AN APPROACH TO THE ORGANIZATION OF THE ARTIFICIAL ARM CONTROL

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Summary

Control problems of the artificial arm are treated where natural neuromuscular mechanisms of the arm cannot be used.

In the first part of the paper the organization of the neuromuscular control of the human arm is described briefly in order to present a basis for the following two conclusions: communication between patient and his assistive device should be established through preserved muscle function complexes rather than through isolated muscles in order to use efficiently higher control levels of natural neuromuscular hieararchy; an assistive control should also be hierarchically organized to be efficiently included into the hierarchy of the human neuromuscular control system.

In the second part of the paper a multi-level organization of the assistive control system is presented with the following two basic control levels: level of regulators and level of artificial synergy. The regulator level provides the manipulability of the arm. The synergy level performs functional mapping of human control signals into the signals to be sent to the regulator level as the reference inputs. By means of this mapping an arm kinematic system, which is more functional than the original one, can be established that enables one to set coordination of the arm components in some basic movements. This results in reduction of the number of control signals.

As example an organization of the artificial synergy for a mechanical arm having five degrees of freedom is presented.

Introduction

The most complex problems in the design of an orthotic/prosthetic arm are encountered in cases where the patient is not able to use any neuromuscular mechanisms of his own arm. Research efforts are being made to build artificial arms able to restore manipulative abilities to the handicapped. The restoration of the natural functions is absolutely limited by the capacity of the communication channel between the patient and his arm substitute. The only possible way to establish such a channel is to use neuromuscular complexes and exteroceptive receptors normally having no function

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This work has been supported by the Social Rehabilitation Service, Washington, D. C., under Grant No. SRS-YUGO-25-68.

in the upper-extremity system. However, it imposes many principal and technical difficulties, but there exist enough possibilities to synthesize systems capable of performing at least those movements that are most important in rehabilitation. Artificial arm systems built so far have not begun to exhaust these possibilities.

The patient and his assistive device compose a unified complex system. The synthesis of such a system, to be successful, requires the solutions of many problems involving mechanical design, power sources, control, and communication.

In the first part of the paper a simplified analysis of the conctruction of movement of the natural arm is presented. The discussion of communication problems that follows is based on that analysis. The construction of normal movement may be considered as a unique guide when organization of an assistive system is conceived.

In the second part of the paper the organization of semiautomatic control is discussed. In semiautomatic control the patient produces arm movements assisted by a multi-level control system.

This approach is demonstrated by an illustrative example.

On Neuromuscular Control of the Human Arm

During the last few decades many attempts have been made to form a general model of human motor activity [2], although the neurophysiology at present does not provide a complete understanding of motor activity control mechanisms. A sketch of such model is given in Figure 1. The part of the nervous system that

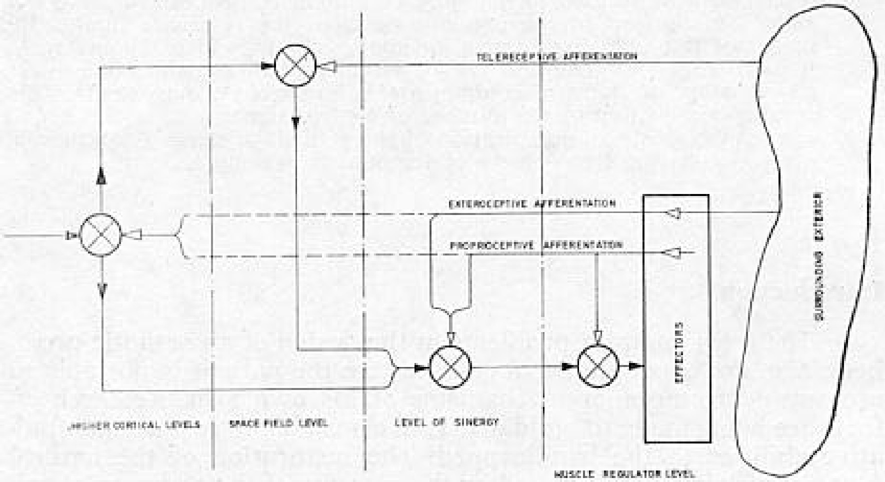

Fig. 1. Structure of the neuromuscular control mechanism

controls the activity of skeletal muscles has an extremely complex hierarchical organization. Lower levels of this structure are directly connected to skeletal muscles. The lowest level comprises some sort of muscle regulator systems. Feedback loops of this regulator are closed through the interneuron pools of spinal cord, where proprioceptive afferentation meets the efferentation of muscle motor units. The mechanism of neuromuscular tonus sets the parameter values of the muscle regulators, e. g. muscle elasticity coefficient and sensitivity of synaptic regions of interneurons, thus enabling the system to perform a specific movement.

The next higher level is the level of synergy whose anatomical substrates are subcortical brain nuclei. Feedback loops within this control level are closed through proprioceptive and exteroceptive afferentation. Information brought by the afferent system is not used here in the original, but in a refined synthetic form. In general, this information maps the body interior. The level of synergy has at its disposal a rich repertoire of automatisms that are used as components in the production of complex movements. This repertoire of automatisms is formed through an onthogenetic process where muscle efforts are minimized.

Immediately above the level of synergy is the first cortical level, the level of space field. The basic afferentation of this level is telereceptoric, prevalently visual. Within this level a synthetic image of the surrounding exterior is formed representing the first immediate model of the outside world. This model is geometrical with definite metric properties.

The higher cortical levels are hierarchically superior to the level of space field. The receptor information at these levels are highly synthesized and they appear in the form of generalized perceptions.

The production of upper extremity movements begins at the level whose position in the hierarchy is determined by the nature and complexity of the movement. Higher cortical levels lead those movements whose conceptual side is prevalent. Two most important categories of upper-extremity movements being led by the level of space field are displacement movements (approaching, grasping, and object moving) and ballistic movement. Movements that are led by the level of synergy are rare.

The leading level calls hierarchically subordinate levels to organize the movement in detail. The processes of the leading level are conscious while processes of the remaining subordinate hierarchical structure are not.

On Communication of the Man with his Artificial Arm

For the arm control purposes with visual feedback, the patient can generate either on-off or continuous signals. At least two artificial arm systems have been built with on/off control of angular velocities in joints applied [1], [12]. Experiments and applications have shown that such systems have been too slow if stable. This serious disadvantage makes the on/off velocity control mode hardly acceptable.

The only practical approach to establishing communication between the patient and his assistive device is to use the patient's available effector and receptor systems. To produce a coordinated movement of the arm it is necessary to generate control signals simultaneously. In generating these signals two alternatives exist: to use side effects of muscle activity, such as EMG signals, or to use effects of relative motion of parts of the body. An anatomical fact might decisively influence the choice of one or other alternative. The model of body interior formed on the level of synergy is philogenetically predetermined to a considerable extent. A definite anatomical dependence exists between this model and neuromuscular complexes of the body. The transfer of the synergistic mechanism from one complex to another is difficult to achieve, if possible at all. On the contrary, the transfer of higher automatisms (organized at a level superior to the level of synergy) from one complex to another is incomparably easier to achieve [3]. This means that straight-forward effects of muscle activity could be applied successfully only in restoring the original function of a residual neuromuscular complex. This is exactly the case with the "Boston" elbow [8] and Philadelphia above-elbow arm [11]. Other attempts to use EMG activity of muscles of different unrelated complexes, to produce independent control signals, were less successful by far. The reason for this is obvious, the patient had consciously to neutralize the natural synergy and organize a new one at a higher cortical level.

When the patient is nat able use any of the original neuromuscular complexes of the upper extremity, the control signals have to be generated by residual motions. Even in case of severe impairment one might find usable residual motions. Such motions are, for example, flexion-extension and elevation-depression of the shoulder with respect to the torso or motions of the head. Synergy necessary for the efficient simultaneous use of such signal sources is to be developed at a level superior to the synergy related to original motions involved. The introduction of exteroceptive feedback using local afferentation on the skin might increase the resolution of generated signals and help in forming new synergistic mechanisms. This can be achieved by the use of transducers exerting reactive pressure on the patient's skin, for example.

Let us comment also on multifunctional use of control sites of the patient. Although attractive at the first glance the technique of multifunctional sites has serious drawbacks: new sites have to be introduced for the selection of functions. The acquisition of higher synergistic skills might be hindered also.

Sources of control signals differ in the resolution that they are able to provide when generating signals. Resolution of the head movements is finer than resolution of shoulder movements. But resolution required in controlling the arm is not the same in all phases of movements. Positioning of the wrist in space has to be done with finer resolution than positioning of the hand with respect to the arm. Therefore signal sources have to match these requirements.

On Organization of Man-Artificial Arm System

Patients considered here usually have preserved the substrates of the level of synergy and the space field level. The functions of the level of synergy, however, cannot be used because there is no practical way to communicate directly with it. The optimism of some investigators concerning the possibilities of straightforward detection, decoding and evoking of neuroelectric signals is not commonly shared.

Functions of the space field level can be used through the level of synergy to control muscle complexes which are not originally involved in upper-extremity movemendts. Existing artificial arms can be classified according to the extent to which natural functions of the space field level are used.

In the Rancho Electric Arm System [1] the arm is positioned with visual feedback with the synergic activities left to the high cortical levels, where they are performed with considerable effort.

In the case of "indirect" control [5], [6] the role of space field level is limited to specifying the desired end position of the arm. The movement is performed in an open-loop mode using a programmed control. However, the limited movement repertoire and its low adaptivity make this method difficult to apply [10].

Artificial arm systems with artificial visual receptors and automatically formed environment models are in the earliest phase of development, and consideration of their applicability would be premature.

In order to organize an efficient man-artificial arm system the artificial arm control has to be implanted into the natural hierarchical structure of the arm control. Hence it is nesessary:

- to use natural level of space field to the highest possible extent;
- to introduce a level of artificial synergy;
- to couple the natural space field level with the level of artificial synergy as close as possible;
- to introduce a regulator level to replace the natural regulator level of muscles in basic functions.

Classification of Arm Movements

The human arm has an extremely rich range of movements. Functional range of movements of an artificial arm is restricted by many factors (some of them have been already mentioned). It is commonly agreed that the following movements are the most important in rehabilitation:

- — displacement of objects in the patient's front space;
- — simple work movements, such as handling switches and joy-sticks;
- — moving objects towards the face and handling them, such as in feeding.

These movements can be functionally partitioned into the following elementary movements:

- — grasping;
- — releasing;
- — arm placing in space (with or without an object grasped);
- — force application to an object.

These elementary operations belong to one of the two categories:

- — wrist placing
- — handling.

Based on this classification degrees of motion of a mechanical arm can be grouped into two characteristic groups, the first being involved in wrist placing and the second in handling.

Mechanical Arm as a Control Object

A mechanical artificial arm is considered as being built of the three concatenated levers (upper arm (U), forearm (F), and hand (H)), which are joined at the three complex arm joints (shoulder (S), elbow (E), and wrist (W)).

Kinematic abilities in hand positioning that approximate the abilities of the human arm can be attained with many arrangements of axis at the joints. As problems of prehension are out of the scope of this paper, the kinematics of the prehensive device will not be considered. In the human shoulder, elbow, and wrist one can recognize 7 degrees of freedom. In the corresponding joints the mechanical arm of the equivalent kinematics has to have seven axes of rotation. Let us denote the angular positions with respect to these by axes $x_1, x_2 \ldots, x_n$ $(n=7)$. The dynamic behaviour of the mechanical arm is fully described by the angular positions: $x_1, x_2, \ldots, x_n$ and angular velocities: $\dot{x}_1, \dot{x}_2, \ldots, \dot{x}_n$ that one can choose as generalized coordinates of the system. The mechanical arm is driven by torques $m_1, m_2, \ldots, m_n$ that actuators apply at the joints.

Presence of external forces, gravity, interaction among arm subsystems, nonlinearity of actuators, and variability of parameters during function make control of the mechanical arm complex. The human operator is not able to control the system of that dynamics by direct manipulation with the driving forces.

Multi-Level Control System of the Artificial Arm

In accordance with the concepts outlined in the previous section a hierarchically organized control system is introduced to assist the operator. Two main control levels of the system are: the regulator level and the level of artificial synergy.

The regulator level is made of regulators enabling an indirect control of the mechanical arm through some chosen reference variables [7]. In principle, angular positions, velocities or accelerations can be chosen as the variables to be regulated. Engineering psychology experiments indicate that the human operator can successfully appreciate velocities and positions while controlling an artificial arm [4]. This conclusion is not valid in the case of acceleration [9]. The simplest measuring and regulating devices are obtained if regulated variables are chosen among the generalized position and velocity coordinates which have been introduced in the preceeding section. Let us designate the regulated variables by $\tilde{x}_1$, $\tilde{x}_2, \ldots, \tilde{x}_n$. Each variable is related to a particular degree of motion, being either the generalized position or the velocity coordinate. The regulator level generates actuating torques so that variables $\tilde{x}_1$, $\tilde{x}_2$, $\ldots$, $\tilde{x}_n$ follow up the given reference variables $x_1(t)$, $x_2(t) \ldots$, $x_n(t)$.

Artificial synergy performs some of the functions normally performed by natural level of synergy. Its organisation follows the classification of movements given in the previous section.

The realization of the arm movements is mainly based on the visual feedback. The patient perceives the state of the system (positions and velocities) within the metrics of his natural space field level. If the conscious effort (engagement of higher cortical levels) is to be minimized while generating control signals, these signals should represent parameters of the system state which the patient perceives easily on his space field level. Then the level of artificial synergy has to map these signals into adequate reference inputs for the regulator level.

The level of artificial synergy should comprise algorithms for automatic coordination of movements analogous to automatisms of natural synergy. If the coordination algorithms introduce additional kinematic relations, total number of independent system variables is reduced. However, the coordination algorithms should not reduce the ability of the patient to perform the movements from the basic repertoire. The complex movements are composed of elementary operations belonging to the category of wrist placing and/or handling.

Let us designate by $u_1, u_2, \ldots, u_m$ $(m<n)$, the variables defined by control signals generated by the patient (n being the number of regulative variables). These variables are brought into correspondence with the manipulative variables representing those parameters of the arm state which the patient perceives most easily. If manipulative variables are designated by $y_1, y_2, \ldots, y_m$ $(y_i \equiv u_i)$ $(i=1, 2, \ldots, m)$ then the following general relation can be stated:

$$y_i = \varphi_i (x_1, x_2 \ldots, x_n), \; i=1, 2, \ldots, m \tag{1}$$

Kinematic relations among manipulative and regulative variables by which the number of manipulative variables is reduced, are:

$$\Phi_i (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m)=0 \; i=1, 2, \ldots, n-m \tag{2}$$

If the relations (1) and (2) are solved explicitly for regulative variables, the following expressions are obtained:

$$x_i = f_i (y_1, y_2, \ldots, y_n), \; i=1, 2, \ldots, n \tag{3}$$

Algorithms of automatic coordination (Eq. 3)) could differ from one type of movements to another. If *s* is a control variable with which the patient selects the desired type of movements and appropriate set of manipulative variables, then the algorithms of synergy level can be described by the following general expressions:

$$x_i = F_i (s, u_1, u_2, \ldots, u_m), \; i=1, 2, \ldots, n \tag{4}$$

An Organization of Artificial Synergy

An example of an organization of the artificial synergy level is presented in this section for a mechanical arm having five degrees of freedom with an axis arrangement shown in Figure 2. This arrangement is somewhat different from those that can be deduced from the human arm. Although it is not simple for realization this arrangement of axis leads to simple algorithms for needed conversion of angular coordinates. At the wrist (W) dorsal-volar flexion of the hand (H) is performed only. The forearm (F) performs its flexion-extension with respect to the upper arm (U) at the elbow joint (E). The shoulder joint (S) is composed of its four simple joints (S_1, S_2, S_3, S_4). The flexion-extension of the upper arm is put into such correspondence to the elbow flexion-extension that an independent control over the shoulder-to-wrist distance is attained. The arm rotates around this axis if rotation at the joint (S_2) is performed.

Elevation-depression and flexion-extension of the arm are performed at the remaining elbow joints (S_2) and (S_1), respectively.

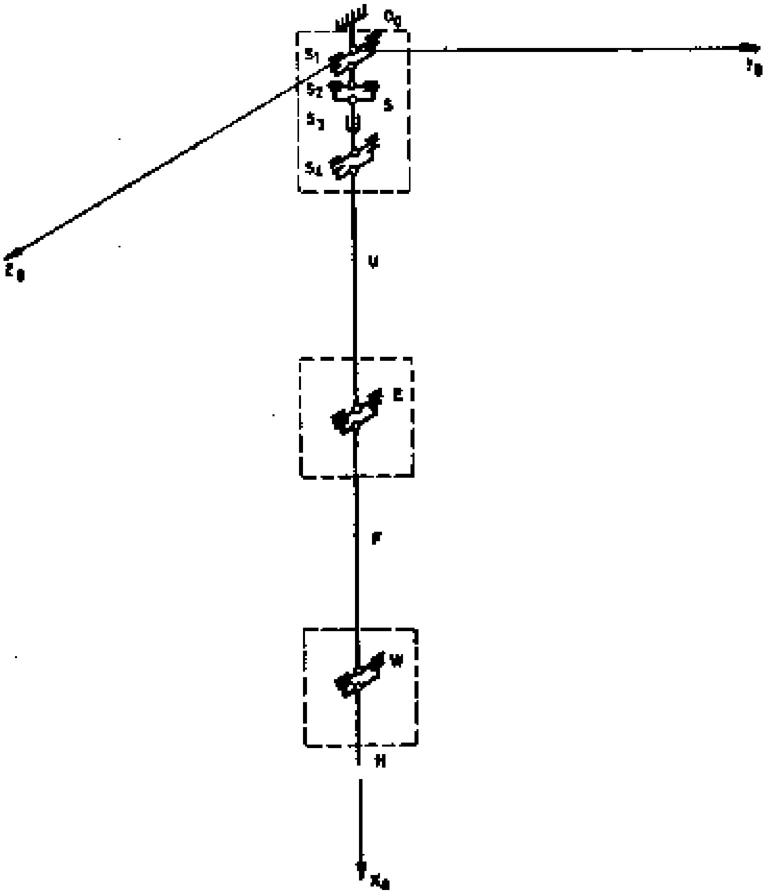

Fig. 2. Configuration of the mechanical arm

Geometry of the described arm is given in Figure 3.

Each of the three phalanxes of the shoulder joint is of the length a. The upper arm and the forearm are of the length l. The coordinate system $(Oxyz)_0$ is fixed to the body. Other coordinate systems are fixed to the arm components as shown in Fig. 3. Axes of rotation are: s_1 at the joint S_1, s_2 at S_2, s_3 at S_3, s_4 at S_4, e at E and w at W. The angular positions of the shoulder joints S_1, S_2, S_3 are denoted by γ_s, β_s and α_s, respectively. The angular positions of the elbow and wrist are denoted by γ_E and γ_W, respectively. The flexion of the upper arm in the shoulder joint S_4 is related* to the elbow

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* The relation can be achieved by mechanical means or by processing the signals sent to the regulator that controls.

flexion and equals to $-\frac{1}{2}\gamma_E$. Therefore the generalized position and velocity coordinates are: $\gamma_S, \beta_S, \alpha_S, \gamma_E, \gamma_W$ and $\dot{\gamma}_S, \dot{\beta}_S, \dot{\alpha}_S, \dot{\gamma}_E, \dot{\gamma}_W$

In arm control the attention of the human operator is concentraited on the wrist. Therefore, suitable manipulative variables to choose are those that describe the wrist velocity. The wrist placing in space is reduced in this case to tracing the wrist trajectories.

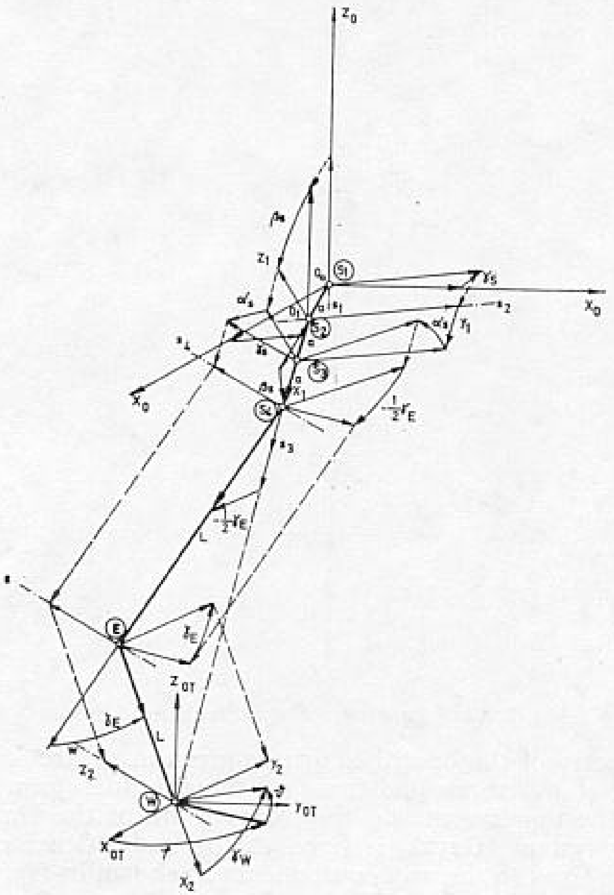

Fig. 3. Geometric description of the mechanical arm

Assume that the wrist velocity is described by its intensity v_W and two angles φ and ϑ that describe the direction of the velocity vector in respect to $(Oxyz)_0$ system. Their values v_W, φ, ϑ are assigned by means of control signals u_1, u_2, u_3, respectively.

The wrist velocity can only be attained by assigning the appropriate values to the velocity coordinates γ_S, β_S, γ_E that one can choose to form the first characteristic group of regulative variables. The algorithm C_0 for the conversion of the wrist velocity coordinates is described by the following expressions:

$$\dot{\gamma}_s = v_w \frac{\sin(\varphi-\gamma_s)\cos\vartheta}{a+\cos\beta_s\left(2a+2l\cos\frac{1}{2}\gamma_E\right)}$$

$$\dot{\beta}_s = v_w \frac{\sin\vartheta\cos\beta_s-\cos(\varphi-\gamma_s)\cos\vartheta\sin\beta_s}{2a+2l\cos\frac{1}{2}\gamma_E} \tag{5}$$

$$\dot{\gamma}_E = -\frac{v_w}{l}\frac{\sin\vartheta\sin\beta_s+\cos(\varphi-\gamma_s)\cos\vartheta\cos\beta_s}{\sin\frac{1}{2}\gamma_E}$$

Derivation of these expressions is given in Appendix A.

The hand orientation at the attained position of the wrist is obtained by assigning the values to α_s and γ_W which are chosen to form another characteristic group of regulative variables.

Synergy algorithms for coordination are based on two rules.

The first rule describes the tangential coordination of the characteristic groups of variables in wrist placing movements. The rule reads: at every point of the wrist trajectory the hand is colinear to the direction of the wrist velocity. The implementation of this rule gives the following advantages: in object reaching movements the hand is brought to the position ready for grasping; control of wrist placing in space is easier as the hand points the direction of motion; number of manipulative variables is reduced from five to three. The algorithm C_1 for the tangential coordination that maps the manipulative variables φ and ϑ into the regulative variables α_s and γ_W, is described by the following expressions:

$$\gamma_W = \arccos\left[\cos\beta_S\cos(\varphi-\gamma_S)\cos\vartheta+\sin\beta_S\sin\vartheta\right]-\frac{1}{2}\gamma_E$$

$$\alpha_S = \operatorname{arc\,tg}\frac{\sin\beta_S\cos(\varphi-\gamma_S)\cos\vartheta-\cos\beta_S\sin\vartheta}{\sin(\varphi-\gamma_s)\cos\vartheta} \tag{6}$$

These expressions are derived in Appendix B.

The second rule describes the translatory coordination in another type of movements useful in object handling. The rule reads: the hand keeps a constant initial orientation in respect to $(Oxyz)_o$ system irrespectively to wrist movements in space. This rule is useful in specific object handling as in feeding. The translatory coordination is obtained again with the algorithm C_1 with the variables φ and ϑ kept constant at their initial values $\varphi(t_0)$, $\vartheta(t_0)$.

In construction of the arm movements where the hand is kept immobile with respect to other arm components the algorithm C_o is used only, while variables α_S and γ_W are kept constant at their initial values $\alpha_S(t_0)$ and $\gamma_W(t_0)$. This type of movement is useful in some basic functions of the arm.

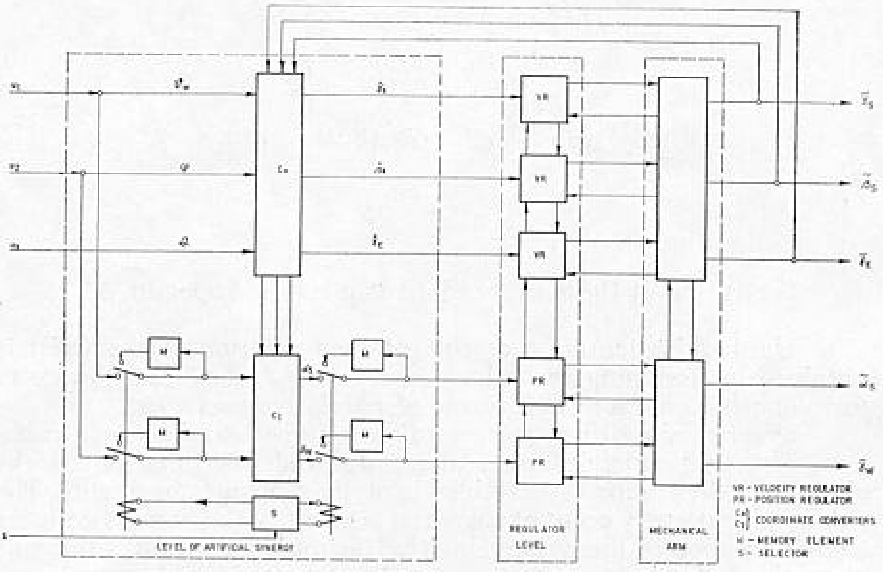

Fig. 4. Block-diagram of the artificial arm control system

The synergy level comprises the following components: angle converters (C_0 and C_1), a switching network (S) for the selection of types of movements and memory elements (M) to store φ, ϑ, α_s and γ_W when necessary. The organization of the arm entire control system is given in Figure 4 by the block diagram.

Conclusion

Hierarchical organization of the neuromuscular mechanism of the human arm is accepted as a general guide in organization of an artificial arm control. From the analysis of the construction of the human arm movements two conclusions are stated: communication between the patient and his assistive device should be established through preserved neuromuscular complexes rather than through isolated muscles in order to use efficiently higher levels of natural neuromuscular hierarchy; an assistive control should also be hierarchically organized to be included efficiently into the hierarchy of the human nervous system.

It should comprise the following control levels: regulator level and level of artificial synergy. The regulator level provides the manipulativity of the mechanical arm. The synergy level performs functional mapping of patient's control signals into the reference inputs to the level of regulators. In this manner automatic coordination in movements can be achieved. If additional kinematic relations are introduced the essential functional degrees of freedom can be established to correspond to the reduced number of patient's control signals.

The same approach, in the authors' opinion, is applicable to general problems of interface synthesis in systems comprising human operator.

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APPENDIX A

Conversion of Coordinates in Velocity Control of Wrist Placement

The wrist position in space with reference to basic system $(Oxyz)_0$, Figure 3, is described by the following coordinates:

$$x_{OW} = \cos\gamma_S \left[a + \cos\beta_S \left(2a + 2l\cos\frac{1}{2}\gamma_E\right) \right]$$

$$y_{OW} = \sin\gamma_S \left[a + \cos\beta_S \left(2a + 2l\cos\frac{1}{2}\gamma_E\right) \right]$$

$$z_{OW} = -\sin\beta_S \left(2a + \cos\frac{1}{2}\gamma_E\right) \tag{A.1}$$

Expressions for the wrist velocity are obtained by taking derivatives with respect to time of Expressions A.1:

$$\dot{x}_{ow} = -\dot{\gamma}_E l \cos\gamma_s \cos\beta_s \sin\frac{1}{2}\gamma_E - \dot{\beta}_s \cos\gamma_s \sin\beta_s \left(2a + 2l\cos\frac{1}{2}\gamma_E\right) - $$
$$- \dot{\gamma}_s \sin\gamma_s \left[a + \cos\beta_s \left(2a + 2l\cos\frac{1}{2}\gamma_E\right)\right]$$

$$\dot{y}_{ow} = -\dot{\gamma}_E l \sin\gamma_s \cos\beta_s \sin\frac{1}{2}\gamma_E - \dot{\beta}_s \sin\gamma_s \sin\beta_s \left(2a + 2l\cos\frac{1}{2}\gamma_E\right) + $$
$$+ \dot{\gamma}_s \cos\gamma_s \left[a + \cos\beta_s \left(2a + 2l\cos\frac{1}{2}\gamma_E\right)\right]$$

$$\dot{z}_{ow} = \dot{\gamma}_E l \cos\beta_s \sin\frac{1}{2}\gamma_E - \dot{\beta}_s \cos\beta \left(2a + 2l\cos\frac{1}{2}\gamma_E\right) \tag{A.2}$$

The wrist velocity with reference to the basic system $(Oxyz)_0$ is defined by its intensity v_W and direction angles φ and ϑ. Hence the velocity components are described by the following expressions:

$$\dot{x}_{OW} = v_W \cos\varphi \cos\vartheta$$

$$\dot{y}_{OW} = v_W \sin\varphi \cos\vartheta \tag{A.3}$$

$$\dot{z}_{OW} = -v_W \sin\vartheta$$

If Equations A.3 are introduced into Equations A.2 and solved for $\dot{\gamma}_S$, $\dot{\beta}_S$, and $\dot{\gamma}_E$, Equations 5 are obtained that describe the conversion of coordinates of the algorithm C_0.

APPENDIX B

Conversion of Hand Orientation Angles

The unit vector that describes the hand orientation in the basic system $(Oxyz)_0$, Figure 3, has the following components:

$$\begin{aligned} x_0 &= \cos\varphi \cos\vartheta \\ y_0 &= \sin\varphi \cos\vartheta \\ z_0 &= -\sin\vartheta \end{aligned} \tag{B.1}$$

Let us express the components of this vector in the system $(Oxyz)_1$. The system of coordinates $(Oxyz)_1$ may be considered being derived from the basic system $(Oxyz)_0$ by rotation around s_1-axis for an angle γ_S, translation along x-axis for a distance a and rotation around s_2-axis for an angle β_S. When these rotations are encountered into the conversion of coordinates (Eqs. B.1) the following expressions are obtained:

$$\begin{aligned} x_1 &= \cos\beta_s \cos(\varphi-\gamma_s) \cos\vartheta + \sin\beta_s \sin\vartheta \\ y_1 &= \sin(\varphi-\gamma_s) \cos\vartheta \\ z_1 &= \sin\beta_s \cos(\varphi-\gamma_s) \cos\vartheta - \cos\beta_s \sin\vartheta \end{aligned} \tag{B.2}$$

The components of the hand orientation vector in the system $(Oxyz)_2$ are:

$$\begin{aligned} x_2 &= \cos\gamma_W \\ y_2 &= \sin\gamma_W \\ z_2 &= 0 \end{aligned} \tag{B.3}$$

The system of coordinates $(Oxyz)_1$ may be considered derived from $(Oxyz)_2$ by translation along x-axis for a distance $-l$, then by rotation around e-axis for an angle $-\gamma_E$, translation along x-axis once again for a distance $-l$, rotation around s_4-axis once again for an angle $-\frac{1}{2}\gamma_E$, translation along x-axis for a distance $-a$, rotation around s_3-axis for angle $-\alpha_s$ and translation along x-axis once again for a distance $-a$. When these rotations are brought into the conversion of the coordinates (Eqs. B.3) other expressions for

x_1, y_1, z_1 are obtained:

$$x_1 = \cos\left(\frac{1}{2}\gamma_E + \gamma_W\right)$$

$$y_1 = \cos\alpha_S \sin\left(\frac{1}{2}\gamma_E + \gamma_w\right)$$

$$z_1 = \sin\alpha_S \sin\left(\frac{1}{2}\gamma_E + \gamma_w\right)$$

If Equations B.2 are introduced into Equations B.4 and solved for α_S and γ_W, Equations 6 are obtained that describe the conversion of coordinates of the algorithm C_1.