FINITE STATE CONTROL OF QUADRUPED LOCOMOTION

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Introduction

Recent efforts directed toward the development of powered prosthetic and orthotic appliances have been concentrated to a large extent in the area of upper-extremity control. This is due, in part, to the difficulties involved in realizing the relatively high frequency response and complexity of motion required of lower-extremity devices and, in part, to the much greater power requirements inherent in locomotion. Moreover, the need for powered lower-extremity appliances is perhaps less apparent than in the upper-extremity case, while at the same time the problem of artificially stabilizing biped locomotion has quite likely discouraged research which might otherwise have been undertaken.

This paper is intended to attract more attention to the possibility of realizing useful powered devices for lower-extremity control. It is addressed not to the entire problem, but to one of the crucial problems, that of automatically controlling the actuators of a powered limb to achieve a stable gait. The approach taken is called »finite state control« and proceeds along the lines proposed in a recent paper by Tomović and McGhee[1]. The finite state point of view regards locomotion as a process involving a distinct and discrete sequence of events rather than as the continuous unfolding of a complicated dynamic process. With such a model for locomotion, an automatic controller can be designed to implement a set of decision rules or a *finite algorithm* which specifies the way in which the sequence of events must occur in order to produce a desired gait. The dynamics of legged locomotion systems are suppressed by this artifice and the controller for an artificial limb consequently takes on the form of an extremely small special purpose digital computer[1].

Adoption of the finite state approach to a control problem raises a number of questions. First of all, it is necessary to decide how to discretize the process being controlled. Such a discretization is certainly in general not unique and some basis is required for making a choice between competing models. An index of desirability which is often used in the study of finite state machines is system simplicity expressed in terms of the total number of states in the machine[2]. Another criterion, suggested for bioengineering systems by Tomović and McGhee[1], is minimization of the exchange of information between the various subsystems of a finite state system. Both of these criteria have been taken into account in the research reported in this paper. Once a finite state model has been developed for the controlled process, there still remains the problem of choosing a control algorithm. Finally, when an algorithm has been selected, its validity must be verified in the context of the dynamic behavior of the controlled system. The latter step can sometimes be accomplished by application of the analytic tools of control theory, usually with the aid of computer simulation, and sometimes by an experimental investigation.

In the case of natural locomotion systems, an exact analytic model for the system dynamics is difficult if not impossible to obtain. Even for an artificial system, the complexity of the equations describing articulated limbs with powered joints coupled together through artificial hip and pelvic structures discourages a purely analytic investigation of dynamic behavior. As a consequence, an experimental approach to the determination of system stability has been taken in the research reported here. In order to reduce the difficulty of achieving stable locomotion, it was decided to restrict this research initially to quadruped systems. An artificial quadruped has been built and furnished with a special purpose digital computer capable of producing the sequences associated with a wide variety of gaits. This quadruped has successfully walked, thereby demonstrating the adequacy of a finite state model for quadruped locomotion. A more detailed study of the dynamic properties of this machine is now being carried out. It is hoped that the results obtained will be capable of some generalization and extension to the biped locomotion problem.

Quadruped Gaits

Muybridge[3] was perhaps the first to make a scientific study of the gaits of quadrupeds. By means of information obtained from high speed motion pictures, he was able to categorize the »pattern of footfalls« of various quadrupeds into a number of distinct patterns or »gaits«. More recently, Hildebrand[4] has greatly extended Muybridge's results. It appears that Tomović[5] was the first to point out that such a characterization of modes of locomotion results directly from an idealization of a leg to a two-state device. These two states are, of course, on the ground and in the air, respectively. In order to treat gaits in terms of the behavior of a finite state machine, it is convenient to call one of these two states the »1-state« and the other the

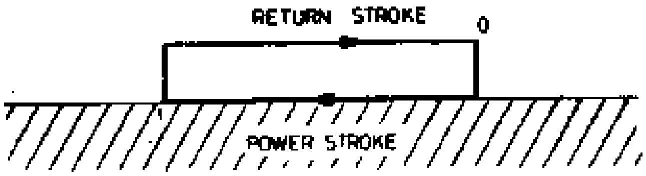

Figure 1. Motion of a foot with respect to the body for an idealized leg

»0-state«. Figure 1 illustrates one of the two ways of numerically designing leg states. By numbering the feet of a quadruped in some standard order, the »state« of the quadruped is then given at any time by a four digit binary number. Figure 2 displays the sequence of states associated with a typical quadruped walk along with the duration of each state[6] Similar information relating to other gaits can be found[3,4,7].

column number	corresponding foot
1	left front
2	right front
3	left rear
4	right rear

a) Correspondence of Feet to Columns of State Sequence

state of legs	duration (seconds)
1 1 1 0	3/16
1 0 1 0	6/16
1 0 1 1	3/16
1 0 0 1	6/16
1 1 0 1	3/16
0 1 0 1	6/16
0 1 1 1	3/16
0 1 1 0	6/16

b) Sequence of States

Figure 2. Sequence of states associated with a quadruped walk

From the point of view of finite state machine theory, Figure 2 defines a *binary sequence generator*. Design techniques for such sequence generators are well established[9]. A particularly attractive realization of the desired sequence is provided by a shift register machine as shown in Figure 3. Such a shift register can be easily constructed from standard two-state electronic »flip-flop« modules. By varying the frequency of the clock oscillator shown on Figure 3, the period of a gait can be changed without altering the sequence of leg states. If the interconnection between flip-flops is made with removable patch cords, both the sequence and relative duration of successive states can also readily be modified. The quadruped which has been constructed is furnished with a programmable sequence generator of this type.

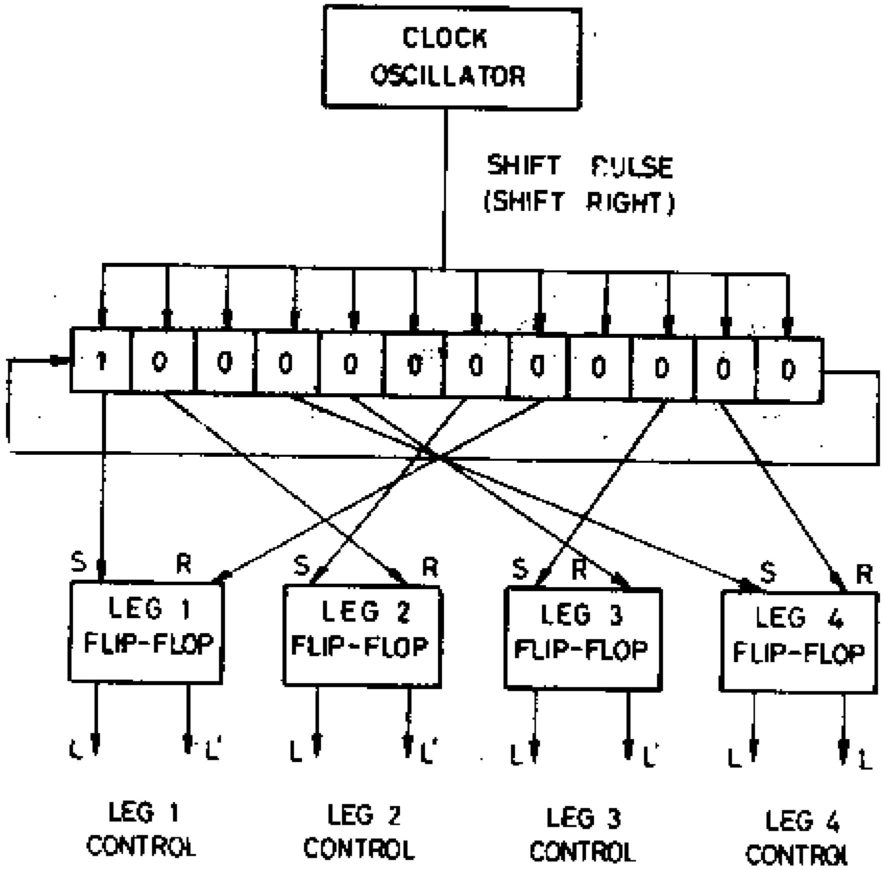

Figure 3. Shift register realization of a sequence generator for a quadruped walk. $S = 1$ sets $L = 1$ and $L' = 0$. $R = 1$ sets $L' = 0$ and $L' = 1$. $L = 1$ commands a leg state of 1. $L = 0$ commands a leg state of 0.

Leg Sub-Automaton

While assignment of just two states to a leg is sufficient to permit a classification of gaits, more states are required of the leg is to function in a matter analogous to the behavior of a natural quadruped leg. In particular, raising of a foot and subsequent return of the foot to a fully extended position in the direction of motion requires that the leg be furnished with some means of shortening the hip-to-foot distance in order to prevent the foot from striking the ground. This ground clearance is provided largely by flexing the knee in natural quadruped (as well as biped) locomotion. Since a goal of the research program reported here is to contribute to a deeper understanding of biped control problems, the artificial quadruped which has been built is furnished with legs possessing both knee and hip joints. Each of these joints is driven by a D.C. electric motor through a self-locking worm gear drive. Either motor can be connected to a positive, negative, or zero voltage source corresponding to forward rotation, rearward rotation, and locked states, respectively. Thus, as shown in Table I, each joint is a three-state device.* With two such powered joints, a leg is therefore a machine with nine possible *internal* states. With respect to the internal functioning of a leg sub-automaton, the previously assigned states 0 and 1 correspont to *output* states.

Table I. States of joints for leg sub-automata

State Number	Joint Motion
0	locked
1	forward
2	rearward

Attainment of the desired leg output state sequence corresponding to a particular gait depends upon the realization of a suitable control algorithm for the internal states of the leg. Such algorithms are certainly not unique and the superiority of one algorithm over another is again a question which may require dynamic analysis, simulation, or experimentation for its resolution. To begin with, there is the question of to what extent the sequence generator should control the output states of each leg. A *fully synchronous* control results if the sequence generator outputs control both the placing and lifting of feet; such a leg can be called *bi-stable* since the beginning of both the 0 and 1 output states is determined by the sequence generator. *Partially synchronous* control results if either the 1 or 0 state is initiated by the sequence generator but the other state begins when some other condition such as a limit switch opening or closing occurs; a leg designed for this type of control is *mono-stable*. An *asynchronous* control algorithm does away with the clock oscillator altogether and relies entirely upon discrete feedback signals to accomplish the desired mode of locomotion.

Tomović and McGhee[1] present a design for an asynchronous biped leg controller. The quadruped reported here makes use of a mono-stable leg in conjunction with a shift register sequence generator. The shift register output, L_1, initiates the rearward stroke of leg 1. The return stroke begins when some selected limit switch is encountered. A finite state controller for this mode of operation can be designed by following the synthesis procedure[1]. The first step of this synthesis requires that a set of binary feedback signals corresponding to *decision points* be defined. The feedback signals used in building the mono-stable leg described above are as follows:

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* Tomović and McGhee[1] have pointed out that at least four actuator states appear to be needed in order to obtain a more or less natural biped gait. This does not seem to be the case for quadruped gaits, at least so long as questions of efficiency are disregarded.

Hip angle limits, h_1 and h_2. Both of these signals are equal zero when the thigh of the leg is fully extended to its forward mechanical limit. As this thigh is rotated backwards, first h_1 and h_2 changes from zero to one. The change in h_2 occurs before the rearward mechanical limit of the hip drive mechanism is encountered.

Knee angle limits, k_1 and k_2. Both of these feedback variables are zero when the knee is fully straightened. As the knee is flexed, first k_1 and then k_2 changes from zero to one.

Table II. Knee and hip joint state sequences for a mono-stable leg.

Controller Inputs					Actuator States		Comment
L	h_1	h_2	k_1	k_2	H	K	
1	0	0	0	0	2	0	Start of power stroke
1	1	0	0	0	2	0	
-	1	1	0	0	0	2	End of power stroke
-	1	1	1	0	1	2	Start of return stroke
-	1	0	1	0	1	2	
-	1	0	1	1	1	0	Knee fully flexed
-	0	0	1	1	0	1	Thigh fully extended
-	0	0	1	0	0	1	
0	0	0	0	0	0	0	Stable state

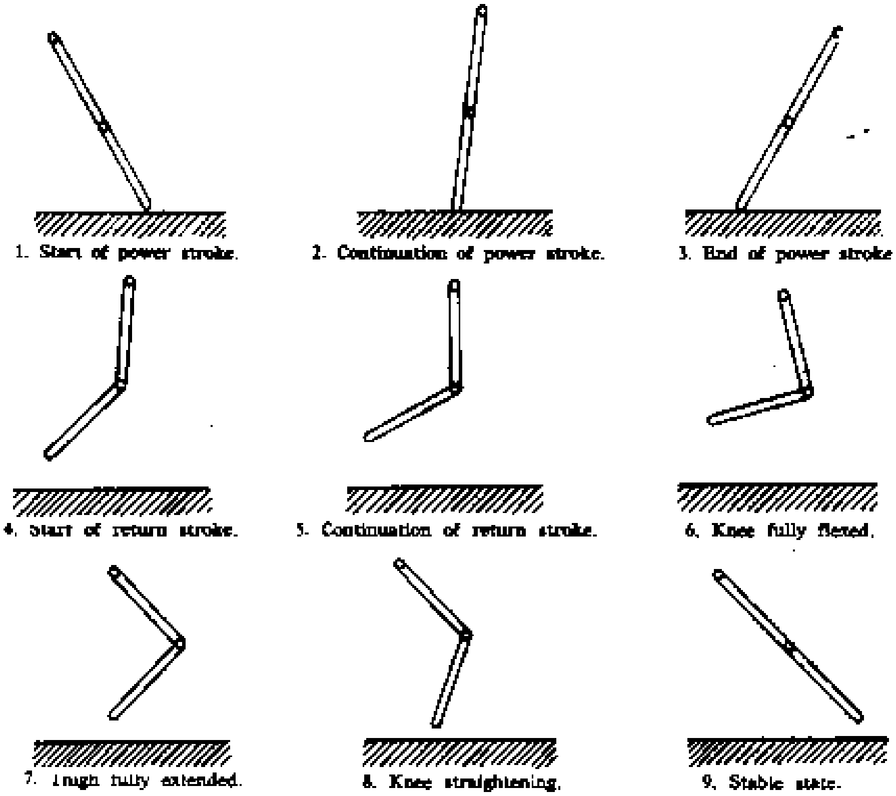
L = Sequence generator output

H = Hip state

K = Knee state

With these definitions, one variety of mono-stable leg control is described by Table II. Figure 4 portrays the same sequence of events graphically. Blanks in Table II correspond to events of Figure 4 which take place regardless of the value of the sequence generator leg control signal, L.

The information contained in Table II is sufficient to permit the synthesis of a finite state controller for each leg[9]. Figure 5 is a state graph for such a controller. The appearance of a prime on a variable indicates electronic inversion; i.e., $x' = 0$ when $x = 1$ and conversely. The numbers appearing in parentheses in Figure 5 refer to actuator states. For the experimental machine, this controller was constructed



1. Start of power stroke.
2. Continuation of power stroke.
3. End of power stroke
4. Start of return stroke.
5. Continuation of return stroke.
6. Knee fully flexed.
7. Thigh fully extended.
8. Knee straightening.
9. Stable state.

Figure 4. Sequence of events for a mono-stable leg

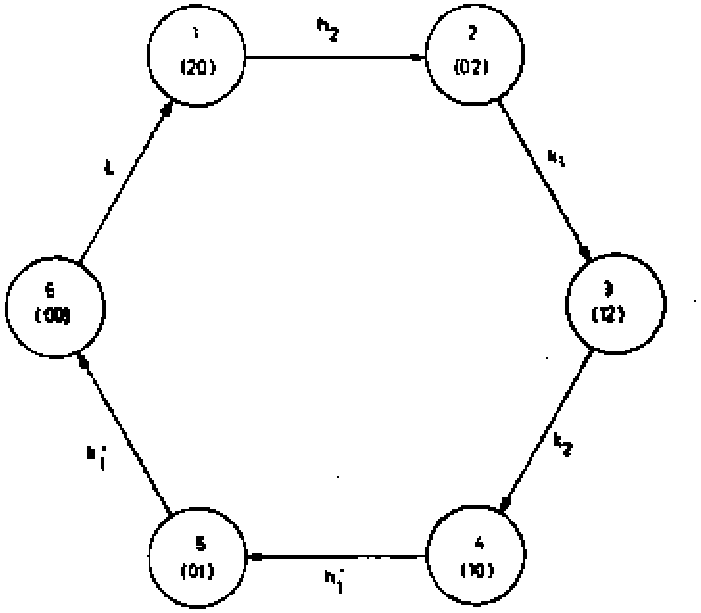


Figure 5. State graph for a finite-state leg controller

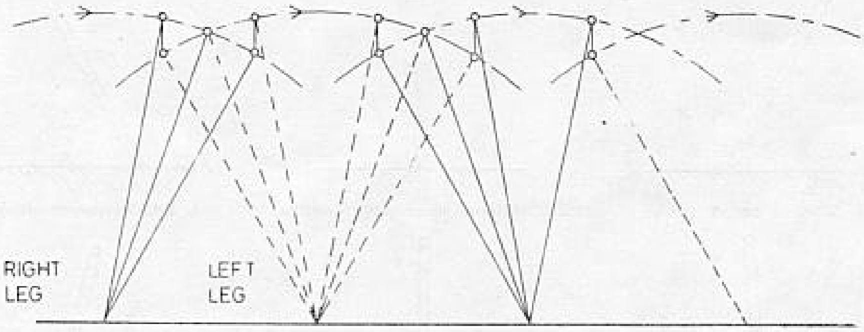


Figure 6. Idealized quadruped pelvic action

from conventional electromagnetic relays. The coupling of the controller to a two joint leg results in a leg sub-automaton subject to the higher level control imposed by the sequence generator. Figure 7 shows such a leg in a flexed state.

Pelvic Structure

Low speed gaits in natural quadrupeds are normally accomplished with locked knees during the rearward stroke of legs in contact with

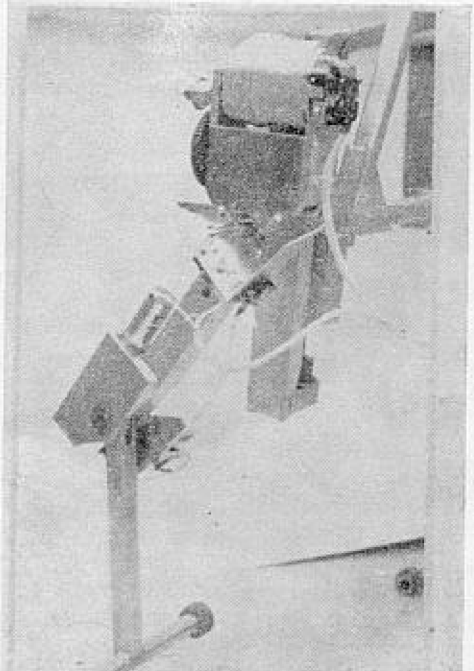


Figure 7. An experimental two-joint leg

the ground. Such gaits usually contain phases in which both feet of a pair of legs (front or rear) are simultaneously on the ground. Consideration of the kinematics of such motion shows that forward movement of an animal is impossible in such a case unless some provision is made for differential displacement of the two hip sockets in a vertical plane. Figure 6 illustrates this situation. In order to satisfly this requirement, the experimental quadruped is furnished with two identical pelvic structures, one for the front pair of legs, and one for the rear pair. The pelvic structures make use of linear ball bearings to obtain free motion in a vertical direction. Rotation in a horizontal plane is constrained by a shackle and a lever-arm system which also permits the attachment of springs providing a variable spring constant both for differential motion and for motion of either leg independently with respect to the body of the quadruped. Figure 8 shows some details of this suspension system.

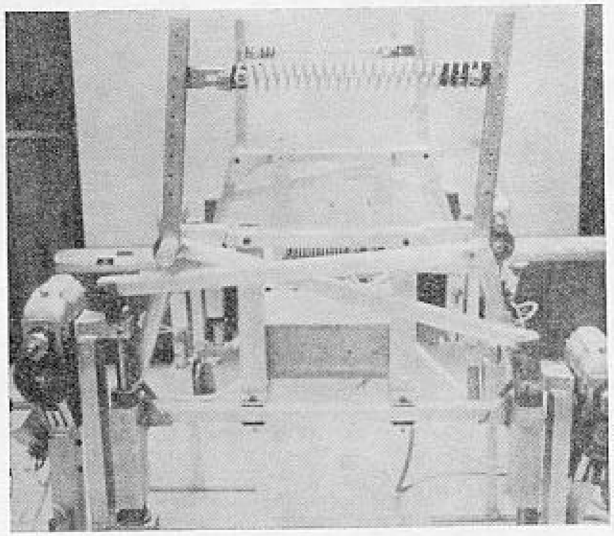


Figure 8. An experimental pelvic structure

Complete Locomotion Machine

The completed machine assembled from the components which have been described is shown in Figure 9. As can be seen, it is furnished with four identical legs and two pelvic structures. The pelvic structures are joined by two aluminum struts which also carry the relay controllers for each pair of legs. Power and control signals are furnished from a trailing cable which connects to a utility cart carrying power supplies and the sequence generator as shown in Figure 10. The entire machine is approximately four feet long and four feet high and weighs one hundred pounds.

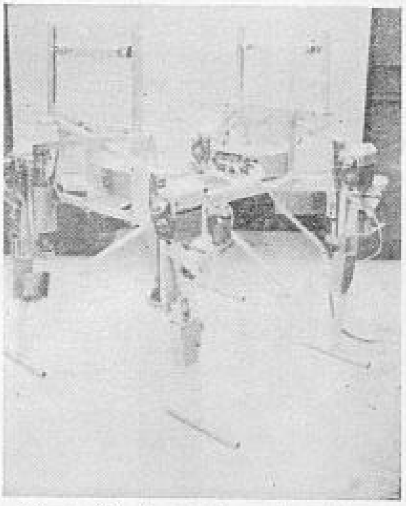


Figure 9. A quadruped walking machine

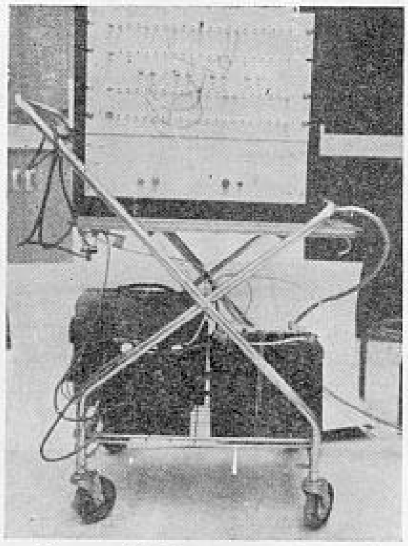


Figure 10. Power supply and sequence generator

Tests with this machine to date have produced a stable gait which corresponds approximately to a quadruped trot[4]. The stability of this gait results from the wide feet shown in Figure 9 which cause the quadruped to be statically stable in all phases of the gait. The action of the passive pelvic structure has not proved to be entirely satisfactory; the normal quadruped walk described in Figure 2 has yet to be attained. The gait which has been realized was chosen to permit pelvic rotation to be severely restricted, thereby reducing the sensitivity of the machine dynamics to the spring constants and damping factors of the pelvic structures. Further experimentation supplemented by dynamic analysis is required to determine if, indeed, a passive pelvic structure can be used to implement a quadruped walk. It is possible that an active pelvis will be needed to accomplish this gait.

Summary and Conclusions

This research program has demonstrated by means of an experiment that stable quadruped locomotion can be obtained by making use of a finite control algorithm. This result has important implications in the fields of prosthetics and orthotics and perhaps also in the design of vehicles for off-the-road transportation. Further work is needed to establish connections between the parameters of this and similar machines and their dynamic stability properties. The possibility of active control of artificial pelvic structures should be investigated. Other types of control algorithms, such as asynchronous control, ought to be studied. While many practical difficulties remain to be solved even before an entirely satisfactory quadruped locomotion machine can be demonstrated, the inherent simplicity of finite state control seems to justify further research in this direction.

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References

1. Tomović, R., and McGhee, R. B.: A Finite State Approach to the Synthesis of Bioengineering Control Systems. IEEE Trans. on Human Factors in Electronics, Vol. HFE-7, No. 2, June 1966.
2. Miller, R. E.: Switching Theory, Vol. II: Sequential Circuits and Machines. John Wiley and Sons, New York, 1965.
3. Muybridge, E.: Animals In Motion. Dover Publications, Inc., New York 1957 (first published in 1887).
4. Hildebrand, M.: Symmetrical Gaits of Horses. Science, Vol. 150, No. 3696, November 5, 1965.
5. Tomović, R.: A General Theoretical Model of Creeping Displacement. Cybernetica, Vol. 4, No. 2, 1961.
6. McGhee, R. B., and Meisel, W. S.: Abstract Locomotion Automata. Report No. 146, Electronic Sciences Laboratory, University of Southern California, Los Angeles, California, October 1965.
7. McGhee, R. B.: Abstract Quadruped Locomotion Automata. Report No. 130, Electronic Sciences Laboratory, University of Southern California, Los Angeles, California, April 1965.
8. Bartee, T. C., Lebow, I. L., and Reed, I. S.: Theory and Design of Digital Machines. McGraw-Hill Book Co., New York, 1962.