ON THE DYNAMIC STABILITY OF LEGGED LOCOMOTION SYSTEMS

R. B. McGhee and M. B. Kuhner

Summary

While legged locomotion is obviously very efficient and versatile and is widely encountered in nature, only limited success has been attained in constructing legged vehicles. The failure of such vehicles to appear is to a large extent due to the lack of a well developed mathematical theory of legged locomotion. Another manifestation of this lack is the absence of electronically controlled lower-extremity prostheses and orthotic devices. The purpose of this paper is to summarize the available body of theoretical knowledge relating to stability and control in legged machines and animals and to suggest areas where additional research ought to be undertaken. The paper includes a discussion of a hierarchy of mathematical models beginning with finite state models and concluding with rigid body dynamics. Some new results on balancing mechanisms for inherently unstable systems are also included.

Introduction

Locomotion in terrestrial animals is most often accomplished by means of a periodic alternation of forward and backward motions of articulated limbs which both support and propel the animal. While such systems are obviously very efficient and versatile, only limited success has been attained in constructing machines based upon the same principles. Rather, one finds that nearly all vehicles for ground transportation make use of wheels as a substitute for legs. Contrary to common opinion, this circumstance is not primarily due to any inherent advantages of wheeled and tracked systems [1, 2], but results mainly from the fact that the control problems involved in limb coordination have not yet been solved by artificial means in an entirely satisfactory manner. This, in turn is to a large extent due to the lack of a substantial mathematical theory of legged locomotion.

This research was supported in part by the U. S. Air Force Office of Scientific Research under Grant No. AF-AFOSR-1018-67B and in part by The Ohio State University as Engineering Experiment Station Project EE 28.

Another manifestation of the lack of a theoretical basis for legged locomotion system design and analysis is the absence of electronically coordinated lower extremity prostheses and orthotic devices. While there seems to be no inherent reason why electronic control should not be applied to lower-extremity systems [3] as it has been to upper extremity devices [4, 5], the primitive state of knowledge of the dynamics and control principles involved in normal legged locomotion has retarded such developments.

The purpose of this paper is to summarize the existing body of mathematical theory relating to stability and control of legged systems to the extent that it is known to the authors and to suggest some areas in which the theory might be improved by further research.

Finite State Theory

The earliest systematic study of human and animal locomotion principles is apparently due to Muybridge [6, 7] who invented a type of motion picture camera which was successfully used in 1877 to obtain the first photographic record of the successive phases of a number of quadruped gaits. In his earliest work, Muybridge was interested primarily in the sequence in which feet are lifted and placed during the steady forward motion of a quadruped. His investigations ultimately revealed a total of eight distinct patterns or "gaits" employed by various animals, some of which were previously unknown to either horsemen or zoologists.

Since only sequences of foot lifting and placing were considered by Muybridge, his results can be cast in mathematical form by assigning just two states, say 1 and 0, to each leg with one state corresponding to a leg being in a supporting phase and the other state corresponding to a leg being in the air (transfer phase). Such a "finite-state" model for locomotion was first proposed by Tomović [8, 9, 10] who also suggested along with McGhee that finite state methods could be extended to a consideration of the problem of joint coordination in a powered lower-extremity prosthesis [3].

In order to test some of the conjectures in [3], a project was initiated in 1965 at the University of Southern California to design and construct an artificial quadruped based entirely upon finite state principles [11]. One of the first questions considered in this effort was the problem of choosing a gait for the machine. Literature research revealed that the work of Muybridge had been greatly extended by Hildebrand [12, 13] who both introduced a more elaborate mathematical model for gait analysis and obtained far better data using modern high speed motion picture equipment. Hildebrand focused much of his work on a class of quadruped gaits, called *regular symmetric* gaits [14], characterized by two special properties: 1) the fraction of a cycle during which a leg is in a support phase is the same for all legs and 2) the two legs in either the front or rear pair are employed symmetrically; i.e., the time between the contact of one such leg with the ground until the contact of the other is equal to one-half of the period of the gait. Hildebrand proved that there are exactly forty-four such gaits theoretically possible. Moreover, he also showed that precisely sixteen of these have the further property that feet are never lifted or placed in synchronism but rather each cycle of locomotion consists of exactly eight distinct events (four foot liftings and four foot placings). Gaits with this property have been called *connected* gaits [14]. Figure 1 is a finite state representation of a particular connected quadruped gait called a *crawl*. While Muybridge was aware of only four connected regular symmetric gaits, Hildebrand has observed eleven of the sixteen theoretically possible gaits of this type in use by one or another species of quadruped [12].

The work of Muybridge and Hildebrand on gait enumeration has been extended and refined by McGhee [14] who discovered that the total number of theoretically possible connected quadruped gaits is equal to 5040. A more recent calculation by the authors has shown that if the *singular* gaits involving simultaneous lifting or placing of more than one foot are taken into account, the total number of distinct quadruped gaits is equal to 65,428, a number much larger than suspected by previous investigators.

While an opportunity remains for much more work on enumeration and classification of gaits for *n*-legged machines and animals, any investigation restricted to lifting and placing sequences can hardly be expected to reveal a great deal about dynamic stability. Clearly, any study of stability must include some spatial properties of gaits as well as their temporal properties. This fact was recognized early in the artificial quadruped program and eventually lead to a kinematic theory of quadruped stability [5].

Kinematic Gait Models and Static Stability

Tomovic [8] observed that "creeping gaits" used by low order animals employing a large number of legs could be achieved by a finite state control system since the dynamic stability problem is solved in such animals by keeping most of the legs on the ground at any given time and in a position such that the animal is always statically stable. This kind of solution to the stability problem is also obviously possible for both quadruped and biped locomotion providing that each leg is furnished with a sufficiently large foot and that these feet are properly overlapped during a locomotion cycle. This approach is typically used in walking toys. It is easily observed, however, that living quadrupeds do not employ this principle. Rather, the feet of most quadruped animals are so small as to approximate a point in comparison to body dimensions. Since quadrupeds are capable of arbitrarily slow locomotion, it is evident that their stability in low speed gaits must result from the placing and lifting of feet in such a way that the vertical projection of their center of gravity is always contained in the "support pattern" [15] determined by the feet in contact with the ground. Figure 2 illustrates such a sequence of support patterns for a singular quadruped crawl.

An analysis of static stability can be carried out mathematically by defining a set of parameters which characterize the time dependent positions of each leg relative to the center of gravity of an animal or a legged vehicle. McGhee and Frank [15] have accomplished such an analysis for quadrupeds by idealizing each foot to a point and considering only constant velocity motion in a straight line. The resulting *kinematic gait model* consists of a $(4n-1)$ dimensional vector of real numbers for an n-legged system. The principal result obtained in [15] is that of the 5040 theoretically possible connected quadruped gaits, only 3 have the property that the feet can be placed so that the system is statically stable at all times. Furthermore, among these three, there exists a unique optimum gait which maximizes the *degree* of static stability. This gait is the crawl illustrated in Figures 1 and 2. The crawl gait

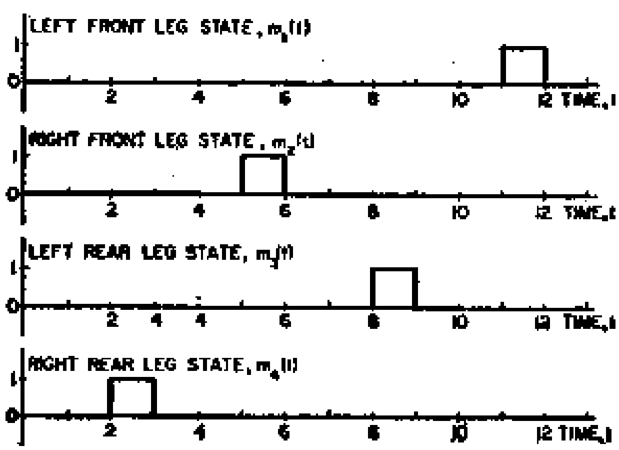

Fig. 1. Binary leg state functions for a quadruped crawl

appears to be the one favored by all quadrupeds for very low speed motion, apparently as a result of its superior stability properties. It was also selected as one of the gaits successfully attained by the University of Southern California artificial quadruped for the same reason [11, 16].

Preliminary investigations indicate that bipeds, including both birds and human beings, also stabilize low speed locomotion by keeping their center of gravity within the support zone formed by their feet. This requires, of course, that biped feet be somewhat larger relative to body dimensions than quadruped feet and this also seems to be typically true of bipeds. It is very clear, however, that stability in all but the lowest speed gaits results from some

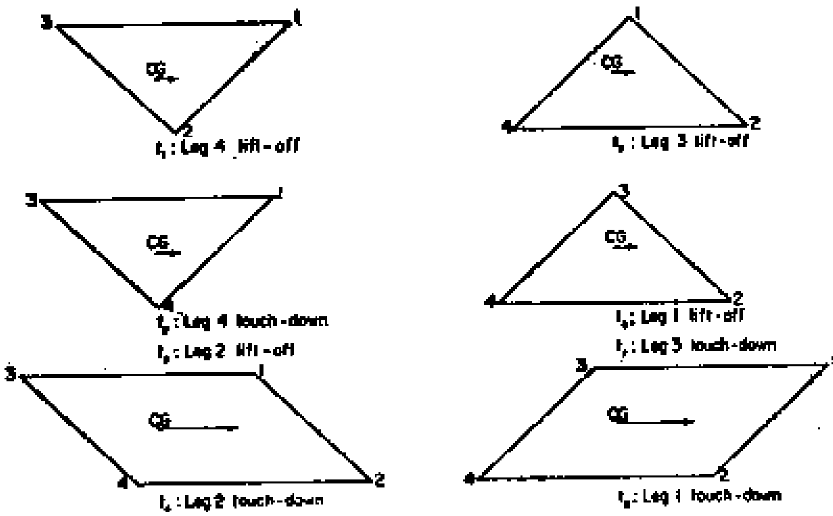

Fig. 2. Support pattern sequence for a singular quadruped crawl. (Arrows indicate center of gravity motion during each phase.)

mechanism other than static stability for both bipeds and quadrupeds. To study these gaits it is therefore necessary to introduce further complexities into the mathematical model for a locomotion system.

Dynamic System Models and Passive Stability

The exact differential equations of motion for a vehicle with articulated limbs are extremely complicated. A precise description of animal dynamics is still more involved and, in fact, is probably unobtainable as a result of the distributed and compliant nature of muscle systems, internal organs, etc. Nevertheless, in keeping with the spirit of mathematical modelling in general, it is possible to make certain approximations which yield equations of motion which are at least amenable to computer simulation. Perhaps the simplest set of equations which still embody much of the essentials of many types of locomotion systems results from treating a machine or animal as a single rigid body to which are attached a specified number of massless legs. The resulting equations of motion can be written in a body centered coordinate system as follows [17]:

$$\dot{u} = vr - wq + \frac{f_x}{m} - g \sin\theta \tag{1}$$

$$\dot{v} = wp - ur + \frac{f_y}{m} + g \cos\theta \sin\Phi \tag{2}$$

$$\dot{w} = uq - vp + \frac{f_z}{m} + g \cos\theta \cos\Phi \tag{3}$$

$$\dot{p} = [(I_{yy} - I_{zz})\, qr + L]/I_{xx} \tag{4}$$

$$\dot{q} = [(I_{zz} - I_{xx})\, rp + M]/I_{yy} \tag{5}$$

$$\dot{r} = [(I_{xx} - I_{yy})\, pq + N]/I_{zz} \tag{6}$$

In these equations, θ and Φ are body elevation and roll Euler angles, respectively, u, v, w are translational velocities in body coordinates and p, q, r are body angular rates in roll, pitch, and yaw respectively. The quantities f_x, f_y, and f_z are forces applied to the body by the legs while L, M, N are the applied moments. These forces and moments are to be determined by an appropriate feedback control law so as to produce stable locomotion with a specified gait. The other symbols appearing in (1) through (6), namely g, m, I_{xx}, I_{yy}, and I_{zz}, are all constants corresponding respectively to gravitational acceleration, vehicle mass, and the three body moments of inertia about the principal axes x, y, z.

A digital computer program has been written to carry out the following sequence of instructions for an arbitrary legged locomotion system governed by (1) through (6):

1. Starting with an initial *body state vector* [17]

$$\vec{x} = (x_E, y_E, z_E, u, v, w, \theta, \Phi, \psi, p, q, r)^T \tag{7}$$

in which x_E, y_E, z_E is the body center of gravity location in an inertial reference frame, Φ is the body azimuth angle, and all other quantities are as previously defined, compute leg lengths and angles for all legs in contact with the ground.

2. Using a feedback control law, determine leg actuator forces and moments as well as reaction forces and moments for each leg. Transform these forces to body coordinates and sum to obtain the forces and moments appearing in (1) through (6). Also determine any change in foot position required to maintain the specified gait.

3. Evaluate $d\vec{x}/dt$ and integrate over one time increment, h.

4. Return to 1.

It should be noted that (1) through (6) provide only six components of $d\vec{x}/dt$ in step 3. The other six components are obtained by transforming the velocity components u, v, and w from body coordinates to inertial coordinates to obtain x_E, y_E, and z_E and by transforming the body angular rates p, q, and r into Euler angle rates θ, Φ, ψ. All of these steps are part of the existing simulation program [17].

The simulation program described above has been used to investigate the utility of a simple type of "model reference" control system [17]. This method of feedback control assumes that the feet of a machine or animal can be placed in the locations determined by an ideal constant velocity kinematic gait model and then computes control torques and forces from the differences between ideal and actual leg angles and lengths. The particular control law which has been simulated amounts to driving the leg angles and lengths to their correct kinematic values and then allowing deviations from these values as a result of both compliance and damping in each of two hip angles per leg and in leg length [17]. As would be expected, gaits which are statically stable at all times are easily realized by such a control scheme. A less obvious result, also obtained from simulation, is that even with such a simple control law, dynamic stability can be obtained in gaits which are statically unstable in some phases. Specifically, referring to Figure 2, if each front foot is lifted before the rear foot on the same side is placed, then a quadruped employing this gait (which is the normal quadruped *walk* [11]) will be statically unstable and will start to fall toward the unsupported side. However, providing the duration of this phase is not too long, when the rear foot is placed in a supporting position, the machine may be able to recover its balance. Simulation shows that it is not difficult to find spring constants and damping rates which dissipate the kinetic energy accumulated during each unstable phase so as to render the overall gait stable. Because a control law of the sort described is equivalent to an automotive type suspension and drive system composed of springs, dampers, and fluid couplers, a gait which can be so stabilized is said to be *passively stable*.

It appears that not all gaits are passively stable. Thus far all attempts to stabilize the quadruped trot [17] by the above type of elementary model reference control have failed. (The success reported in [17] was later found to have resulted from a programming error.) This is not too surprising since a trot, which involves support only by one diagonal pair of legs at a time, contains no statically stable phases [17]. It appears that active stability control is necessary for such a gait.

Active Stabilization Systems

The problem of stabilizing a quadruped or a biped gait which is not passively stable bears some resemblance to the problem of stabilizing an inverted pendulum. Figure 3 shows such a pendulum constrained to rotate in the x-y plane. As can be seen, if the base of the pendulum is considered to be moveable, and if the mass m, is allowed to rotate relative to the supporting arm, l, then this system possesses three degrees os freedom: b, Φ_1, and Φ_2. If it is possible to apply a torque at the lower end of the supporting arm, then

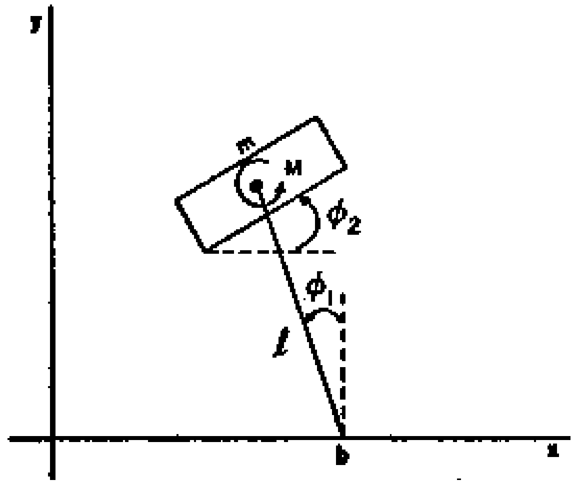

Fig. 3. An inverted pendulum system

there is no difficulty in stabilizing such a system in an upright position. Vukobratović et al [18] have shown that it also is not difficult to stabilize such a pendulum by automatically moving its base according to a feedback control law which computes b from Φ_1 and $\dot{\Phi}_1$. However, if b is fixed and no torque can be applied to the base of the pendulum, then it is not immediately obvious that a stabilizing mechanism can be found. To investigate this question, it will be convenient to assume that the supporting arm of the pendulum is both massless and perfectly rigid. The pendulum then becomes a constrained system and the Lagrangian formulation of classical mechanics [19] provides an organized way of obtaining the equations of motion.

Suppose that the mass m in Figure 3 is supported by a frictionless bearing located at its center of gravity and further that a torquing device is present to produce an arbitrary torque, M, between the supporting arm and m. Let the sense of M be such that a positive value tends to increase Φ_2. Finally, for convenience, let the origin of the x-y coordinate system be the base of the pendulum. Then, evidently, the coordinates of the center of mass are

$$x = -l \sin \Phi_1 \tag{8}$$

$$y = l \cos \Phi_1 \tag{9}$$

The potential energy is consequently given by

$$U = m\, g\, l \cos \Phi_1 \tag{10}$$

Since l is constant, from (8) and (9)

$$\dot{x} = -l\dot{\Phi}_1 \cos\Phi_1 \tag{11}$$

$$\dot{y} = -l\dot{\Phi}_1 \sin\Phi_1 \tag{12}$$

so the kinetic energy of the system is

$$T = \frac{1}{2} I \dot{\Phi}_2^2 + \frac{1}{2} m(\dot{x}^2 + \dot{y}^2) \tag{13}$$

$$= \frac{1}{2} I \dot{\Phi}_2^2 + \frac{1}{2} m l^2 \dot{\Phi}_1^2 \tag{14}$$

where I is the moment of inertia of the pendulum mass. Combining (10) and (14), the *Lagrangian* of the pendulum is given by

$$L = T - U \tag{15}$$

$$= \frac{1}{2} I \dot{\Phi}_2^2 + \frac{1}{2} m l^2 \dot{\Phi}_1^2 - mgl\cos\Phi_1 \tag{16}$$

According to the sign convention on M, the virtual work done by a virtual displacement of Φ_1 and Φ_2 is

$$\delta M = M(\delta\Phi_2 - \delta\Phi_1) = -M\,\delta\Phi_1 + M\,\delta\Phi_2 \tag{17}$$

Consequently, from the Lagrange equations of the second kind for nonconservative forces [19], it follows that the equations of motion for the inverted pendulum system are obtained from (16) and (17) by evaluating the expressions

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\Phi}_1} - \frac{\partial L}{\partial \Phi_1} = -M \tag{18}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\Phi}_2} - \frac{\partial L}{\partial \Phi_2} = M \tag{19}$$

Carrying out the indicated differentiations produces the coupled set of differential equations:

$$m l^2 \ddot{\Phi}_1 - mgl\sin\Phi_1 = -M \tag{20}$$

$$I\ddot{\Phi}_2 = M \tag{21}$$

To complete the analysis of the inverted pendulum system, it is necessary to choose a feedback control law. The general *linear* control law for this system can be written as

$$M = k_1\Phi_2 + k_2\dot{\Phi}_2 + k_3\Phi_1 + k_4\dot{\Phi}_1 \tag{22}$$

where the k_i are control system gain constants. Substitution of this equation into (20) and (21) produces the *controlled system equations*

$$m l^2\ddot{\Phi}_1 + k_4\dot{\Phi}_1 + k_3\Phi_1 - mgl\sin\Phi_1 = -k_2\dot{\Phi}_2 - k_1\Phi_2 \tag{23}$$

$$I\ddot{\Phi}_2 - k_2\dot{\Phi}_2 - k_1\Phi_2 = k_4\dot{\Phi}_1 + k_3\Phi_1 \tag{24}$$

The question now to be answered is whether or not there exist any values for the gain constants which stabilize this system. One way of settling this question is to make an assumption that Φ_t is a small angle so that it approximates its own sine and then to apply the Laplace transform to the resulting linearized version of (23) along with (24). When this has been done, the Routh criterion [20] can be used to determine if any parameter values exist such that all roots of the system characteristic equation have negative real parts. While the details of this calculation are too lengthy to present here, the result is favorable; stabilizing parameter values do exist. Figure 4 presents a numerical solution to (23) and (24) obtained with the parameter values shown on the figure. The stability of the system is evident.

The system treated above resembles a legged locomotion system in the respect that only forces and not moments can be applied to the "leg" by the supporting surface. The inverted pendulum particularly suggests a quadruped trot since a quadruped employing this gait is free to rotate about the line joining its two supporting

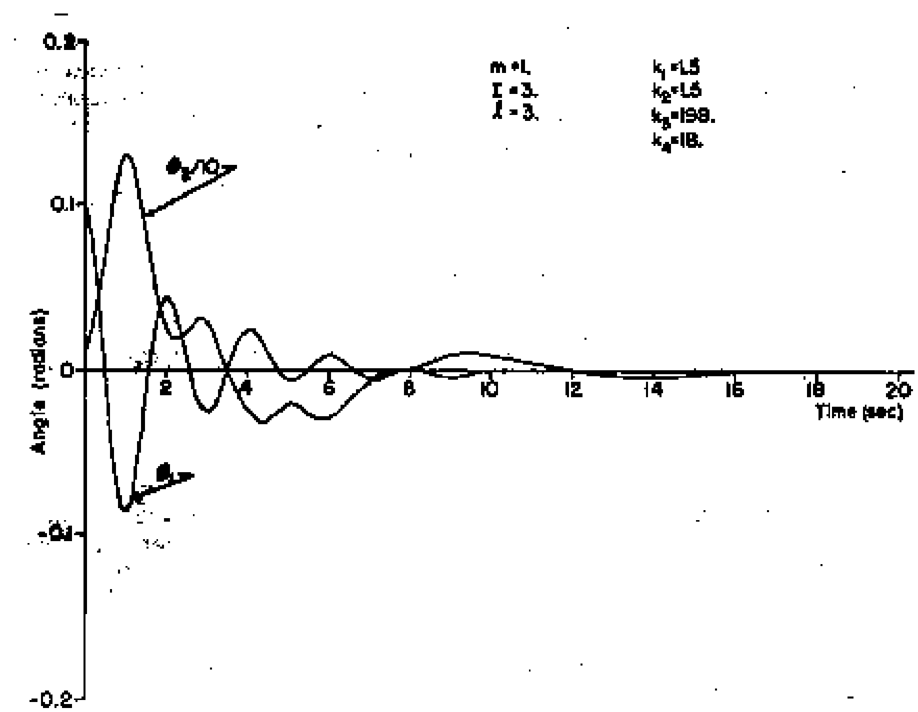

Fig. 4. Inverted pendulum transient response with linear feedback control

legs during any phase of locomotion [17]. The same observation can be made with respect to a biped walk during those phases when both feet are in contact with the ground. It is reasonable to hope, therefore, that a control law resembling (22) could stabilize either of these gaits. This question has been answered affirmatively for the trot by means of computer simulation. Specifically, to the "passive suspension" control law described earlier, a supplementary torque was added to the actuator controlling leg rotation normal to the nominal plane of motion. This torque was proportional to a linear combination of body displacement and body velocity normal to that plane. The roll behavior of the trot after this addition to the control law is qualitatively the same as the pendulum response shown in Figure 4.

Improved Dynamic Models

So far as is known to the authors, all analytic and computer studies of legged locomotion systems to date have either ignored leg masses, as in the equations presented in this paper, or have artifically constrained motion to a plane [21]. To the extent that legs can be represented as rigid segments joined by hinges or gimbal systems, these restrictions are unnecessary. It is possible to construct precise mathematical models for leg motion in three dimensions. The Lagrangian formulation of mechanics again appears to furnish the best method for introducing forces of constraint into the equations for such systems. Current research in legged locomotion at Ohio State University includes an effort to improve the existing general legged system simulation to incorporate leg mass for legs with a two degree of freedom hip joint and a one degree of freedom knee joint. While a great deal can be learned about quadruped locomotion with massless leg models, it is clear that a realistic study of human gait dynamics must include a leg with nonnegligible mass and at least the above three degrees of freedom.

Conclusions

The design and analysis of both natural and artificial legged locomotion systems can be put on a firm scientific basis by careful application of the laws of mechanics and an exploitation of modern control theory, finite state machine theory, and computer simulation. While the present body of results relating to such a quantitative mathematical theory is rather small, there is no inherent reason why this situation should prevail in the future. Serious efforts by physicists and engineers in cooperation with physiologists, physicians, and others in the fields of prosthetics, orthotics, and vehicle design should result in a theoretical understanding which will eventually lead to improved devices and machines for aiding and accomplishing legged locomotion.

REFERENCES

1. Liston, R. A., "Walking Machines", *Journal of Terramechanics*, Vol. 1, No. 3, pp 18—31, 1964.
2. Mosher, R. S., "Exploring the Potential of a Quadruped", SAE Paper No. 690191, presented at the International Automotive Engineering Conference, Detroit, Michigan, January, 1969.
3. Tomović, R., and McGhee, R. B., "A Finite State Approach to the Synthesis of Bioengineering Control Systems", *IEEE Transaction on Human Factors in Electronics*, Vol. 7, No. 2, pp 65—69, June, 1966.
4. Rakić, M., "Report on the Further Development of Belgrade Hand Prosthesis", *External Control of Human Extremities*, Yugoslav Committee for Electronics and Automation, Belgrade, 1967, pp 90—96.
5. Bottomley, A. H., "Progress with the British Myoelectric Hand", *External Control of Human Extremities*, Yugoslav Committee for Electronics and Automation, Belgrade, 1967, pp 114—124.
6. Muybridge, E., *Animals in Motion*, Dover Publications, Inc., New York, 1957, (first published in 1899).
7. Muybridge, E., *The Human Figure in Motion*, Dover Publications, Inc., New York, 1955 (first published in 1901).
8. Tomović, R., "A General Theoretical Model of Creeping Displacement", *Cybernetica*, IV, pp 98—107 (English translation), 1961.
9. Tomović, R., and Karplus, W. J., "Land Locomotion-Simulation and Control", *Proceedings of the Third International Analogue Computation Meeting*, Opatija, Yugoslavia, pp 385—390, September, 1961.
10. Tomović, R., "On the Synthesis of Self-Moving Automata", *Automation and Remote Control, Vol. 26*, pp 297—304 (English translation), February, 1965.
11. McGhee, R. B., "Finite State Control of Quadruped Locomotion", *External Control of Human Extremities*, Yugoslav Committee for Ecletronics and Automation, Belgrade, 1967, pp 221—231.
12. Hildebrand, M., "Symmetrical Gaits of Horses", *Scince, Vol. 150*, pp 701—708, November, 1965.
13. Hildebrand, M., "Analysis of the Symmetrical Gaits of Tetrapods", *Folia Biotheoretica, Vol. 4*, pp 9—22, 1966.
14. McGhee, R. B., "Some Finite State Aspects of Legged Locomotion", *Mathematical Biosciences, Vol. 2, No. 1/2*, pp 67—84, February, 1968.
15. McGhee, R. B., and Frank, A. A., "On the Stability Properties of Quadruped Creeping Gaits", *Mathematical Biosciences, Vol. 3, No. 3/4*, pp 331—351, October, 1968.
16. Frank, A. A., "Automatic Control Systems for Legged Locomotion Machines", Ph. D. Dissertation, University of Southern California, Los Angeles, California, May, 1968.
17. Frank, A. A., and McGhee, R. B., "Some Considerations Relating to the Design of Autopilots for Legged Vehicles", *Journal of Terramechanics, Vol. 6, No. 1*, 1969.
18. Vukobratović, M., Marić, M., and Gavrilović, M., "On Stability of Locomotion Automata", *External Control of Human Extremities*, Yugoslav Committee for Electronics and Automation, Belgrade, 1967, pp 218—220.
19. Greenwood, D. T., *Principles of Dynamics*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965.
20. Gille, J. C., Pelegrin, M. J., Decaulne, P., *Feedback Control Systems*, McGraw-Hill Book Company, Inc., New York, 1959.
21. Vukobratović, M., and Juricić, D., "Contribution to the Synthesis of Biped Gait", *IEEE Transactions on Biomedical Engineering, Vol. 16, No. 1*, pp 1—6, January, 1969.