DISSIPATIVE PROPERTIES AND OPTIMAL CONTROL OF MANIPULATORS

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Summary

Problem of determining the programs of the optimal movements arises if the manipulator is to be controlled by a computer. The criteria of optimality may be diverse. In cases when the power supply has a limited capacity, it is worthwhile to find an optimal control which requires minimal energy consumption.

Energy consumption in performing the working operations depends essentially on kinematic peculiarities of the manipulator and its dissipative characteristics. In this paper an analysis of dissipative properties of manipulators with different drivins systems is given according to which a classification of manipulators is made. Determination of control programs for realization of simple working operations with minimal energy consumption is considered as an example.

Introduction

Within the last few years application of manipulators has become frequent and versatile. Early manipulators were stationary in general, intended for servicing rooms or boxes. Modern manipulators operating automatically are often mounted on carriages enlarging thus significantly their servicing space and the capability of solving complex tasks. Operating often in very complex and specific environments such systems are equipped with autonomous power sources of limited capacity.

In such cases the rational use of power resources is of particular importance. Thereby there arises a question whether the energy consumed by the manipulator depends on control and is it possible to choose and realize such control of the manipulator under which minimal energy is consumed for performing the desired movement.

Modern manipulators may be considered as complex mechanisms with many degrees of freedom, each of which is provided with its actuating system [1]. They are general purpose devices intended to perform versatile working operations posed by the technological process itself. These operations consist of grasping and displacing the objects within the working space and can be grouped into the following two types: constrained and free operations.

The first type consists of cases where it is necessary to realize a certain preset trajectory or to apply a certain law of motion to the object (for example, uniform turning of the handle for a certain angle). The second type includes those operations in which it is necessary to realize a desired end position of an object only or some of its phase coordinates (for example, lifting the load to a height). Duration of free operations can be unspecified also.

In any case an operator or a computer has to choose and realize those motion laws of the manipulator joints which accomplish the required operation. This choice is arbitrary to certain extent even in cases of strict operations. Namely, the grasping device can be brought to an object at many angles in order to grasp it from the relevant sides. Of course, a relevant manipulator configuration and a motion law correspond to each of these alternatives. If the manipulator possesses the maneuverability, its joints may take different positions in space even during one and the same position of the grasping device and thus the maneuvring manipulators extend to the operator even greater possibilities of selection. In the case of free operations, the operator has also to choose a law for moving the object from its initial to its final state.

The choice of motion programs, i. e. the laws of moving the manipulator joints in space, is determined by additional considerations, for example, referred to fulfilment of a required accuracy or to an economic operation. The operator or the control computer seeks for a program of motion which is optimal with respect to these additional requirements. As mentioned before, in operation requiring considerable power consumption, an economic operation may be considered important to achieve if energy resources are limited.

The energy consumed by the manipulator depends on the program of motion. Thus during a useful operation performed by the manipulator (for example, lifting the load to a required height), torques and velocities of every particular actuator are determined by the selected program of motion and in general energies consumed for accomplishment of such an operation may differ remarkably. These differences are due to the following two properties of the manipulator: existence of potential energy in the handled object (for example, when the object is lifted before handling), and multijoint nature of kinematic structure of the manipulator.

The first property results in the fact that the energy consumption is affected significantly by the characteristics of actuators to dissipate a power, the potential energy of the object, in particular. This property will be called dissipative. Power dissipation is characteristic for actuators in general but in manipulators this property is most important due to their specific operation.

Multijoint kinematic manipulator hardware leads to the fact that power dissipation in actuators during working operations is possible in above-mentioned sense. The quantity of the dissipated energy varies according to the motion programs and positions of the manipulator links in space; a choice of an optimal program can reduce these power losses to minimum.

Before proceeding to optimal control solving it is necessary to clarify peculiarities of controlling the flow of energy to the manipulator actuators and to consider influences of different factors on energy dissipation. These problems are treated in the first part of the paper. For such an analysis is was necessary to establish a novel point of view as far as the actuators are concerned and to make their classification. As an illustration of the influence that the parameters of the manipulator kinematics and dissipative characteristics of its actuators have on the optimal programs, an example of optimization of the motion in plane is considered.

Maneuverability

The manipulator has to be provided with at least seven degrees of freedom of motion for grasping and orientation of objects in space. In modern manipulators this number of degrees of freedom or a larger one is realized by means of different kinematic structures. Since the possibilities of choosing the motion programs depend on the manipulator kinematics, it is expedient to introduce a criterion characterizing the manipulator ability of being oriented in space in many different ways while performing an operation. This property will be called the maneuverability [1].

The manipulator maneuverability represents the mobility of its mechanism while its grasping device assumes a fixed position. An increase in the number of degrees of maneuverability makes it possible to realize movements of more complex classes, to decrease dead zones of the mechanism and increase operator's capabilities in performing complex technological tasks.

Kinematic organizations of some modern manipulators is given in Figure 1. The mechanical copying manipulators have seven degrees of freedom, but they loose their mobility and assume particular configurations when their grasping devices are kept fixed. For every position of the grasping device there exists a corresponding unique configuration of the manipulator in space (Fig. 1a).

Maskot manipulator [2] has also seven degrees of freedom (Fig. 1b). Two different positions of their joints correspond to a single position of its grasping device; in other words, it can realize two configurations (second configuration of the manipulator is shown by dotted lines). We shall say that Maskot has a half degree of maneuverability. It enables the operator to avoid some obstacles in the operating region. A simplified kinematic structure of the human arm is shown in Figure 1c where the hand is shown as a grasping device having a single degree of freedom. A manipulator with such kinematics can realize an infinite set of configurations that envelopes a certain volume due to which it offers more choices to the operator in performing the complex movements. In this case it

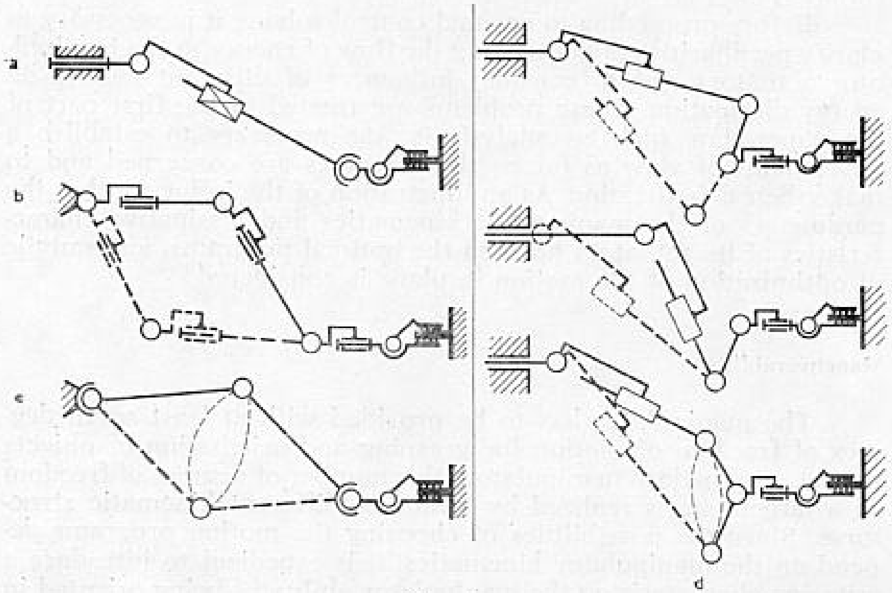

Fig. 1.

is worthy of paying attention to the peculiarity in distributing the kinematic joints of the mechanism resulting in the fact that the joints having three degrees of freedom (shoulder and wrist joint) appear to the end joints in the chain. Both possible alternatives of distribution of joints change essentially its maneuverability although without changing the total number of degress of freedom.

Manipulator "Zuk" (Beetle) [3] with nine degrees of freedom has three degrees of maneuverability. With the fixed grasping device it appears as a mechanism of the first family whose kinematic structure contains two kinematic joints of the class 1 Y and five joints of the class Y. The three degrees of maneuverability are illustrated in Figure ld. The high maneuverability of the manipulator enables an efficient operation even in cases in which other manipulators are useless. It is necessary to notice that the large number of degrees of freedom of an open-loop mechanism itself does not determine its maneuverability. It depends on how the kinematic joints are distributed.

The existence of the maneuverability results in the fact that the manipulator links can move in many different ways for the same law of movement of the grasping device with an object in space. The optimal programs of motion have to include the programs for all links and depend essentially on the maneuverability.

Dissipative Characteristics

Characteristics of actuators that will be considered in this section are associated with the control of the power flow which is the case for any actuating device not necessarily applied to manipulators.

Under dissipative characteristic we assume an inability of the actuating system to store a potential energy. Let us explain this characteristic on a biological system. By lifting a load by his arm, the man accomplishes a work while the load gets a potential energy. Now, let the man lower slowly the load: his arm muscles accomplish a work again. At the same time the potential energy of the load is dissipated as it cannot be accumulated by the muscle inherently (as distinct from a spring, for example). Similar dissipative characteristics to some extent one can find in the manipulator actuators. Let us emphasize that this section deals with properties of actuators that are manifested in those cases in which the load appears to be a source of potential energy (a lifted load, for example).

Let us define the following two typical modes of operation of the manipulator: mode of static load and mode of negative work. The latter mode is characterized by the fact that the potential energy of the load decreases. As a basis for classification of manipulator actuators, their dissipative characteristics are chosen since they are typical for the above-mentioned operations.

Actuators which consume energy from the source in the regime of static load are classified as actuators of the type A. In actuators of this type power dissipation depends directly on time. Turning back to the biological case, it is obvious that the muscle as an actuator can be classified to that type. Indeed, when holding the load in a fixed position, the man expends an energy as a function of time. An electric motor with a non-selfbraking reduction gear box, a follow-up feedback system [4], and the actuators of the Maskot manipulator [2] are few examples. Actuators of the type A will be called dissipative in time.

Let us refer all other actuators to type B. This group is divided into two subgroups according to its properties in the regime of negative work. In actuators of the type B1 there exists a power dissipation coming from the user, but this dissipation is also accompanied by a power consumption from the source. Good examples are an electric motor with a selfbraking reduction gear box designed with a big margin with respect to the selfbraking boundaries and a hydraulic actuator with throttle control and buffer. It is necessary to notice that all the known manipulator actuators of the type A possess the properties of type B1 in the regime of negative work. Therefore those properties should be considered implied when speaking of type A actuators in the text to follow.

Let us refer to the type B2 the actuators which do not dissipate power from the source in such regimes. These properties one can find for example in electric motors with reduction gear box realized close to the selfbraking boundary; in the regime of negative work relatively small amount of power is necessary to start such an actuator. Another example is a throttle controlled hydraulic actuator with buffer and an actuating piston with a spring on one side. It can be noticed that the presence of a buffer in the hydraulic actuator or a selfbraking reduction gear box in the electric motor appears as a characteristic feature of actuators having dissipative properties of the type B.

The above analysis makes it possible to distinguish several idealized types of actuators with specific amplifying and dissipative properties. Finally, a real actuator can be only approximately classified to one of the enumarated types. However, an analysis of idealized types enables one to explain the basic factors which affect the energy consumed by the manipulator.

Optimization Problem Statement

Multilink kinematic structure as well as the presence of maneuverability offer the operator an ability to select different programs of motion. Thereby different positions of the manipulator in space are obtained as well as the laws of motion of its joints, and consequently power losses. Even in the case of vertical load lifting operation which is characterized by the positive sign of the useful energy produced by the manipulator in total, certain actuators can operate in the regime of negative work and dissipation of energy will occur in them. This is caused by the fact that the actuators are connected by the manipulator links into a unified kinematic mechanism due to which certain relations are imposed onto the movements during a particular operation. This is the reason why often none program is capable of avoiding power dissipation entirely. However, optimal programs reduce them to a minimum.

Before a problem of optimal control of the manipulator is defined in general case, let us consider an example which illustrates the effect of maneuverability and the mechanism of the manifestation of the dissipative properties. Let the manipulator having a kinematic structure of the "arm" (Fig. 2) lift a load to the height H moving in a vertical plane. The regime of motion is quasistatic, i. e., at any time instant the conditions of static equilibrium are met; inertia of the manipulator components and mobility of the manipulator components and mobility of the shoulder joint are neglected. Let the actuators have the properties of the type B2. Then for each actuator the dissipation of energy can be described by the following expression:

$$E = \frac{1}{2} \int M\omega \, (\mathrm{sign}\, M\omega + 1) \, dt$$

where M is the torque developed by the actuator, and

ω is the angular velocity.

Assume that a task is stated to lift a load vertically (Fig. 2a). Then due to the only one degree of maneuverabillity two programs are completed, the manipulator takes the positions (A, B_1, C_1) and (A, B_2, C_1), respectively.

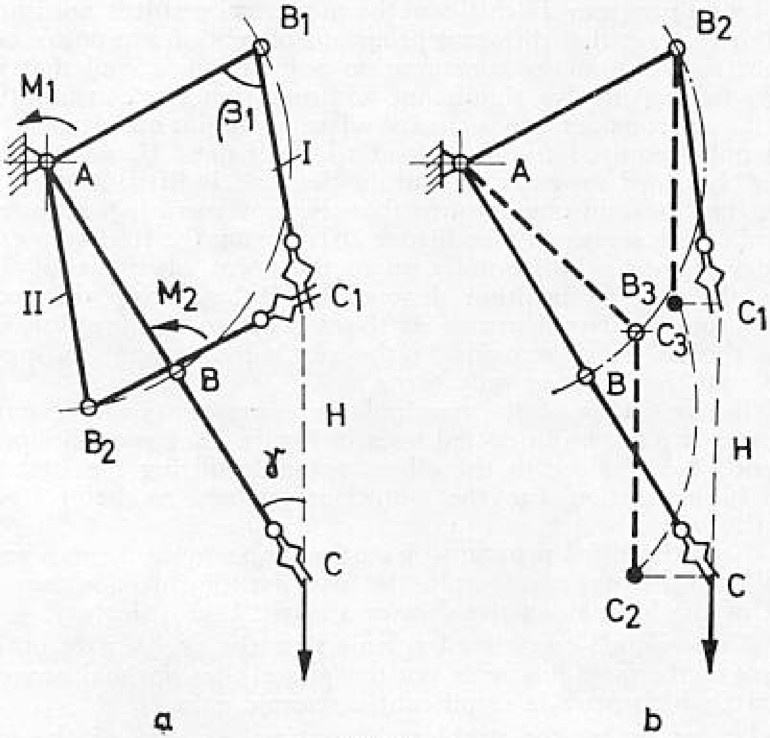

Fig. 2.

When moving under the first program, the manipulator necessarily has to lower its elbow to the angle β_1. Hence, a dissipation of energy occurs in the elbow actuator since along this part the movement is performed against the torque.

These power losses should evidently be compensated for by an additional energy consumption of the shoulder actuator as the entire useful work done by the manipulator is nonnegative and equal to GH. Thus, the energy consumption will exceed the useful work by an amount of energy dissipation. For a particular case: $\gamma=\frac{\pi}{6}, \frac{H}{R}=0.73$ and the full energy consumption is $E_1=1.43$ GH if the first program of motion is implemented.

When the second program is applied, power dissipation will appear already in the shoulder actuator, while the elbow actuator will consume an energy equal to the sum of useful work and dissipated energy. For that particular case one can evaluate: $E_2 = 1.72$ GH.

For the numerical example considered the first program of motion has appeared to be more economical although for some other initial conditions energy dissipation could be even less with the second program. In this case the numerical result is not important but the fact that different programs of motion are not of equal worth from the energy consumption point of view, and that both programs can involve significant additional energy consumptions.

Let us consider now a case of a free end-point operation where it is only required to lift a load to the height H. Assume that under the third program of motion the load is lifted along a line other than straight one. Assume that the movement is performed in the following sequence (see Figure 2b): during the first step grasping device moves horizontally up to the point when the BC lever takes the vertical position denoted by B_3C_2; during the second step a rotation is performed at the A joint to the position B_2C_3, while the BC lever remains in the vertical direction; during the third step the BC lever only turns.

The positions of the manipulator components at the end of each step are shown in dotted lines in Figure 2b. Power dissipation evidently takes place in the elbow actuator during the first step only. Its evaluation for the numerical values as before yields $E_3 = 1.275$ GH.

Thus the third program of motion appears even more economical than the first two, despite the fact that they provide the movement of the load along the shorter straight line trajectory.

The example considered reveals that the problem of optimal control of the manipulator is not trivial and that optimal programs do exist which provide significant economic gain.

Let us define the problem of optimal control of the manipulator for the general case of the free end-point operations. Let us choose the system of generalized coordinates x_i, each describing the displacement of the corresponding actuator of the manipulator. Power N_i, that is consumed from the source, is apparently function of speed and torque M_i developed by the i-th actuator on the output component. Then the energy consumption is:

$$E = \sum_{i=1}^{n} N_i \, dt = \sum_{i=1}^{n} \Phi(\dot{x}_i, M_i) \, dt \qquad (1)$$

where n describes the number of the manipulator degrees of freedom. The torques M_i are determined by the positions of some controllers or by control signals (electric motor voltage supply, valve opening, etc.) denoted by u_i and simply called controls. Further more, M_i depends on the phase state of the actuator,

$$M_i = \varphi_i \; (x_i, \; u_i) \qquad (2)$$

As far as u_i is concerned it is necessary to point out that they are inherently constrained; we shall consider that u_i assumes values from the bounded control set U.

Eliminating M_i from Eqs. 1 and 2 and introducing $x_n + 1 = x_i$ we obtain E as a function of control and phase coordinates:

$$E = \sum_{i=1}^{n} F_i \; (u_i, \; x_{n+i}) \; dt \qquad (3)$$

As it follows from the Section 3 and the above example, functions $\frac{\partial F}{\partial x}$ can have discontinuities if the actuators possess the properties of the type B. However, in many cases it is permissible to approximate F_i by some smooth functions having continuous derivatives with respect to x, within the phase space X, particularly because the dissipative characteristics of the type B are idealised to some extent, since they can be achieved only approximately in realistic actuators. In the text that follows such approximation is assumed.

Therefore the task of the optimal control is to determine u(t) which the functional (3) assumes its minimum value, at the constraints imposed by the system of nonlinear differential equations describing the movement of the manipulator and the object. In the vector form these equations can be written as follows:

$$\dot{x} = f \; (x, \; u) \qquad (4)$$

where x is a 2n-dimensional vector of the phase space X. Control vector u(t) is to be chosen from the class of the stepwise continuous functions and for any t there exists a bounded control set U. The functions F and f together with their partial derivatives with respect to the components of the vector x exist and they are continuous in direct product U×X. In the case of a free-end point operation the initial value x(0) and the end value x(T) of the system state are given while time T is not fixed.

In this manner the problem of optimal control of the manipulator is reduced to a two-point boundary optimization problem with control being constrained. This problem can be solved at least in principle, by means of the well known optimization methods, for example, by the dynamic programming or the maximum principle. However, it appears extremely difficult due to the large dimensionality of the system.

The major difficulty in applying the maximum principle [5] consists in evaluation of the initial values for the variables of the adjoint system:

$$\dot{\psi}_i = \sum_{j=1}^{n} \frac{\partial F_j}{\partial x_i}$$

This procedure can be facilitated if the following method is applied. Assume that the solution of the variational problem with fixed boundary values is known for the set of Eqs. 3 and 4 and a functional

$$J_1 = \int \Phi\ (x, u)\ dt$$

where Φ is continuous and differentiable with respect to the variables of the vector x. Then we shall solve the problem for the set of Eqs. 4 and the following functional:

$$J = (1-k) \int \Phi\ (x, u)\ dt + kE$$

As already stated, if $k=0$ the solution of that problem is known and as a consequence the initial conditions of Ψ_i are known, too. Due to the well known theorem on continuous dependence of the solutions of differential equations on their right hand sides, for a small increment of k the initial values of Ψ_i increase slightly, too, yielding the solution of the boundary value problem. It follows that $\Psi_i(0)$ for $k=\delta k$ obtain values cloce to $\Psi_i(0)$ for $k=0$ and the procedure of choosing the initial values for $k=\delta k$ is considerably simpler. The entire computation procedure consists of sequential solving a sequence of problems for increasing values of the parameter k up to $k=1$. In the latter case the solution of the problem (3, 4) we are interested in, is obtained. During this procedure the extremum appears to be deformed in the phase space due to which this method would be called the method of deformations.

Conclusion

The optimal control in its basic sense is important in the automatic control of the manipulators by means of computers. In that case, precomputed optimal programs of simple movements have to be stored into the computer memory constituting a set of basic movements out of which more complex movements are synthesized. As shown, the realisation of such basic movements can lead to a significant economic gain. Analysis of optimal control may also appear, worthy for the synthesis of basic novements of the human operator.

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