VOLUMETRIC CRITERIA IN SERVICE ANALYSIS OF MANIPULATORS

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Summary

A volumetric method of kinematic analysis of manipulator systems is presented. It enables one to get integral performance criteria of manipulator kinematic properties. One of such criteria is the coefficient of service that characterizes some manipulator properties related to a set of trajectories.

1. Manipulators are devices whose purpose is to reproduce mechanically certain manipulative functions of the human arm. They found their wide application in many fields of science and technology. Upper-extremity prostheses are special manipulators intended to restore functions the human has lost due to amputation. Modern systems with myoelectrical control give the patient an ability to produce complex movements, to control the gripping force refinely and to obtain precise information about it. In other words, they make it possible to realize complex control systems similar to those applied in manually controlled industrial manipulators.

The characteristics caused by the fact that control signals applied to prosthesis appear in myoelectric code instead of the motion codes as with the conventional manipulators, become the more essential the more complex the functions to be reproduced are. These characteristics, however, will not be discussed here.

The following three problems are fundamental in the theory of manipulators [1]:

(1) design of fast response actuators (to be applied in upper-extremity prostheses, i. e. "mechanical arms") which are simple to control and also provided with load sensors or sensors of other external effects; (2) design of systems for transmission and processing of control signals that are delivered by reference systems (control arm of the manipulator or myoelectric control sites); (3) problem of man — manipulator (patient — prosthesis) interfacing.

These three problems are significant even in the theory of robots, however, with differences due to constraints of the control computer as compared to the human operator.

Basic features of manipulators are determined by the complexity of their arms which can be considered as open multistage kinematic systems. They have many degrees of freedom, due to which construction of purposefull movements and their kinematic and dynamic analysis are difficult. Modern methods of the theory of mechanisms make it possible to analyze the manipulator kinematics when motion laws of links of the control arm are given. However, an ordinary approach, based on the analysis of particular trajectories of motion does not give an insight into general kinematic properties of manipulators. Hence, it is worthwhile to apply a method simultaneously with the trajectory one which enables one to evaluate general properties of manipulators that characterize certain features in processing a set of trajectories. The goal of the paper is to introduce this method which will be named the volumetric method.

2. The following notation will be used throughout the text that follows (Fig. 1): S, E, W — shoulder, elbow, wrist — flexible joints by which different kinematic structures can be constructed; l_1, l_2, l_3 — lengths of upper arm, forearm and hand, respectively, where l_3 describes the distance between the wrist joint (W) and the joint (P) of prehension. Other notations are apparent from the figure.

By the working space of a manipulator we shall define the three-dimensional space of its configurations, i. e. the space bounded by a surface that encloses the entire set of prehension points. The boundary radius of the working sphere is determined by the sum $l_1+l_2+l_3$.

If the arm length is assumed constant, an effective working-space of the manipulator is determined by the design constraints imposed on lengths of its links and rotations in its flexible joints. Difficulties encountered when solving the problems of optimal arm "partitioning" and "distributing" of inherent constraints among particular flexible joints become more serious due to the following less evident circumstances.

The operating part of the manipulator — its prehensive device, can reach any point of the working space, although functional kinematic capabilities of the mechanism appear to be different at different points of this space. Thus, for example, at the space boundary the only one manipulative degree of freedom of the prehensive device is its rotation with respect to the longitudinal axis of the arm. Within the working space the prehension capabilities are much richer. Therefore, certain features characterizing the manipulator capabilities to realize different operations are related to each point of the particular manipulator.

In this manner, by solving the problems pertaining to the arm partitioning and distribution of the rotation constraints to particular joints, not only the working space but the functional properties of the system within this space are determined.

Though the analysis of these properties does not submit detailed data on the manipulator ability to realize particular operations, it enables one to estimate its general kinematic properties. One of the most important properties enabling one to estimate kinematic abilities of the manipulator, when performing the working movements will in this paper be named the manipulator "service" [2]

3. During the operation of the manipulator it is necessary to possess the ability of directing and placing its prehensive device with respect to the manipulated object in many different ways. However, structural and design constraints do not permit the bringing of the prehensive device to any point of the working space at any desired angle. At each point of the space it is possible to determine a certain solid angle Ψ within which the prehensive device can be brought to that point. This angle is called the solid operating angle or angle of service.

Let us call the ratio $\theta = \Psi/4\pi$ the coefficient of service at a set point of the working space.

Thus, for any manipulator it is possible to determine a certain scalar field over its working space by means of bringing a set of coefficient values of service to correspond to a set of working space points. The coefficient values can vary from zero, for boundary points of the working space, to unity, for points enabling full service and located within the so-called 100% or full service zone.

Applying the concept of the coefficient of service it seems purposeful and practically justified to characterize the overall performance of the manipulator by the service coefficient mean value.

$$\theta = \frac{1}{V} \int_{(V)} \theta \, dV$$

evaluated over the entire working space V, or over some particular regions of that space, which are of prior interest to that or other groups of working operations. We shall call these values the servicing characteristics or manipulator "service".

In evaluating the coefficient of service at a given point C of the space let us assume that the manipulator has gripped by its jaws an infinitesimally small object at that point. Let us assume also that the manipulator arm moves in space in such a manner that its prehensive device rotates around the fixed point C. Thus the connection between the object and the prehensive device is analogous to the connection imposed by a ball joint at that point between the prehensive device and the fixed support. As a result, a spatial mechanism is obtained. Its structure and size determine completely the angle Ψ, within the boundaries of which the prehensive device can be placed and turned, and consequently, the value of the service coefficient θ at point C of the manipulator working space.

Thus, the task of determining the service coefficients θ can be reduced to the analysis of a set of spatial mechanisms with an arbitrary number of degrees of freedom. The number of entities forming this set is determined by the number of points for which the service coefficients are evaluated.

4. Design constraints imposed on the angles of rotation of the manipulator subsystems at all kinematic joints, influence significantly the kinematic properties of the manipulator and consequently, its service. This is shown by a diagram in Figure 2 where the boundary angles of rotation at the *i*-th kinematic joint are denoted by φ_i^*. Subscripts to the letters S, E, W in italic notation denote the number of degrees of freedom of the corresponding joints. The structure of the manipulator presented in this paper is analogous to the structure and relations among the essential parts of the human arm.

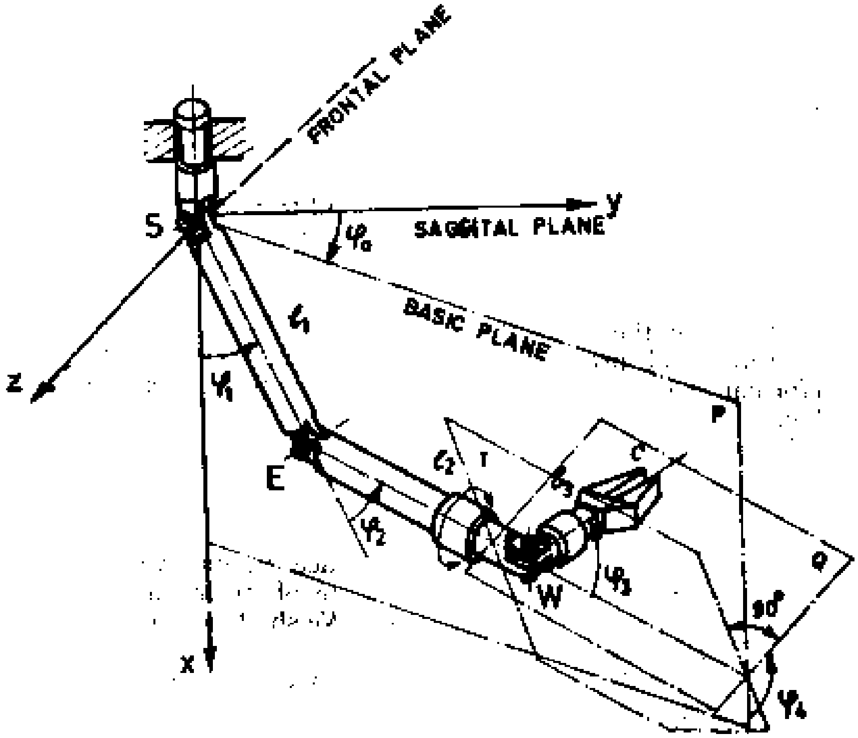

Fig. 1.

When performing operations with a fixed object located in space at the point C, kinematic capabilities that characterize manipulator properties at that point depend on dimensions of its subsystems, having constraints imposed on the kinematic joints, and on parameter R which characterizes the location of point C in space

$$\bar{\theta}=\bar{\theta}\,(l_i, \varphi_i^*, R)$$

To each value of the parameter R corresponds a certain configuration of the mechanism and a particular value of the coefficient of service.

For illustration, Figure 3 shows a kinematic diagram of the mechanism for the case when its subsystems are placed in a so-called basic plane (see Fig. 1). The process of an arbitrary reorientation of the prehensive device can now be considered as the rotation of joint W with respect to the fixed point C, so that all trajectories of the point W are on the sphere l_3. Thereby, the parameter R represents the support length SC of the four-link spatial mechanism. By changing the location of point C along the x-axis of the basic plane, one can evaluate a series of values of the coefficients of service. Displacement of the point C along the x-axis towards the shoulder, as well as away from it, is constrained by the two boundary values corresponding to two extreme positions of the mechanism. In one of them, the prehensive device takes the farthest position from the shoulder; then point C takes the position on the boundary of the manipulator working space ($R_{max}=l_1+l_2+l_3$). The second extreme position of the mechanism defines the inner boundary of the working space. Figure 3 shows a diagram which does not include the actual constraints imposed on the relevant rotations of the joints (see Fig. 2). These constraints, of course, have an essential influence on the domain of feasible configurations of the mechanism.

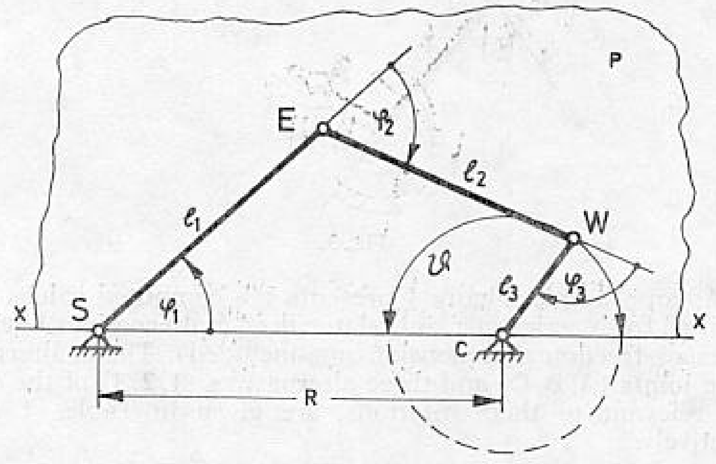

Fig. 2.

By turning the basic plane around the x-axis it is possible to determine successively the coefficients of service for any point of the space, obtaining thus the required servicing characteristic θ of the given manipulator, i. e. its service.

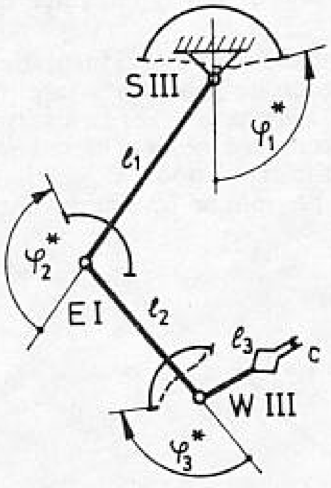

Fig. 3.

As an example, Figure 1 presents the computed values of the quantity θ for a series of manipulator design alternatives having six degrees of freedom (prehension not included). Three alternatives for the joints (A, B, C) and three alternatives (1, 2, 3) of the constraints, relevant to their rotations, are given in Tables 1 and 2, respectively.

Table 1

	A	B	C
l_1	98	88	108
l_2	83	93	73
l_3	25.5	25.5	25.5

Computations for all alternatives have been performed with a uniformly distributed net of points covering the entire manipulator working space. It is evident that in the case when it is necessary to evaluate nonlinear properties of the manipulator more accurately, the net of locations of point C of the prehensive device can be chosen ununiformly, having it condensed, for example, in those regions of the working space which are found preferable when performing the working operations.

Table 2

	1	2	3
φ_1^*	$+45_{-90}$	$+45_{-90}$	$+90_{-90}$
φ_2^*	$+135_0$	$+135_0$	$+90_{-90}$
φ_3^*	$+90_{-90}$	$+150_{-30}$	$+90_{-90}$
φ_4^*	$+90_{-90}$	$+90_{-90}$	$+90_{-90}$

Coefficients of service θ calculated for all alternatives are given in Table 3.

Table 3

	1	2	3
A	0.569	0.701	0.403
B	0.498	0.675	0 374
C	0.628	0 726	0.414

The calculations carried out show that the alternative 2C is the most preferable with respect to the service charaterisitics. Obviously, it is not difficult to make a program aiming to determine more precisely the dimensions of components and motion boundaries within the regions permissible by design solutions with the purpose of determining an optimal design alternative. In this case, there are many reasons to presume that the optimal alternative, with respect to the coefficient of service, will have, in average, better functional properties in comparison with other design alternatives.

As illustrated in Table 3, it is possible to deal with an essential improvement of these properties hardly obtainable by other methods.

The volumetric method makes possible the determination, together with the coefficient of service, a number of particular characteristics that reflect other interesting properties of the manipulator, as for example:

a) ratio of the number of net points in the working space, in which the prehensive device can be oriented along the positive direction of x-axis, to the total number of points in the net;

b) ratio of the number of points in which the prehensive device can be oriented along the positive direction of z-axis, to the total number of points of the net, etc.

Interest in particular characteristics and regions of the working space is raised by the functional purposes of the manipulator and its exploitation conditions. At the same time, the service coefficient, as well as any particular characteristics obtained by means of the volumetric method, give a distinctive picture of general properties of the hardware.

5. Application of the volumetric method is not only restricted to the analysis of kinematic properties of the mechanical hardware of the manipulator. By means of this method it is possible to construct the coefficients of service of the manipulator control arm. This method enables one to estimate roughly the characteristics of the human operator which are analogous to the coefficient of service (of course, with respect to large arm parts only). A comparison of the manipulator could probably make it possible to derive a number of important conclusions concerning the compatibility of the biological and technical properties of the operator — manipulator system. Comparisons can concern, for example, size and configurations of the working space, their particular regions, etc.

By comparing the corresponding characteristic of the healthy arm with the arm prosthesis, it is possible to evaluate, in the first approximation, the extent to which the manipulative functions lost due to amputation, etc., are restored.

The volumetric method can also be efficiently applied in estimating the precision properties of manipulators. The point is, that the facts presented above are related to an ideal manipulator composed of absolutely rigid joints and components without back-lash effects, provided with drivers having absolutely stiff characteristics. Real designs do not have such properties. As a result, the working spaces and service characteristics of control and executive arms appear to be inadequate. In fact, the working space of the executive arm and its particular regions are "deformed" as compared to their estimated configurations. In case of the copying manipulator, the operator during the control process performs additional movements compensating discrepancies due to influences of the mentioned factors. These compensating actions appear only as a part of the procedure of movement control performed by the operator, and cannot be separated from its remaining parts. However, for robots the impact of their global characteristics, concerning their accuracies, is far more interesting. Deformations of the working spaces and their particular regions, possible configuration degenerations in certain singular positions of the mechanism. Those, for example, close to the extreme ones, can essentially effect the functional abilities of the manipulator and therefore must be taken into account in programming.

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