DYNAMIC CHARACTERISTICS OF PNEUMATIC, ELECTRIC AND HYDRAULIC ACTUATION OF PROSTHETIC AND ORTHOTIC DEVICES

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The majority of externally powered prostheses or orthotic aids in clinical use are pneumatically operated open-loop systems in which the velocity of movement is a function of command signal. A small number of more sophisticated closed-loop control systems, some of which incorporate adaptive control, are in use but these have been restricted to the terminal device where the power requirements are generally small and have been electrically operated[1,2]. It is generally accepted that in order to give a greater degree of independence to those with severe deformities or total abscence of the upper limbs, more versatile upper limb prostheses must be developed having three or more independent movements apart from prehension of the terminal device. The multi-function or multi-degree of freedom arm will undoubtedly bring with it a fresh crop of problems amongst which will be the shortage of control sites and the need for the patients to be able to learn to control two or three simultaneous movements of the prosthesis.

Most workers now accept that, apart from prehension, closed-loop position control is desirable for the other movements of the prosthesis. This has the great advantage that it helps to unload the visual feedback channel since the patient can now learn to associate the displacement of the control site with the position of the controlled device. For easy simultaneous operation of a multi-degree of freedom system, however, there is considerable evidence to suggest that a further requirement is necessary; namely, that the time response of the systems must be sufficiently fast for the operator to be unaware of the delays involved. As the complexity of the task increases so does the need for fast response.

The paper sets out to obtain a comparison of the dynamic response of three closed-loop position control systems; namely hydraulically, pneumatically and electrically operated. The absolute values of the time delays associated with each type of system will depend largely on the geometry and loading of the system, and for this reason no attempt is made to determine actual value of time constant, attention simply being given to the relative magnitudes of the quantities.

A typical position control system is shown in Figure 1.

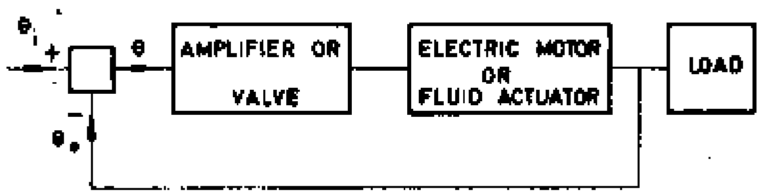

Figure 1. Typical position control system

By making certain linarising assumptions it can be shown that the open and closed-loop transfer functions of either a fluid powered or electrically powered device can be expressed in the form:

$$G(D) = \frac{\theta_o}{\theta} = \frac{1}{\tau_1 D (1 + 2\nu\tau_2 D + \tau_2^2 D^2)} \quad . \quad . \quad . \quad . \quad . \quad 1$$

$$H(D) = \frac{\theta_o}{\theta_i} = \frac{1}{(1 + T_1 D)\ (1 + 2\nu T_2 D + T_2^2 D^2)} \quad . \quad . \quad . \quad 2$$

Provided that τ_1 and τ_2 are dissimilar in magnitude it can be shown that

$$T_1 \approx \tau_1 \quad \text{and} \quad T_2 \approx \tau_2$$

Thus the exponential and complex delay time constants in the closed-loop transfer functions (which are the significant quantities as far as the human operator is concerned) can generally be estimated from the more easily determined integrator and complex time constants of the open-loop transfer function.

Even if τ_1 and τ_2 are comparable and the approximation is no longer valid, it is still true that a comparison of the values of the open-loop time constants will give a useful guide to the relative values of the closed-loop quantities. It is worth noting at this point that although the second term in the denominator of each transfer function is referred to, for convience, as a complex delay, there is in fact no need for the damping ratio ν to be less than unity and aperiodic responses can be obtained if required.

The dynamic response of the three types of system is compared in three different ways, firstly by the complex delay time constants ($T_2 = \tau_2$), secondly by the exponential/integrator time constants ($T_1 = = \tau_1$) and thirdly by the time required for the system to reach saturation velocity following a large step input.

The Complex Delay

a) Fluid Power Systems

By making certan linearizing assumptions it can be shown[3,4] that the open loop transfer function of either a hydraulic or pneumatic position servo of the type shown in Figure 2 can be expressed in the form:

$$G(D) = \frac{1}{\tau_1 D(1 + 2\nu\tau_2 D + \tau_2^2 D^2)}$$

where $\tau_2 = \sqrt{\dfrac{Vm}{2A^2\beta}}$ and β = bulk modulus of compressibility
A = cross sectional area of piston
V = ½ entrapped volume
m = inertial load

Thus it can be seen for a given geometry of actuator and inertial load that the complex delay time constant is inversely proportional to the square root of the bulk modulus of the fluid. Now in the case of hydraulic fluids the bulk modulus would be typically about 10^5 lbf/in² while for a pneumatic system β will be equal to the working pressure which is unlikely to exceed the region 100—300 lbf/in².

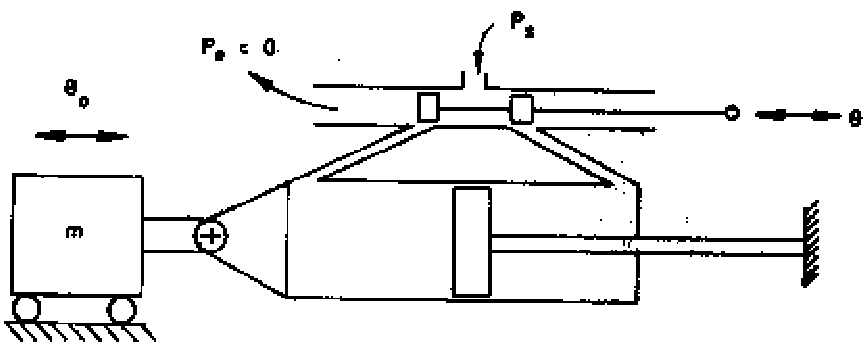

Figure 2. Fluid powered position servomechanism

Thus the ratio of the complex delay time constants will normally be

$$18.25 < \frac{\tau_{2\,pneu}}{\tau_{2\,hyd}} < 31.6$$

b) Electrically Powered System

On the grounds of efficiency and weight, a permanent magnet motor will normally be chosen in this application and armature control must therefore be employed. Referring to Figure 1 and assuming the armature resistance and inductance to be R and L respectively and the load characteristics to consist of inertial and viscous damping of coefficient f, the following equations may be written:

$$K\theta - K_1 B D \theta_o = R(1 + T_3 D)\, i \quad . \; . \; . \; . \; . \; . \; . \quad 3$$

where K is the gain of the amplifier (volt/radian)

K_1 is a constant relating to the back E.M.F. of the motor

B is the flux density

$T_3 = L/R$, and

i is the armature current.

Also if the output torque is given by $K_2 B\, i$

$$K_2 B\, i = I D(1 + T_4 D)\theta_o \quad . \; . \; . \; . \; . \; . \; . \; . \quad 4$$

$$\text{where } T_4 = \frac{I}{f}$$

These equations may be combined to give

$$\frac{\theta_o}{\theta} = \frac{1}{\tau_1 D(1 + 2\nu\tau_2 D + \tau_2^2 D^2)}$$

$$\text{where } \tau_1 = \left(\frac{fR}{KK_2B} + \frac{K_1 B}{K}\right)$$

$$\text{and } \tau_2 = \sqrt{\frac{LI}{fR + K_1 K_2 B^2}}$$

For the type of application considered here, the viscous damping applied to the load (f) will be kept as small as possible (both for mechanical reasons and for reasons of efficiency); also R will normally be small for reasons of electrical efficiency, thus the first term in the denominator of the expression for τ_2 may be neglected in comparison with the second and the expression simplified to

$$\tau_2 \approx \sqrt{\frac{LI}{K_1 K_2 B^2}}$$

Now the inductance of the armature windings will be proportional to the square of the number of turns and the permeability of the core material (μ), the constants K_1K_2 will each be proportional to the number of turns in the winding, thus τ_2 will be approximately equal to

$\sqrt{\frac{I}{C\cdot\frac{B^2}{\mu}}}$ where C is a constant depending upon the construction of the motor. Thus the complex time constant of the electrical system is dependent upon the flux density. Unfortunately, magnetic materials are subject to saturation and the maximum value of B that can be achieved is limited. Blackburn, Rethof and Shearer have drawn attention to this and suggest that for modern magnetic materials the best combinations would only yield figures that would be equivalent in a fluid system to a bulk modulus of 250 lbf/in². It is therefore reasonable to argue that the best value of $\tau_{2\,elec}$ that could be achieved would be a lower one than that for a pneumatic system $\tau_{2\,pneu}$ but appreciably higher than that for a hydraulic system $\tau_{2\,hyd}$. That is $\tau_{2\,hyd} \ll \tau_{2\,elec} < \tau_{2\,pneu}$, the probable order of magnitude of the ratios being

$$\tau_{2\,hyd} : \tau_{2\,elec} : \tau_{2\,pneu} :: 1 : 19.7 : \begin{vmatrix} 18.25 \\ 31.6 \end{vmatrix}$$

The Exponential/Integrator Time Constant

The most convenient means of considering the integrator time constant, τ_1, is by noting from equation (1) that this is a measure of the maximum or saturation steady state velocity achieved by the output when a constant error is applied to the system thus $(D\,\theta_o) = \frac{\theta_{max}}{\tau_1}$.

θ_{max}, in the case of a fluid power system, will be determined by the maximum valve opening and in the case of an electrical system by the maximum output voltage from the amplifier. Theoretically there is no reason why the saturation velocity should not be increased to any required value since in the case of the fluid-powered device it will be dependent upon the size of the valve orifice and, in the case of the electrical device, upon the maximum or saturation output voltage of the amplifier. Practical considerations, however, will impose limits upon these quantities, but probably of greater importance will be the effects of static loading upon the saturation velocity. Normally in fluid powered devices the static load applied to the jack is a small proportion of the maximum force available (i.e. stall load) given by p_s. A where p_s is the supply pressure. In these circumstances, static load causing a pressure difference, p_d, across the ram which is small compared with p_s has little effect on the flow through the valve and hence upon the saturation velocity. The penalty to be paid for this state of affairs is the extremely low efficiency obtained from the device. For a displacement l of the jack the work output will be given by p_d Al while the work input will be p_sAl, hence the efficiency will be determined by $\eta_e = \frac{p_d}{p_s}$, and if p_d is made negligible compared with p_s the effiicency tends to zero.

Clearly this will be an unacceptable penalty for prosthetic requirements where the energy store must be carried by the patient and it will be necessary for p_d to be made significant compared with p_s. Thus static load will have, a significant effect upon the saturation velocity. Assuming turbulent flow through the valve orifice the saturation velocity will be reduced in the ratio $\sqrt{\frac{p_s - p_d}{p_s}}$ or $\sqrt{1 - \eta_e}$.

A similar state of affairs exists in the case of the electrical device, the effects of load torque C_L can be seen by adding this quantity to the right hand side of equation (4) when it will be found that the saturation velocity $(D\,\theta_0)_{ss}$ is reduces from $\frac{\theta}{\tau_1}$ to $\frac{\theta}{\tau_1} - \frac{R\,C_L}{K K_2 B \tau_1}$. Once again the effect on the saturation velocity will depend upon the ratio of the load torque, C_L, to the maximum torque available from the motor. For the electrical system it is less easy to make concise arguments on the grounds of efficiency, it is apparent that the load torque must be made a significant proportion of the maximum available motor torque simply on the grounds of actuator weight. Owing to the relatively low saturation flux density of magnetic materials, electric actuators are incapable of giving the same high power/weight ratios that can be obtained from fluid power devices and Blackburn, Rethof and Shearer show that this must be at least an order of magnitude lower.

Thus it can be seen that in all three cases, static load will have a significant effect on the saturation velocity and hence slow down the integrating action of the device. An alternative way of visualizing this effect is to recognize that as a greater proportion of the maximum available force or torque from the actuator is used to overcome static loading so less will be available to produce acceleration of the inertias involved and a deterioration of the time response must occur. This visualization suggests an alternative method of comparing the integrating action of the devices which is now considered.

An Alternative Method of Comparing the Integrator Time Constant

In this analysis a measure of the integrating time constant will be taken as the rise time of each of the systems to reach saturation velocity having been subjected to maximum accelerating force. This is provided in the fluid systems by opening the valve wide and in the electrical system by subjecting it to maximum driving voltage. Each system will operate on a load equivalent to half its stall load—accelerating this load against gravity and overcoming its inertia. The resulting system response characteristics will be compared with regard to rise time and actuator efficiency, having been required to reach the same saturation velocity (i.e., in the electrical case, after gearing).

This procedure requires a fairly intensive analysis of the systems' dynamics and these will be dealt with individually first. Since the usual linearizing assumptions constraining analysis to small loads and small displacements are not possible in this case, nonlinear equations arise in describing the fluid systems. The equations describing the action of the electrical system, however, remain linear under these conditions, and this system will be dealt with first.

a) Electric Actuator

By assuming that both f and L are negligible compared with the other parameters, equations 3 and 4 may be combined to give a first order linear differential equation for $D\theta_o$:

$$D(D\theta_o) + \frac{K_1 K_2 B^2}{IR}\, D\theta_o = \frac{\Theta\, K_2 B}{IR} - \frac{C_L}{I}$$

where Θ is maximum amplifier output ($K\theta_{max}$).

The solution, assuming zero initial angular velocity is

$$D\theta_o = \left(\frac{\Theta}{K_1 B} - \frac{C_L R}{K_1 K_2 B^2}\right)\left(1 - e^{-\frac{K_1 K_2 B^2}{I R} t}\right) \quad . \quad . \quad . \quad . \quad 5$$

which indicates an exponential time constant

$$T_1 = \frac{IR}{K_1 K_2 B^2}$$

and a saturation velocity

$$(D\theta_o)_{ss} = \frac{\Theta}{K_1 B} - \frac{C_L R}{K_1 K_2 B^2}$$

b) Hydraulic Actuator

The valve is opened wide to attain saturation velocity as quickly as possible which means that the valve coefficient is independent of valve opening and relates flow to pressure difference across the valve. The equations of motion referring to Fig. 3 and assuming the fluid to be incompressible are:

$$Q_1 = A\dot{x} = C_v (P_s - P_1)^{1/2}$$

$$Q_2 = -A\dot{x} = -C_v P_2^{1/2}$$

$$A(P_1 - P_2) = m\ddot{x} + m g$$

where P_s = supply pressure

A = piston c/s area

C_v = valve flow coefficient

Q_1 and Q_2 are flow rates into the jack chambers

P_1 and P_2 are the pressures in the jack chambers

x is system displacement vertically up

m is the mass of the load

g is acceleration of gravity.

These equations can be combined to eliminate Q_1 and Q_2 giving the nonlinear equation

$$2\,A^2\,\dot{x}^2 = C_w^2\left[P_s - \frac{m}{A}(\ddot{x} + g)\right]$$

This can be nondimensionalized by setting

$$X = \frac{x}{\bar{x}} \quad \text{where} \quad \bar{x} = \frac{C_w^2\,m}{2\,A^3}$$

and
$$\tau = ut \quad \text{where} \quad u = \frac{A^2\sqrt{2P_s}}{C_w\,m}$$

giving
$$\dot{X}^2 = 1 - \ddot{X} - \frac{m\,g}{P_s\,A}$$

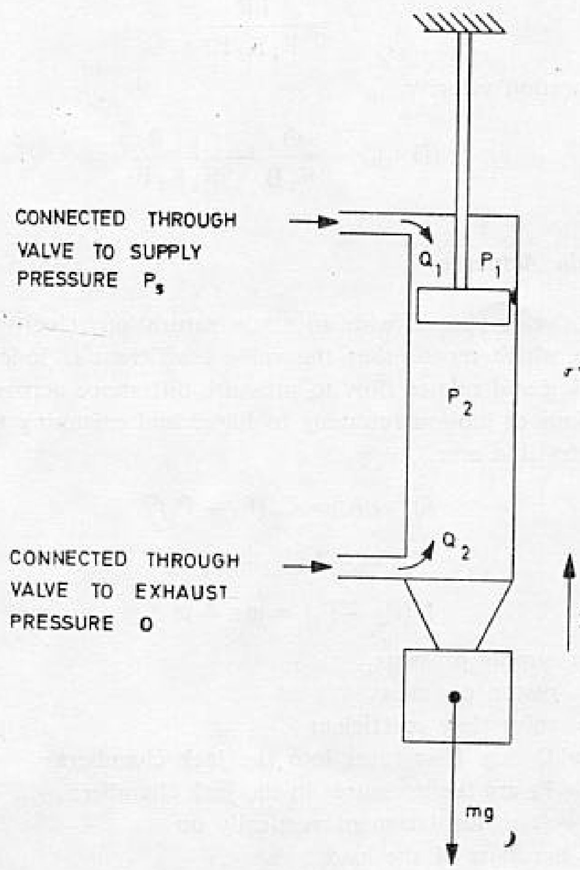

Figure 3. Configuration of fluid powered actuator

Furthermore, if we set $V^2 = 1 - \frac{m g}{A P'}$,

the equation becomes $\dot{X}^2 = V^2 - \ddot{X}$

(where dots now indicate derivatives with respect to τ).

The equation can be written

$$\frac{d \dot{X}}{d \tau} = V^2 - \dot{X}^2$$

which can be integrated to give

$$\tau = \int_0^{\dot{X}} \frac{d \dot{X}}{V^2 - \dot{X}^2} = \frac{1}{V} \tanh^{-1} \frac{\dot{X}}{V}$$

Thus the velocity of the system is

$$\dot{X} = V \tanh V \tau \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6$$

Since the hyperbolic tangent of any increasing argument approaches a maximum value of 1, the saturation velocity is evidently V (in non-dimensional terms); or in terms of the physical parameters,

$$V_{sat} = \mu \bar{x} V = \frac{C_v}{A} \sqrt{\frac{\bar{P}_s}{2}} \left(1 - \frac{m g}{A P_s}\right)^{\frac{1}{2}}$$

The acceleration can be written down directly as

$$\ddot{X} = V^2 - \dot{X}^2 = V^2 (1 - \tanh^2 V \tau) = V^2 \operatorname{sech}^2 V \tau$$

which approaches zero as τ becomes large.

Thus the initial acceleration is

$$(\ddot{X})_{\tau=0} = V^2$$

or in terms of the physical parameters

$$(\ddot{x})_{t=0} = \bar{x} \mu^2 V^2 = \frac{P_s A}{m} - g$$

as would be expected.

c) Pneumatic Actuator

The equations describing the open valve operation of this system involve a certain amount of thermodynamics. Since all the gas flow takes place at less than the critical pressure ratio for choking however, a single set of equations, although complex[4], can be written down.

Refering to the previous diagram, these are:

Valve

$$W_1 = C\frac{P_s}{\sqrt{T_s}}\left(\frac{P_1}{P_s}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{P_1}{P_s}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$W_2 = C\frac{P_2}{\sqrt{T_s}}\left(\frac{P_3}{P_2}\right)^{\frac{1}{\gamma}}\sqrt{1-\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}}}$$

Ram

$$W_1 T_s = \frac{g}{C_p} P_1 \frac{dV_1}{dt} - \frac{g}{\gamma R}\frac{d}{dt}(P_1 V_1)$$

$$W_2 T_s = \frac{g}{C_p} P_2 \frac{dV_2}{dt} - \frac{g}{\gamma R}\frac{d}{dt}(P_2 V_2)$$

Chamber Volumes

$$V_1 = V + Ax$$

$$V_2 = V - Ax$$

Load $$(P_1 - P_2)\, A = m\ddot{x} + mg$$

where the new parameters are

W = weight rate of flow

T_s = absolute stagnation temperature of supply (assumed constant)

C = valve constant

γ = ratio of specific heats $\frac{C_p}{C_v}$

C_p = specific heat at constant pressure

C_v = specific heat at constant volume

R = universal gas constant

Since compressibility of the gas is an important effect, it was not found possible to simplify these equations to a tractable form for analytical solution. With certain minor modifications, therefore, they were programmed for solution on an analogue computer. The results are discussed in the next section along with those of the other two systems.

d) Comparison of Rise Times of the Three Systems

Each of the solutions (equation (5) for electric, equation (6) for hydraulic and the analogue programme for pneumatic) was supplied with data from realistic physical systems and corresponding to the performance requirements mentioned previously — i.e., accelerating vertically upwards to the same saturation velocity an inertial load whose weight is half the system stall load. The results are shown in the graph (Fig. 4). Curve (a) for the electric actuator shows the typical exponential rise of a first order linear system. Curve (b) for the hydraulic actuator is a hyperbolic tangent with positive argument. Curve (c) is the analogue result for a pneumatic system and is oscillatory. The rise times are closely comparable but the difference in the dynamic characteristics is obvious.

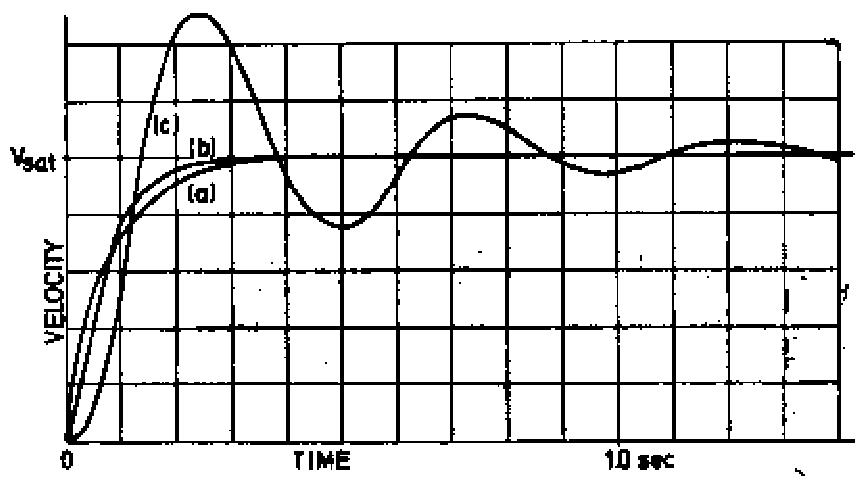

Figure 4. Time response of analyzed systems: a — electric, b — hydraulic, and c — pneumatic

At low velocity the order is first electric, second hydraulic, third pneumatic.

At high velocity the order is reversed, but because of the oscillation in the pneumatic response, the hydraulic is the first to remain

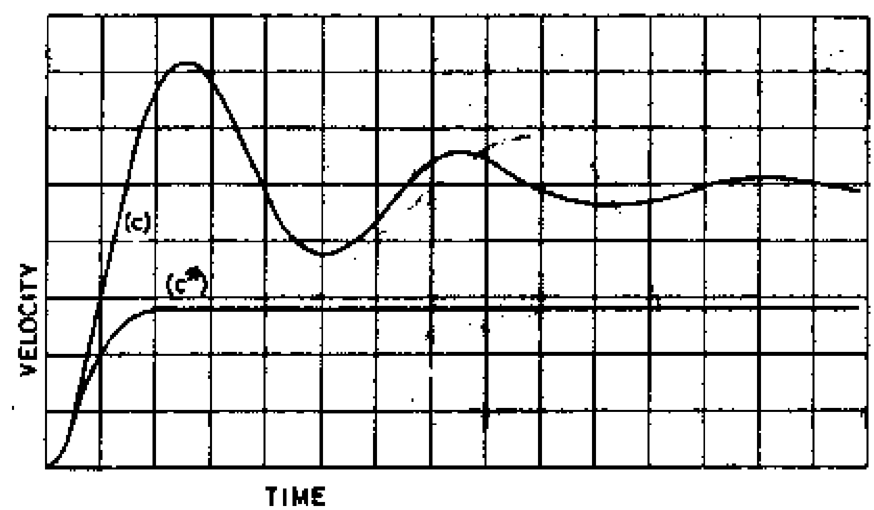

Figure 5. Time responses of pneumatic system: c — oscillatory (without viscous friction), and c* — aperiodic (with viscous friction)

within a small percentage of the final value. The oscillation and the comparatively slow initial acceleration of the pneumatic system are attributable to the high compressibility of the fluid medium. Figure (5) shows the same undamped pneumatic response (curve (c) along with the response of the same system with just enough viscous friction to damp out the oscillations (curve (c*)). The effect of this energy absorption is to reduce the saturation velocity to about 55% of its design value.

e) Comparison of Efficiencies of the Three Systems

Although an exact measure of actuator efficency is not attempted, comparative measures can be made fairly easily to show the order of merit of the three systems. In the fluid actuators a fair measure is the ratio of the work done in raising the load to the energy expended in employing a certain volume of pressurized fluid.

$$\text{i. e.} \quad \eta = \frac{m g x}{P_s A x}$$

With the load specified, the hydraulic actuator efficiency in accomplishing this task is about 50%. The pneumatic efficiency would be about the same, but for the oscillations which require more fluid to be expended. In practice, such an oscillatory characteristic would be intolerable, but the penalty in damping them out is to lower the saturation velocity, thus reducing the numerator of η and reducing efficiency to 25—30%. Experimental efficiency versus torque curves are well documented for electric motors of the appropriate size and it is easiest to refer to one of these in discussing the electrical system. In performing this task the motor will be operating over the point of the curve indicated and thus it is fair to say that its average efficiency will be in the order of 30—40%.

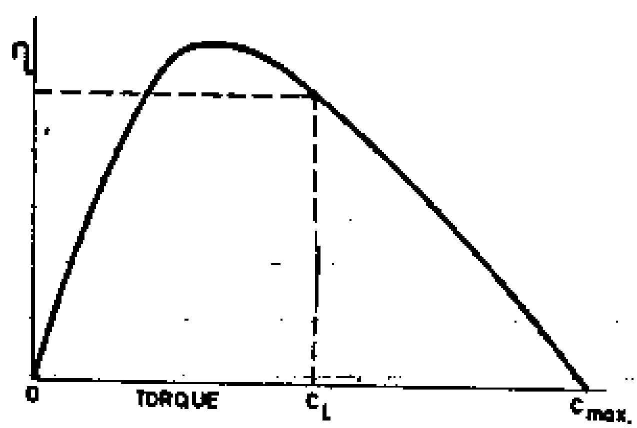

Figure 6. Typical efficiency — torque characteristic for a D.C. electric motor

Thus the order of merit of the systems again appears to be

1. Hydraulic (50%)
2. Electric (30—40%)
3. Pneumatic (25—30%)

Conclusions. The hydraulically operated system clearly offers certain advantages over the other two types of actuation considered. The high stiffness indicated by a low value of the complex delay time constant τ_2 is perhaps its most easily recognized attribute and some idea of its importance has been given by Orloff[5] who considered typical actuators for shoulder and elbow movement. Clearly the shoulder case is most significant since it imposes the greatest reflected inertial load on the actuator, and by considering a typical arm prostheses carrying a 2.5 lb load at the termination, it can be shown that the complex time constant τ_2 will be in the region of 0.5 s for a pneumatically operated system working at 100 lbf/in^2 pressure.

This is by no means negligible compared with the human operator delays and may be expected to adversely affect his performance with the prosthesis. Delays of a similar order may be expected from an elecrically operated system but a reduction in the region of $\frac{1}{30}$ might be expected from a hydraulic system.

The overall response of the system will also be dependent upon the magnitude of the integrator time constant τ_1. Clearly a serious conflict between response and efficiency will occur here but the analysis in the third section of the paper suggests that the hydraulic system offers some advantages over the electrical or pneumatic systems.

To offset the advantages in performance that could be expected from hydraulic operation, it must be pointed out that such a system would impose serious practical problems. Energy cannot be stored hydraulically and either pneumatic or electrical storage with subsequent conversion must be employed. (This disregards the possibility of chemical storage which, although very attractive from a power to weight viewpoint, does not seem practicable at present.) Pneumatic-hydraulic conversion presents fewer practical problems but does not offer the advantages of safety, good power-weight ratio, and ease of recharging that could be obtained with electrical storage if a satisfactory electrical-hydraulic converter can be realized. Leakage will also present considerable problems as will the design of valves and similar ancillary equipment.

The problems associated with the hydraulic system are, however, predominantly of an engineering nature and with the present state of the art should not be insuperable. In view of the potential advantages to be gained the authors would suggest that serious consideration should be given to the hydraulically operated system.

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