LEGGED LOCOMOTION STUDIES

ON THE STABILITY OF BIPED LOCOMOTION

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Summary

The stability of legged machines in locomotion is considered. The general machine has a rigid body to which legs are attached. Locomotion is performed on level smooth surfaces.

The construction of machines is discussed. Types of actuators and their relation to overall machine stability are presented. Control schemes for locomotion of general systems are discussed.

The control and stability properties of a simplified dynamic system are presented. Concepts such as repeatability and cyclicity are introduced for this model.

The general stability of all such machines is separated into three areas: 1) stability of the bodies orientation with respect to earth, 2) stability of the bodies trajectory or path, and 3) the stability of the gait or manner of moving the legs.

The control laws for a full dynamic system with 6 degrees of freedom are presented. Their stability properties are studied by simulation with the introduction of disturbances.

The results of disturbances on such a dynamic system are presented with respect to the type of stability introduced.

Basic Concepts

The machine considered for stability consists of a rigid body to which a pair of "legs" is attached. The legs supply the driving and supporting forces to a "rigid" body. Note that the word "body" refers to that portion of the machine which is to be transmitted from one place to another. The work "rigid" implies that the body's orientation with respect to the various driving elements is geometrically fixed.

The motion is to take place, in general, over a smooth horizontal plane. No resistance to motion from effects such as viscous drag from the media will be considered.

The basic machine and environment are specified. Consider now a more detailed construction of the machine. It is clear that if the body is to be supported by legs of the type and shape classically observed in nature, the joints of the leg must have some form of regulation. There are two ways to consider the regulation:

1) the existence of a force or torque actuator in which a desired force or torque is generated, and

2) the existence of a "position" actuator where the position of each joint is commanded by some predetermined algorithm.

The word "position" means the state of a characteristic coordinate of the system. These two regulation schemes can yield vastly different results when considering stability. Careful definition of the machine must be made to ensure meaningful results.

To study stability, the machine with its regulation must be modelled mathematically. The stability of this model is then to be analyzed. The model complexity is dependent upon the construction, regulation and mode of operation of the machine. For example, if the machine is constructed with a body which can be described by a series of point masses, the number of equations necessary to describe its motion can be greatly reduced by neglecting rotational motion and the resulting moment of inertia. This model is good for low velocities where these approximations hold.

It is obvious as velocity is increased beyond a certain point in any system ,it no longer becomes possible to make such approximations and thus a more general representation must be used.

This type of machine must have control systems in order to function. Two types of controls were mentioned: a force controller and a position controller. The difference is that the force controller supplies a force independent of the position of the controlled element or, in this case, the angle position of a joint, and the position controller holds a position independent of the force applied to the controlled element. If only position controllers are used to command the angles of the legs, the angles must be generated by a program. Such a system has been termed an *algorithmic system* [1]. The stability of such a machine can be considered differently and will not be discussed here. If an actuator is of the force type then it has some force impedance. That is, it can resist a disturbance force independent of its position. The control command to these actuators would be a force or torque commanded by some control law or scheme.

The most general machine, however, has a mixture of the two types of actuators. That means, some actuators are required for precision positioning and others are required to supply forces or torques. Such are the types of systems, along with the physical construction and environment listed above to be studied for stability.

Machine motions to be considered here are those which occur while external force disturbances are present. These disturbances may include parameter variations in a finite time period. Examples of the action of the longitudinal forces on the machine will be considered during shortening or lengthening of the leg, corresponding to stepping on a rock or into a hole respectively.

Since the nature of the problem is cyclic, that is, certain characteristic coordinates tend to repeat in general, these disturbances can be considered variations in the system's state at the beginning of a cycle. This is possible since a perturbation of parameters during a particular cycle results in a difference in the state at the end of this cycle. This, then can be considered a variation in initial conditions of the differential equations describing the motion of the machine.

To correct these variations some control system must exist. One example of such a device is to design a system only to correct differences in the initial conditions at the beginning of each new cycle. Such a control scheme will be illustrated. Other control schemes using continuous information feedback will also be discussed. These control schemes, of course, are used in combination with force and position actuators.

The classical theorems of the stability of dynamic systems [2] are well known. By all means the appraisal of stability and the locomotion systems is subject to these general definitions. It is proposed here, however, that systems possess certain properties in this sense and this necessitates the introduction of three categories of stability for the general analysis of stability of the locomotion systems:

a) *body stability*
b) *body path stability*
c) *gait stability.*

All these categories will be discussed later. The first two types of stability have been classically studied in aircraft and missile systems [3].

There are many possible ways to represent a machine mathematically. Two models representing different gaits will be analyzed. The above concepts will be applied to these models and the qualities of "stability" in the manner defined will be studied. Simulation will be used to obtain results.

Stability of a Simplified System

Suppose a system consisting of point masses supported by a set of legs which are position commanded by an algorithm is considered for stability. Suppose the leg algorithms are such that the motion of the body is completely described. Suppose a gait is chosen such that the two feet remain continuously on the ground and the legs are advanced by sliding one foot forward at a time*

* This has been assumed due to the maximally simple relations between the geometrical parameters of structure. However, such a simplification does not affect the generality of the considered concept.

(Figure (AI.1). In order to have this system dynamically satisfied, a compensating mass must be used.

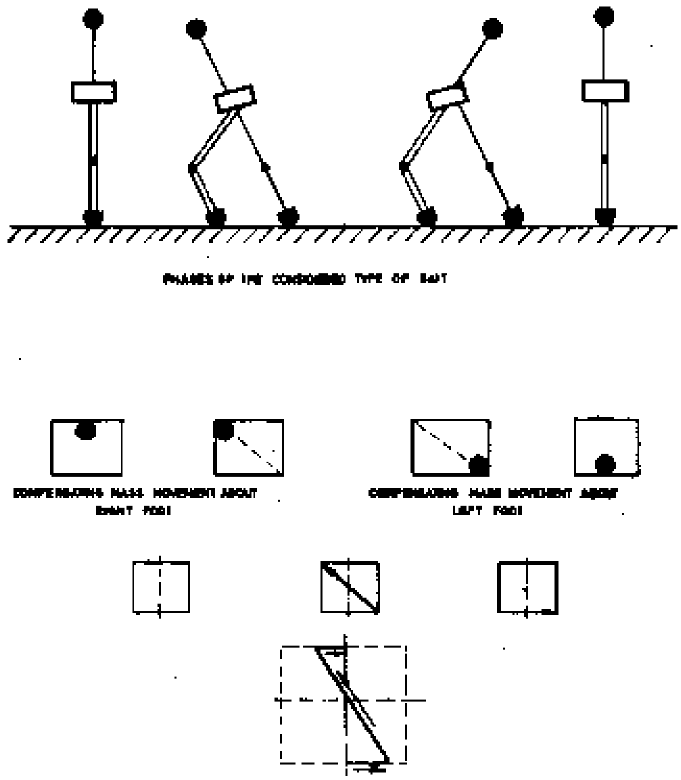


Fig. AI.1. Movement of compensating mass for one step

Based on these asumptions, the mechanical problem is reduced to a dynamic system whose number of degrees of freedom coincides with the number of coordinates characterizing the motion of the compensating mass in space. These are angles in two mutually normal planes in body coordinates (Figure AI.2).

The mathematical model of motion in the form of two nonlinear differential equations with time-variable coefficients is presented in Appendix I. The only connection between the kinematic relations (of the assumed algorithm) and the dynamics of the compensating system, is the angle α. The assumed law of variation in this parameter (system of excitation) is given by

$$\alpha = \frac{\alpha_s}{2}(1-\cos\omega t) \qquad (1)$$

where ω=const.

Notice from Figure AI.3 that a "stationary" gait is determined by this algorithm [4].

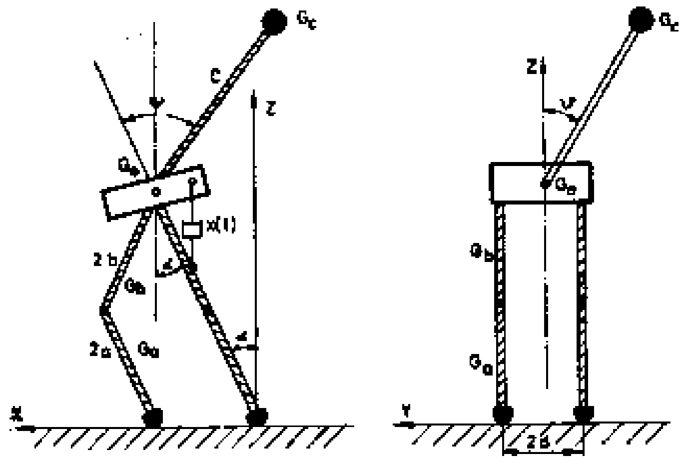


Fig. AI.2. Simplified mechanical model of a biped

If the differential equations are satisfied and a stationary gait exists then there must exist mathematical *conditions of repeatability* for equations AI.1, 2. These are:

$$\begin{aligned} \bar{\psi}_O &= \bar{\psi}_T \\ \vartheta_O &= -\vartheta_T \\ \dot{\bar{\psi}}_O &= \dot{\bar{\psi}}_T \\ \dot{\vartheta}_O &= -\dot{\vartheta}_T \end{aligned} \tag{2}$$

The solution of the basic system (AI.1, 2) with the imposed features of repeatability has been suggested on the basis of variations in the boundary conditions (2):

$$\begin{aligned} \vartheta_0+\Delta\vartheta_0 &= -(\vartheta_T+\Delta\vartheta_T) \\ \bar{\Psi}_0+\Delta\bar{\Psi}_0 &= \bar{\Psi}_T+\Delta\bar{\Psi}_T \\ \dot{\vartheta}_0+\Delta\dot{\vartheta}_0 &= -(\dot{\vartheta}_T+\Delta\dot{\vartheta}_T) \\ \dot{\bar{\Psi}}_0+\Delta\dot{\bar{\Psi}}_0 &= \dot{\bar{\Psi}}_T+\Delta\dot{\bar{\Psi}}_T \end{aligned} \tag{3}$$

Since each of the increments $\Delta\vartheta_T$, $\Delta\bar{\Psi}_T$, $\Delta\dot{\vartheta}_T$, and $\Delta\dot{\bar{\Psi}}_T$ depends on the disturbances in the initial conditions: $\Delta\vartheta_0$, $\Delta\bar{\Psi}_0$, $\Delta\dot{\vartheta}_0$, and $\Delta\dot{\bar{\Psi}}_0$ one obtains the following relations

$$\frac{\Delta_\vartheta \vartheta_T}{\Delta \vartheta_0}\Delta \vartheta_0 + \frac{\Delta_\psi \vartheta_T}{\Delta \psi_0}\Delta \bar{\psi}_0 + \frac{\Delta_{\dot\vartheta} \vartheta_T}{\Delta \dot\vartheta_0}\Delta \dot\vartheta_0 + \frac{\Delta_{\dot\psi} \vartheta_T}{\Delta \dot\psi_0}\Delta \dot{\bar{\psi}}^0 = \Delta \vartheta_T$$

$$\frac{\Delta_\vartheta \bar\psi_T}{\Delta \vartheta_0}\Delta \vartheta_0 + \frac{\Delta_\psi \bar\psi_T}{\Delta \psi_0}\Delta \bar{\psi}_0 + \frac{\Delta_{\dot\vartheta} \bar\psi_T}{\Delta \dot\vartheta_0}\Delta \dot\vartheta_0 + \frac{\Delta_{\dot\psi} \bar\psi_T}{\Delta \psi_0}\Delta \dot{\bar{\psi}}_0 = \Delta \bar\psi_T$$

$$\frac{\Delta_\vartheta \dot\vartheta_T}{\Delta \vartheta_0}\Delta \vartheta^0 + \frac{\Delta_\psi \dot\vartheta_T}{\Delta \bar\psi_0}\Delta \bar{\psi}_0 + \frac{\Delta_{\dot\vartheta} \dot\vartheta_T}{\Delta \dot\vartheta_0}\Delta \dot\vartheta_0 + \frac{\Delta_{\dot\psi} \dot\vartheta_T}{\Delta \psi_0}\Delta \dot{\bar{\psi}}_0 = \Delta \dot\vartheta_T$$

$$\frac{\Delta_\vartheta \dot{\bar\psi}_T}{\Delta \vartheta_0}\Delta \vartheta_0 + \frac{\Delta_\psi \dot{\bar\psi}_T}{\Delta \dot\psi_0}\Delta \bar{\psi}_0 + \frac{\Delta_{\dot\vartheta} \dot{\bar\psi}_T}{\Delta \dot\vartheta_0}\Delta \dot\vartheta_0 + \frac{\Delta_{\dot\psi} \dot{\bar\psi}_T}{\Delta \dot{\bar\psi}_0}\Delta \dot{\bar{\psi}}_0 = \Delta \dot{\bar\psi}_T \quad (4)$$

By substituting $\Delta\vartheta_T$, $\Delta\dot\vartheta_T$, $\Delta\bar\Psi_T$, and $\Delta\dot{\bar\Psi}_T$ from (4) into the condition (3), the following matrix equation is obtained

$$[A]\{\Delta q_0\} = \{q_{T,0}\} \quad (5)$$

where the sensitivity matrix for finite variations in the initial conditions is given by

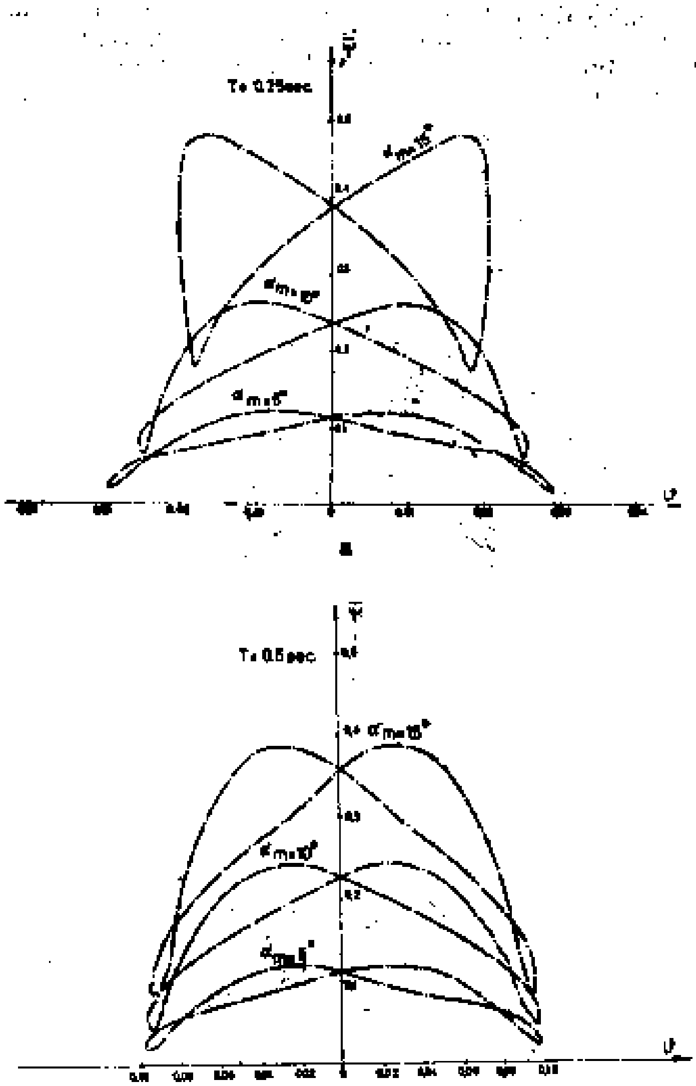
$$A = \begin{bmatrix} \frac{\Delta_\vartheta \vartheta_T}{\Delta \vartheta_0}+1 & \frac{\Delta_\psi \vartheta_T}{\Delta \bar\psi^0} & \frac{\Delta_{\dot\vartheta} \vartheta_T}{\Delta \dot\vartheta_0} & \frac{\Delta_{\dot\psi} \vartheta_T}{\Delta \dot{\bar\psi}_0} \\ \frac{\Delta_\vartheta \bar\psi_T}{\Delta \vartheta_0} & \frac{\Delta_\psi \bar\psi}{\Delta \bar\psi_0}-1 & \frac{\Delta_{\dot\vartheta} \bar\psi_T}{\Delta \dot\vartheta_0} & \frac{\Delta_{\dot\psi} \bar\psi_T}{\Delta \dot{\bar\psi}_0} \\ \frac{\Delta_\vartheta \dot\vartheta_T}{\Delta \vartheta_0} & \frac{\Delta_\psi \dot\vartheta_T}{\Delta \bar\psi_0} & \frac{\Delta_{\dot\vartheta} \dot\vartheta_T}{\Delta \dot\vartheta_0}+1 & \frac{\Delta_{\dot\psi} \dot\vartheta_T}{\Delta \dot{\bar\psi}_0} \\ \frac{\Delta_\vartheta \dot{\bar\psi}_T}{\Delta \vartheta_0} & \frac{\Delta_\psi \dot{\bar\psi}_T}{\Delta \bar\psi_0} & \frac{\Delta_{\dot\vartheta} \dot{\bar\psi}_T}{\Delta \dot\vartheta_0} & \frac{\Delta_{\dot\psi} \dot{\bar\psi}_T}{\Delta \dot{\bar\psi}_0}-1 \end{bmatrix}$$

$$\Delta q_0 = \begin{Bmatrix} \Delta \vartheta_0 \\ \Delta \bar\psi_0 \\ \Delta \dot\vartheta_0 \\ \Delta \dot{\bar\psi}_0 \end{Bmatrix} \qquad q_{T,0} = \begin{Bmatrix} -(\vartheta_T+\vartheta_0) \\ -(\bar\psi_T-\bar\psi_0) \\ -\ \dot\vartheta_T+\dot\vartheta_0) \\ -(\dot{\bar\psi}_T-\dot{\bar\psi}_0) \end{Bmatrix}$$

where T designates the period of one half of the step (Figure AI.2).

Thus, the problem of obtaining the solution of system (AI.1, 2) with repeatability properties is reduced to solving this basic motion system and the sensitivity equations above (5) simultaneously.

The solutions for the motion of systems with an imposed regime of repeatability leads to closed and symmetric trajectories of the compensating mass in the plane of the characteristic coordinates Ψ and ϑ (Fig. AI.3). The trajectory closing, that is, the realiz-

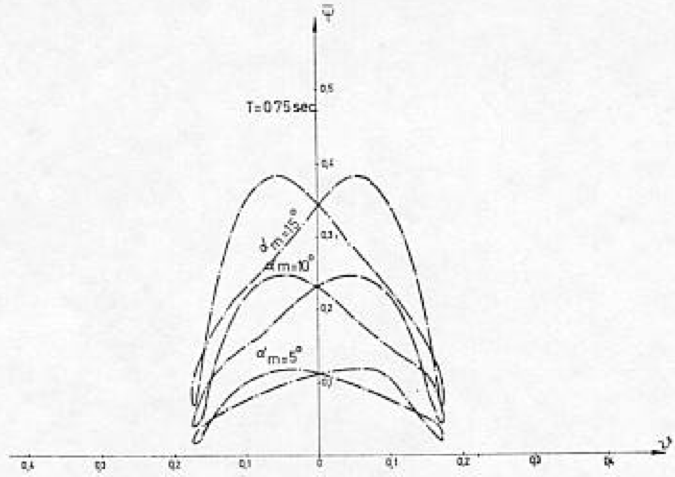


b

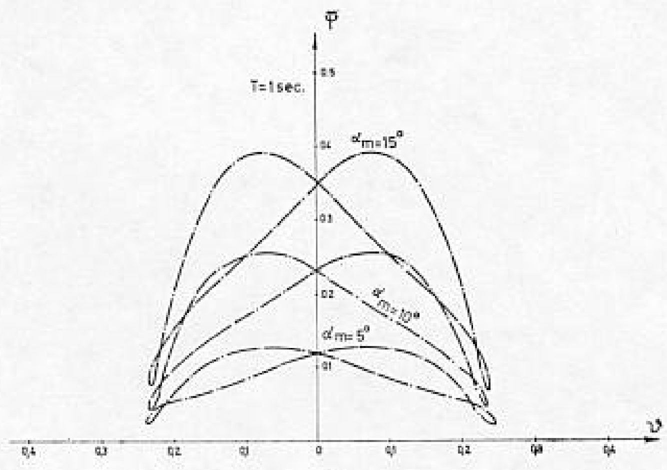
Fig. AI.3a, b. Phase portraits in ψ—$\dot{\psi}$ plane.

ation of repeatability represents the *mathematical conditions of a gait in ideal conditions*. Since the solutions of this system depend exclusively on the "free" parameters **T** and α_m for the fixed values of the other geometrical dynamic parameters, a series of trajectories (Fig. AI.3) is available. Consequently, the algorithm (5) will be convergent in that domain of variation of the parameters **T** and α_m in which the *repeatability conditions* (2) can be realized.

To realize the gait in real conditions* on level ground these repeatability conditions are only *necessary conditions*. Namely, since the motion of such a system has an unstable character, for each pair of allowable parameters T, α_m there exists only one vector



c



d

Fig. AI.3c, d. Phase portraits in ψ— plane.

of initial conditions satisfying (2). It is clear that motion repeatability in this case is not a result of the solution of the mathematical model but of the exactly imposed boundary conditions. Thus an

* The real conditions in this case assume only the variations in the initial conditions and system parameters.

arbitrary perturbation in the system parameters* leads to an instability due to the disturbance of the repeatability conditions, i.e. the cyclic repetition of the movement.

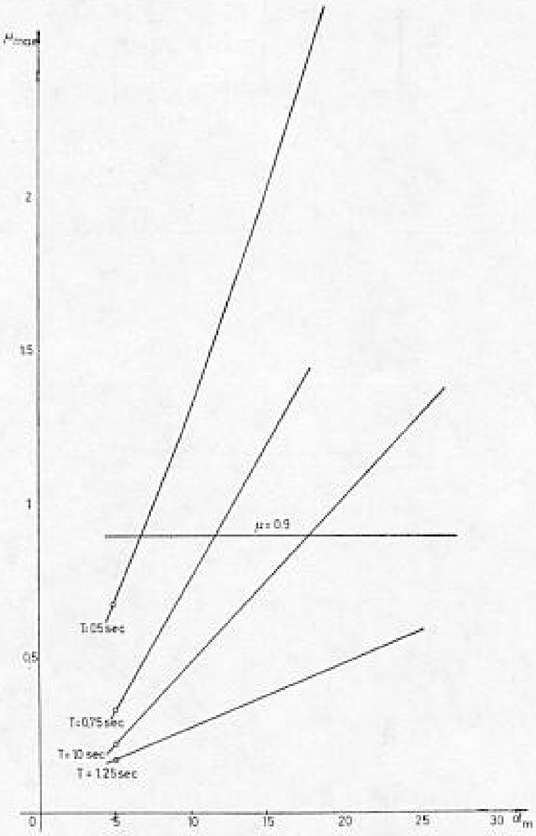


Fig. AI.4. Dependence of friction coefficient on α

A logical conclusion follows: the problem of maintaining the *dynamic equilibrium*** of such systems can be solved (only) by the introduction of regulating or control loops for the purpose of controlling and keeping the conditions of equilibrium in motion. However, in this case the same matrix equation (5) simultaneously

* The "system parameters" means the parameters and initial conditions characterizing the system.

** The dynamic equilibrium here assumes the maintaining of the equilibrium in motion affected by perturbations. Thereby it is not limited only to the case of constant velocity in motion.

solved with the basic equations of motion can be used in its inverse form for calculating deviations from the stationary state. That is:

$$\begin{Bmatrix} \Delta\vartheta_o \\ \Delta\bar{\psi}_o \\ \Delta\dot{\vartheta}_o \\ \Delta\dot{\bar{\psi}}_o \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} -(\vartheta_T+\vartheta_o) \\ -(\bar{\psi}_T-\bar{\psi}_o) \\ -(\dot{\vartheta}_T+\dot{\vartheta}_o) \\ -(\dot{\bar{\psi}}_T-\dot{\bar{\psi}}_o) \end{Bmatrix} \tag{6}$$

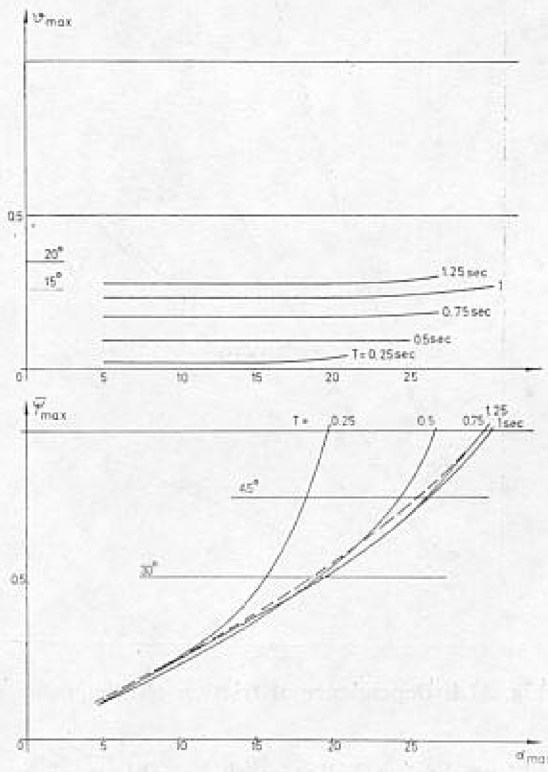


Fig. AI.5. Dependence of Ψ and ϑ on α

Based on this, the following statement can be formulated:

The repeatability conditions of the characteristic coordinates represent the necessary conditions for a stationary gait of such biped mechanical machines. The maintenance of these conditions represents the sufficient conditions for stationary gait.*

* The stationary gait assumes a gait with constant mean velocity during the motion of the locomotion machine.

The results of a simulation of the dynamic behaviour of this biped locomotion system can be represented in the parametric plane of the independent parameters T, α_m (Fig. AI.6). This diagram

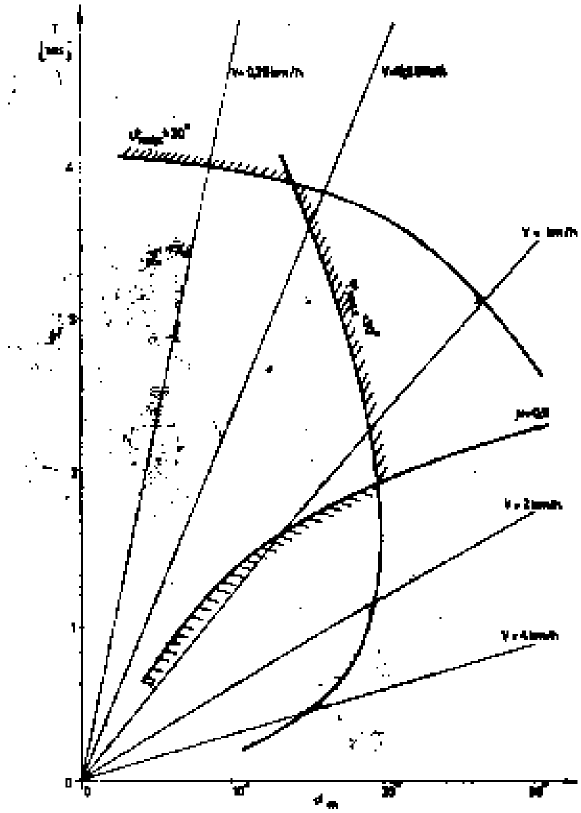


Fig. AI.6. Operating diagram of gait of the considered model

represents, in fact, the necessary conditions of this "*mathematical*" *gait*, or the realization of locomotion under ideal circumstances. Diagrams of this type serve as an *index of capability* for fulfilling the necessary conditions of ideal locomotion. They hold for particular pairs of characteristic values T, α_m with a given type of structure, with definite geometrical — dynamic parameters, and with prescribed kinematic limitations. In other words, every characteristic point M(T, α_m) of such a diagram within the specified limitations, represents a definite possible state of dynamic equilibrium of the locomotion system in ideal working conditions.

The above considerations can be extended also to more complex dynamics of locomotion systems. For a locomotion system represented by a full number of degrees of freedom the problem of necessary and sufficient conditions is reduced to the selection of a certain algorithm and its maintenance by a complete regulating system. This will be treated later.

Stability of a Complete Dynamic System*

This section is concerned with the stability of a system described completely by dynamic equations. The motion of the "body" of such a system is considered for stability. The "body" is consider-

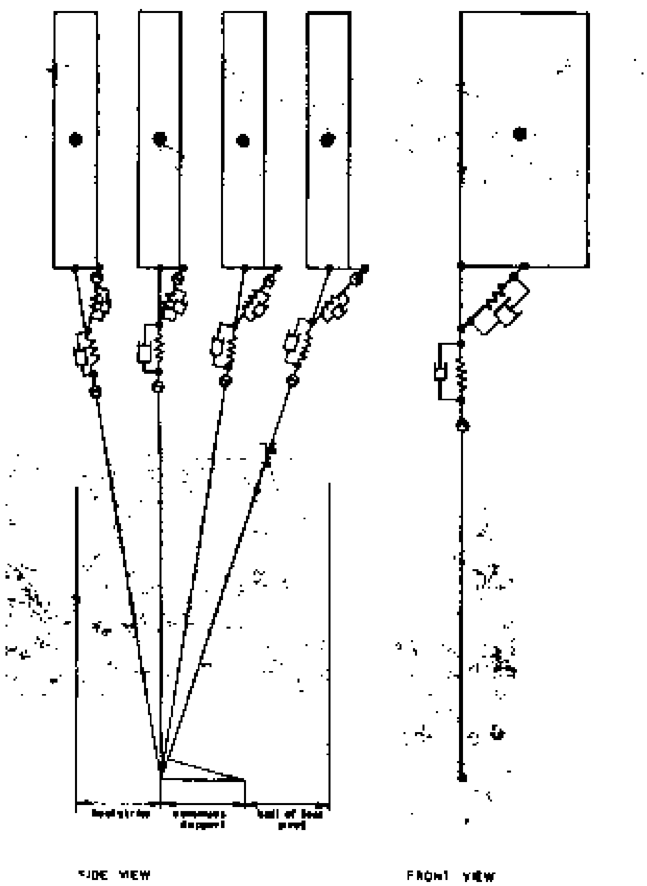


Fig. AII.1. Configuration of dynamic model

ed the rigid portion of the machine to which a set of legs is attached. In general, this body can be described in six degrees of freedom

* Under the term of complete dynamic system here is assumed the full number of degrees of freedom of the locomotion machine's body.

by a set of six nonlinear differential equations of second order. One method to describe this motion is presented in Figure AII.1, 2, 3, 4, 5.

To make such a body move in the six degrees of freedom, forces are applied. These forces must be applied in such a way that "stable" motion results. The three basic types of stability will be discussed here.

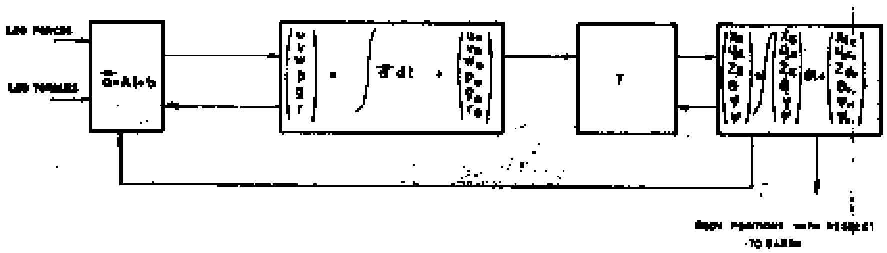


Fig. AII.2. Computation of body angles and positions from applied leg forces and torques

First consider the body angular orientation and altitude. Suppose some systems for the control of these coordinates have been designed. The problem is to study the stability of the body's orientation with respect to earth. Due to the nature of the problem, i.e. the fact that one leg is in support at some times, the body generally experiences some angular displacement regardless of the control

$$T = \begin{vmatrix} T_1 & 0 \\ 0 & T_2 \end{vmatrix}$$

$$T_1 = \begin{bmatrix} \cos\theta\cos\bar{\psi} & \cos\bar{\psi}\sin\theta\sin\varphi - \sin\bar{\psi}\cos\varphi & \sin\bar{\psi}\sin\varphi + \cos\bar{\psi}\sin\theta\cos\varphi \\ \sin\bar{\psi}\cos\theta & \cos\bar{\psi}\cos\varphi + \sin\bar{\psi}\sin\theta\sin\varphi & \sin\bar{\psi}\sin\theta\cos\varphi - \cos\bar{\psi}\sin\varphi \\ -\sin\theta & \cos\theta\sin\varphi & \cos\varphi\cos\theta \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0 & \cos\varphi & -\sin\varphi \\ 1 & \tan\theta\sin\varphi & \tan\theta\cos\varphi \\ 0 & \sec\theta\sin\varphi & \sec\theta\cos\varphi \end{bmatrix}$$

Fig. AII.3. Coordinate transformation to resolve body velocities to earth velocities

scheme. This is also the case with altitude. Thus for analysis purpose, the motion of the center of gravity can be considered to exist in a "region" in the four coordinates of concern. In "Normal" undisturbed locomotion the region can be closed. Keeping in mind these concepts the definition of this form of stability can be made:

Definition 1 — *Body stability*. A biped machine's body is considered stable if there exists a closed region "R" which encloses the undisturbed trajectory of the 3 earth angles and the altitude such that if the machine is disturbed by a disturbance $d \in D$, the trajectory returns to the region "R" as time becomes infinite. D is a class of disturbances.

$$A = \begin{bmatrix} \frac{-\cos\alpha}{m\,\ell\cos\beta} & \frac{\sin\alpha\sin\beta}{m\ell} & 0 & \frac{\sin\alpha\cos\beta}{m} \\ 0 & \frac{-\cos\beta}{m\ell} & 0 & \frac{-\sin\beta}{m} \\ \frac{\sin\alpha}{m\,\ell\cos\beta} & \frac{-\cos\alpha\sin\beta}{m\ell} & & \frac{\cos\alpha\cos\beta}{m} \\ \frac{y_h\sin\alpha}{I_{xx}\,\ell\cos\beta} & \frac{\ell\cos\alpha - y_h\cos\alpha\sin\beta - z_h\cos\beta}{I_{xx}} & \frac{-\cos\beta\sin\alpha}{I_{xx}} & \frac{-z_h\sin\beta + y_h\cos\alpha\cos\beta}{I_{xx}} \\ \frac{\cos\beta - z_h\cos\alpha}{I_{yy}\,\ell\cos\beta} & \frac{x_h\cos\beta \;\; z_h\sin\alpha\sin\beta}{I_{yy}\,\ell} & \frac{\sin\beta}{I_{yy}} & \frac{x_h\sin\beta + z_h\sin\alpha\cos\beta}{I_{yy}} \\ \frac{y_h\cos\alpha}{I_{zz}\,\ell\cos\beta} & \frac{y_h\sin \;\; \sin \; - \; \sin \; - x_h\cos}{I_{zz}} & \frac{-\cos\beta\cos\alpha}{I_{zz}} & \frac{-x_h\sin\beta + y_h\sin\alpha\cos\beta}{I_{zz}} \end{bmatrix}$$

Fig. AII.4. The matrix of inertial components and leg to body transformations

A number of points must be kept in mind in this definition. First, this deals only with the body angular position with respect to earth and the body's altitude. This has no implication on the mode of locomotion or the path of this locomotion. But this stability is necessary before any other form of stability can be considered. Second, this corresponds to the concept of repeatability conditions mentioned above except in a somewhat looser or less precise sense. This is necessary since the system is more complex.

The next coordinates to consider for stability are the linear coordinates in space. Since altitude is already considered, the position of the body in two directions (longitudinal and lateral) and its forward velocity must be considered. Again for stability to exist control of these axes must be implemented. In general, this implies forward velocity and directional control, or velocity vector control. Stability about these axes, however, requires the existence of a nominal trajectory of the center of gravity and after a disturbance, the system must return to this nominal trajectory. The classical definitions of trajectory stability can be used with a slight modification. Since the body is continuously in oscillation, the body will not return exactly to the nominal trajectory but to a region of the nominal trajectory. The following definition provides a basis for such an analysis.

Definition 2 — *Body path stability*. The path of the body in space of a biped machine is considered stable if the "average velocity vector" returns toward its original direction and magnitude after a disturbance $d \in D$. The average velocity vector is:

$$\frac{1}{T}\int_{0}^{T} \vec{v}\, dt = \vec{v}_{ave}$$

where T is the period of a complete cycle. It is both an average in the magnitude and direction. D is a class of disturbances.

$$t = \begin{Bmatrix} T_\alpha \\ T_\beta \\ T_\gamma \\ F_l \end{Bmatrix} \text{ — applied force and torque vector}$$

where

T_α, T_β and T_γ are torques applied in the α, β, and γ directions, respectively

F_1 is the longitudinal leg force.

$$a = \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} \text{ —}$$

where

$\dot{u}$, $\dot{v}$ and $\dot{w}$ are linear accelerations in body coordinates

$\dot{p}$, $\dot{q}$ and $\dot{r}$ are rotational accelerations about the doby axis

$$b = \begin{Bmatrix} vr - wq - g \sin \theta \\ wp - ur + g \cos\theta \cos \varphi \\ uq - rp + g \cos \theta \cos \varphi \\ \dfrac{(I_{yy} - I_{zz})}{I_x} qr \\ \dfrac{(I_{zz} - I_{xx})}{I_{yy}} rp \\ \dfrac{(I_{xx} - I_{yy})}{I_{zz}} pq \end{Bmatrix} \text{ —}$$

where

u, v and w are linear velocities in body coordinates

p, q and r are rotary velocities about the body axis

I_{xx}, I_{yy} and I_{zz} are body moments of inertia about the body axis

g is the gravitation constant

θ and φ are body pitch and roll angles with respect to the earth

Fig. AII.5. The vector quantities of the body acceleration equations

Finally, there is the problem of describing the stability of leg movements. It only makes sense to consider the stability of such movements if an average constant forward velocity is assumed. The velocity is averaged over one cycle. In this case this reduces to the study of the stability of a "stationary gait". A "stationary gait" can be characterized by the following factors at least:

1) average constant forward velocity
2) constant stride
3) constant phasing
4) constant duty factor*
5) constant cycle time.

More parameters are necessary to describe the gait if the arms or other aids are used in locomotion. Note that these parameters are all continuous in their own domain but must be computed discretely after each complete cycle.

Suppose a given stationary gait has "k" continuous characteristic parameters. These parameters represent a point g_0 in k-space. If the gait is "stationary" this point does not move from cycle to cycle. When the system is disturbed this point moves to a new point g_1 in k-space. Then, after "n" steps, if the point g_n approaches g_0 in k-space, stability results. Realistic systems, however, cannot repeat a gait exactly. Therefore g_0 is a volume in k-space. Thus formally:

Definition 3 — *Stationary gait stability.* A stationary gait is considered stable if the characteristic factors of the undisturbed system represented by a k-vector g lie within a volume g_0 and if after a disturbance, the vector g_n returns and remains within g_0, where n is the number of steps after a disturbance.

This definition seems to coincide with the statement about the necessary and sufficient conditions of repeatability above. It differs, however, in the fact that here the leg system characteristic coordinates are concerned whereas the above was concerned with body characteristic coordinates. Here, the system must have control of all the properties of the legs movements. The simplified system above replaced all these control variables by an algorithmic program selected a-priori, thus eliminating problems of this type.

It should be noted that this discussion about the k-dimensional vector is correct in the case when force control exists on elements of the leg system to control these characteristics. If force controls do not exist for all these parameters then they must be determined by some algorithm. For example, suppose the stride length is always set at a particular number s, the phasing is always to be precisely P, and the cycle time is always set at T. Then only velocity and duty factor are variable. Thus the space of g is two-dimensional.

* Duty factor is the measure of the amount of time the leg spends on the ground versus the amount of time it spends in the air.

The implication of this definition should be made clear. It should be noted that for a given average forward velocity, an infinite number of gaits can exist. For example, the stride may be reduced with a corresponding reduction in cycle time, etc.

In any physical system there exists a limit to the excursions of elements of the legs and a limit to the magnitude of forces and torques applied. When determining the stability of a system to perturbations these limits must always be kept in mind. The mathematical representation should include these limits.

This concept of limits leads naturally to the *concept of disturbance capability*. That is how large a disturbance d of a particular class D can be sustained by a system before one of these physical limits is exceeded. There are three kinds of stability and each has its own limits for a particular machine.

The introduction of disturbances to the mathematical model of a dynamic biped machine shows the concepts discussed. The biped machine is represented by the Figure AII.1. The equations

Fig. AII.6. Photograph of mathematical model during locomotion

describing the motion are presented in Figures AII.2 and AII.3. The control schemes for the various body angles and axes are listed in Appendix II.

The motion of this system is shown in Figure AII.6. This is a photograph of the projection of the simulation of this machine as it performs locomotion. A line is drawn from the center of gravity to a line joining the two hip joints and a line is drawn from the hip of the leg in support to its corresponding heel. The leg is not shown in the swing phase.

The response of this system to disturbances will be illustrated. Two classes of disturbances will be investigated. First a longitudinal

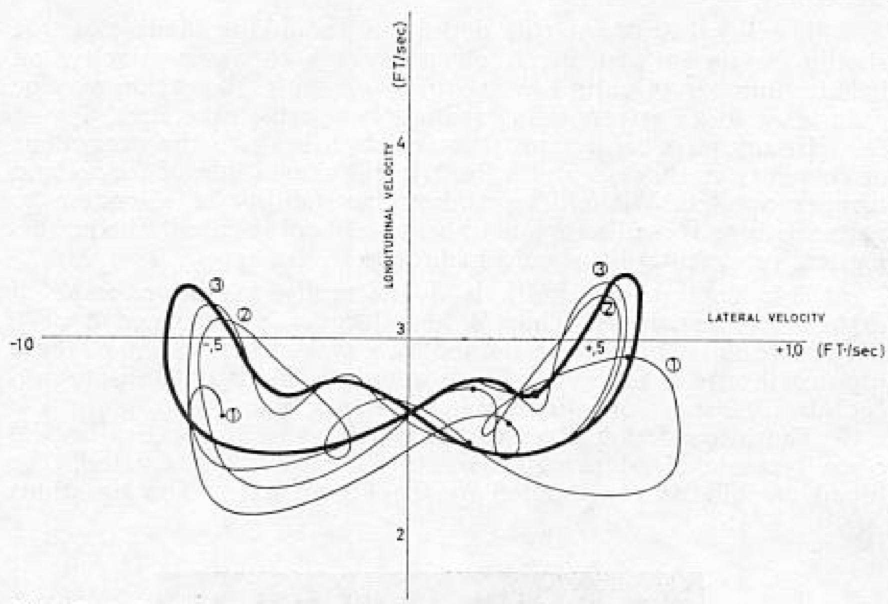


Fig. AII.7. Longitudinal and lateral velocity reactions to a longitudinal force disturbance

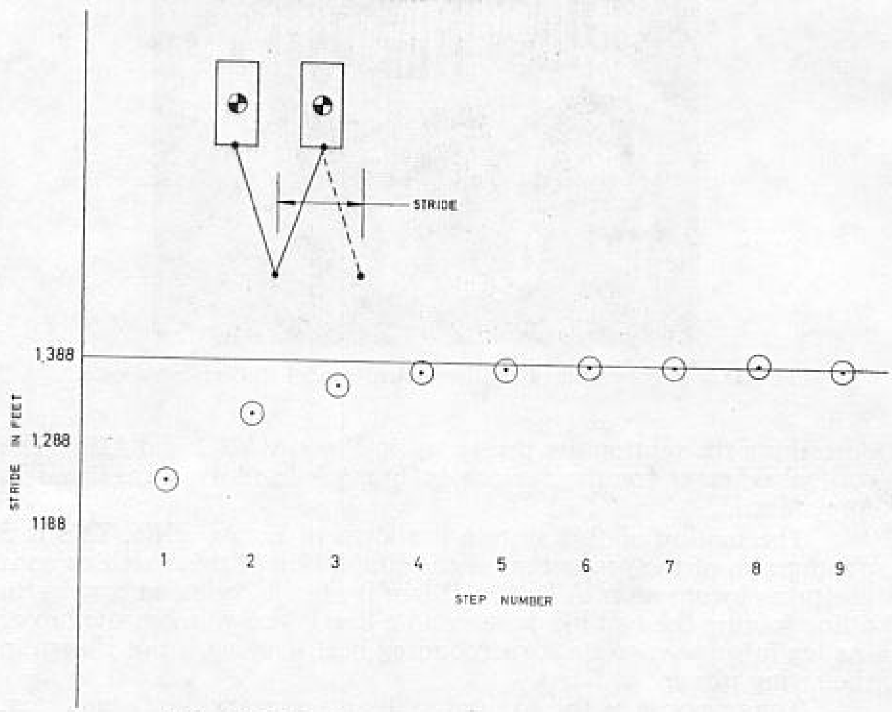


Fig. AII.8. Stride variation after a perturbation force will be introduced to simulate running into an obstacle. A force impulse resulting in a change of forward velocity of 20% is investigated. The resulting characteristic velocities of the succeeding steps are shown in Figure AII.7. The bold line shows the undisturbed motion, the light line shows the motion after a disturbance. The numbers show the converging nature of the control algorithm after each step. The gait also varies in this case. The variation in one of the gait factors (stride) is shown after a disturbance in Figure AII.8.

A second class of disturbances is steps or holes. A step is simulated for one cycle. The induced disturbance in the pitch and roll angles is shown in Figure AII.9. The bold line shows the

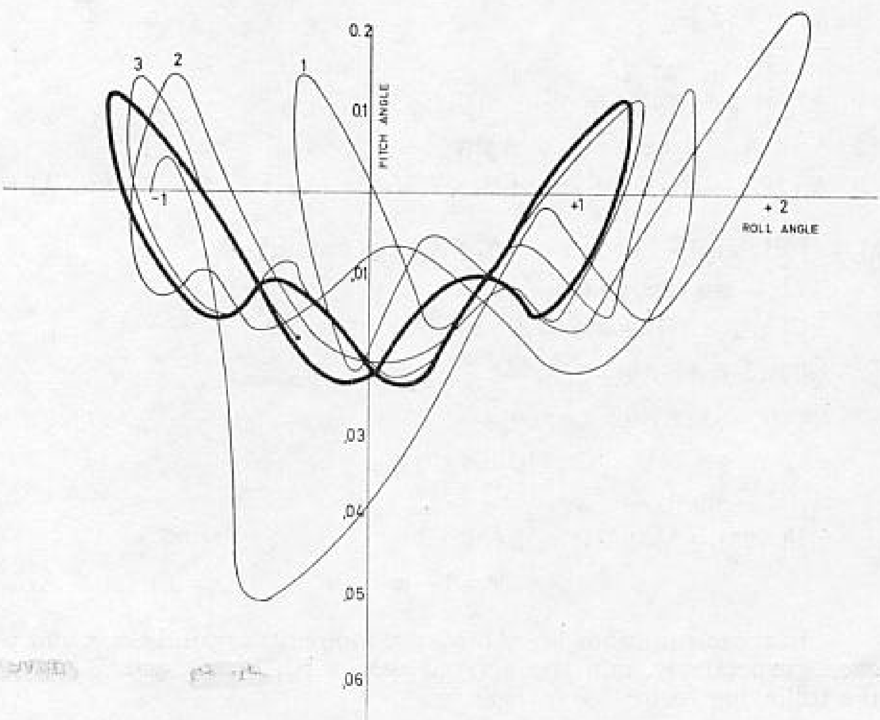


Fig. AII.9. Pitch and roll reactions to a step disturbance

undisturbed motion. The light line shows the motion after a disturbance. The disturbance is 20% of the length of the leg. Again the numbers show how the regulating mechanism drives the system practically back to its normal operation after 3 steps.

APPENDIX I

The dynamic system is maximally simplified by the kinematic connection between the levers of the locomotion system (Fig. AI.1). In the analysis, a gait has been adopted where the extremity tip is in continuous contact with the ground during the motion. The gait phases in which the locomotion model has been considered are presented in Figure AI.2.

The differential equation of motion for the first and second part of the step (Fig. AI.2) can be written in the following concise form:

$$
\begin{aligned}
M_y \equiv\; & \ddot{\vartheta}\, 2B_2 \sin\vartheta - \ddot{\bar{\Psi}}[2B_3 \cos\vartheta + \cos^2\vartheta] + \\
& \ddot{\alpha}[4(A_1+A_2)(A_1+B_1) + 2B_4 \cos\vartheta + \\
& \qquad 2(A_1+2B_1)M_1 + 4(A_1+A_2)(A_1+B_1)M_3 + 2M_4 + M_5] + \\
& \dot{\vartheta}^2\, 2B_2 \cos\vartheta + \\
& \dot{\bar{\Psi}}^2\, 2B_2 \cos\vartheta + \\
& \dot{\bar{\Psi}}\dot{\vartheta}[4B_3 \sin\vartheta + 2 \sin\vartheta \cos\vartheta] - \\
& A_4[2(A_1+B_1) \sin\alpha - \cos\vartheta \sin\vartheta] - \\
& A_4[2(A_1+2B_1)M_1 + 2(A_1+B_1)(2M_2+M_3)] \sin\alpha = 0 \qquad \text{AI.1}
\end{aligned}
$$

$$
\begin{aligned}
M_x \equiv\; & \ddot{\vartheta}[(B_0 \sin\vartheta - 1) \cos\bar{\Psi} - 2(A_1+A_2) \cos\vartheta \sin\alpha] + \\
& \ddot{\bar{\Psi}}(B_0 - \sin\vartheta)\cos\vartheta \sin\bar{\Psi} + \\
& \ddot{\alpha}\, 2(A_1+A_2)(B_0 - \sin\vartheta) \sin\alpha + \\
& \dot{\vartheta}^2[B_0 \cos\vartheta \cos\Psi + 2(A_1+A_2) \sin\vartheta \cos\alpha] + \\
& \dot{\bar{\Psi}}^2(B_0 - \sin\vartheta) \cos\vartheta \cos\bar{\Psi} + \\
& \dot{\alpha}^2\, 2(A_1+A_2)(B_0 - \sin\vartheta) \sin\vartheta \sin\Psi - \\
& A_4(B_0 - \sin\vartheta) - \\
& B_0A_4(2M_1+M_2+M_3) = 0 \\
& \qquad\qquad \bar{\Psi} = \Psi - \alpha \qquad \text{AI.2}
\end{aligned}
$$

In these equations M_x and M_y are moments around the x and y axes, respectively, and the derived values B_0, B_1, B_2 and B_4 have the following form:

(a) for the first part of the step,

$$
\begin{aligned}
& B_0 = A_3, \\
& B_1 = A_3, \\
& B_2 = (A_1+A_2) \sin(\bar{\Psi}+\alpha), \\
& B_3 = (A_1+A_2) \cos(\bar{\Psi}+\alpha), \\
& B_4 = B_3
\end{aligned}
$$

(b) for the second part of the step,

$$B_0 = -A_3,$$
$$B_1 = -A_2,$$
$$B_2 = A_1 \sin(\bar{\Psi}+\alpha) + A_2 \sin(\bar{\Psi}-\alpha),$$
$$B_3 = A_1 \cos(\bar{\Psi}+\alpha) + A_2 \cos(\bar{\Psi}-\alpha),$$
$$B_4 = A_1 \cos(\bar{\Psi}+\alpha) - A_2 \cos(\bar{\Psi}-\alpha)$$

$$A_1 = \frac{a}{c} \qquad M_1 = \frac{m_a}{m_c}$$

$$A_2 = \frac{b}{c} \qquad M_2 = \frac{m_b}{m_c}$$

$$A_3 = \frac{d}{c} \qquad M_3 = \frac{m_o}{m_c}$$

$$A_4 = \frac{g}{c} \qquad M_4 = \frac{J_a}{m_c\, c^2}$$

$$M_5 = \frac{J_o}{m_c\, c^2}$$

a, b, c, d are geometrical characteristics of the biped structure (Figure AI.1). m_a, m_b, m_c, m_o are masses of the appropriate parts of the biped structure. J_a, J_o are inertia moments of the appropriate masses of the biped.

Accordingly, the dynamic equations of motion represent a set of nonlinear, nonhomogeneous differential equations with time-varying coefficients. Due to a fixed kinematic program of the shifting of the lower extremities, the only connection between the system (its output coordinates Ψ and ϑ) and the extremities is the angle α. This represents the law of change in the driving system

$$\alpha = \frac{\alpha_m}{2}(1-\cos\omega t).$$

Parallel with the basic differential equations of motion are the expressions for the system reactions at the contact point of the extremity with the ground.

The reactions recorded in a concise form are represented by the symbols which take the form

$$
\begin{aligned}
\frac{x}{m_c\,c} = {} & \ddot{\theta}\sin\theta\sin\bar{\Psi} - \\
& \ddot{\bar{\Psi}}\cos\theta\cos\bar{\Psi} + \\
& \ddot{\alpha}\{M_1\,2(2A_2+A_1)+M_2\,4(A_1+A_2)+M_3\,2(A_1+A_2)+ \\
& \quad 2(A_1+A_2)\cos\alpha\}+ \\
& \dot{\theta}^2\cos\theta\sin\Psi + \\
& \dot{\bar{\Psi}}^2\cos\theta\sin\bar{\Psi} - \\
& \dot{\alpha}^2\{2(A_1+2A_2)M_1+2(A_1+A_2)(2M_2+M_3+1)\}\sin\alpha + \\
& \dot{\theta}\dot{\bar{\Psi}}\,2\sin\theta\cos\bar{\Psi}
\end{aligned}
\qquad \text{AI. 3}
$$

$$
\frac{y}{m_c\,c} = -\ddot{\theta}\cos\theta + \dot{\theta}^2\sin\theta \qquad \text{AI.4}
$$

$$
\begin{aligned}
\frac{z}{m_c\,c} = {} & -\ddot{\theta}\sin\theta\cos\bar{\Psi} - \\
& \ddot{\bar{\Psi}}\cos\theta\sin\bar{\Psi} - \\
& \ddot{\alpha}\{M_1\,2A_1+M_2\,2(2A_1+A_2)+(M_3+1)2(A_1+A_2)\}\sin\alpha - \\
& \dot{\theta}^2\cos\theta\cos\bar{\Psi} - \\
& \dot{\bar{\Psi}}^2\cos\theta\cos\bar{\Psi} + \\
& \dot{\theta}\dot{\bar{\Psi}}\,2\sin\theta\sin\bar{\Psi} - \\
& \dot{\alpha}^2\{M_1\,2A_1+M_2\,2(2A_1+A_2)+2(M_3+1)(A_1+A_2)\}\cos\alpha + \\
& A_4(2M_1+2M_2+M_3+1)
\end{aligned}
\qquad \text{AI.5}
$$

The simultaneous solving of the motion system AI.1, 2 with the sensitivity system (6) for a series of angles α_m and step periods T has resulted in a series of portraits in the plane representing space shifting of the compensation lever of the locomotion system (Fig. AI.3).

Figure AI.4 illustrates dependences of friction coefficient (angle tangents between the components of the system reactions) according to the expressions (AI.3, AI.4, AI.5), in the function of angle α_m for a series of values of the step period T.

Figure AI.5 shows the results concerning the dependence between the angles $\bar{\Psi}$ and θ and the angle α_m for different values of the step period.

The dependence of the step period on the angle α for limited values of the angles of compensation lever and an adopted friction coefficient* has been given as the final diagram based on the

* Coefficient $\mu=0.9$ corresponds to the contact between the ground made of concrete and the extremity rubber tip.

preceding diagram dependences. The curves of constant speeds are given in the working diagram for definite pairs of T and α_m. Thus the working diagram limited by the curves ϑ_{max}; Ψ_{max} and μ=const. has been obtained (Fig. AI.6).

APPENDIX II

The motion of a rigid body in space with respect to a body and earth coordinate systems is given by Figures AII.1 and AII.2. Figure AII.1 shows the method of computing the velocity and position of the body axes system from body axes accelerations. Figure AII.2 provides the transformations necessary to obtain velocities relative to earth from body axes velocities. To compute the acceleration vector the vector equation

$$a=At+b$$

is used. The quantities a, A, t and b are defined in Figures AII.4 and AII.5. The Figure AII.4 provides the transformation from leg coordinates to body coordinates. Figure AII.5 provides the definition of the quantities t and b. The vector t is the force and torques applied in leg coordinates. The vector b represents the velocity coupling and gravity terms.

The generation of the forces and torques is given by Figure AII.5. Here the control laws for the various leg coordinates are given.

1. Longitudinal control torque

$$F_\alpha=c_\alpha(\alpha-\alpha_c)+c_{\dot\alpha}(\dot\alpha-\dot\alpha_c)$$

Δt — time within a cycle

$\dot\alpha_c$ — desired angular rate — a function of velocity

α_c — computed from desired body angle and foot position.

2. Lateral control torque

$$F^\beta=c_\beta(\beta-\beta_0)+c_{\dot\beta}(\dot\beta).$$

3. Yaw control torque

$$F_\gamma=c_\gamma(x_E-x_{lhE})+c_{\dot\gamma}(\dot x_E-\dot x_{lhE})$$

x_{lhE} — position of hip number one with respect to earth

4. Leg force along its length

$$F_l = c_l (z_E - z_{ED}) + c_l(\dot{l})$$

5. Leg placement in forward direction

$$x_F = x_E + d_x$$

d_x — preselected constant corresponding to α_0

6. Leg placement in lateral direction

$$y_F = y_E + d_y$$

d_y — preselected constant corresponding to $\beta = \beta_0$

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