |\alpha| \leq \alpha_0$

The last relation is quite near approximated by the circle:

$$\gamma^2 + \alpha^2 = \alpha_0^2$$

The search for an optimal speed function resulted in:

$$\frac{d\alpha}{dt} = \alpha_0 \frac{2\pi}{T} \sin \frac{2\pi}{T} t \qquad 0 \leq t \leq T$$

Generated $\varphi_1^*(t)$, $\varphi_2^*(t)$ and obtained trajectories $\varphi_1(t)$, $\varphi_2(t)$ are shown in Figure 4. They are very close what confirms that parameters of the tracking system were well chosen. In the same figure the four characteristic positions of the extremity during the movement are given.

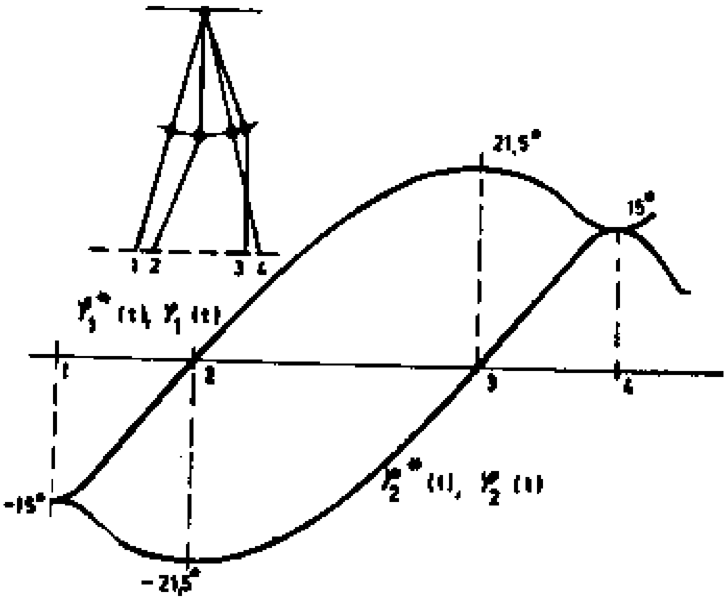

Figure 4.

On the computer model the movements with duration T different with respect to the quasi-period of the extremity oscillations were simulated with control system functioning satisfactorily.

References

1. R. Tomović: A Control Theory of Prosthetics. Second IFAC Congress, Basel, 1964, Butherworth's Publication, London.