

BIOMECHANICAL APPROACH TO PROSTHESIS DESIGN OF LOWER
EXTREMITIES WITH EXTERNAL POWER SOURCES

I. S. Moreinis

Summary

Investigation of human locomotion, with the help of modern instrumentation, mathematical models, and electrophysiology, allows to determine the losses of power during locomotion of normal test subjects as well as amputees. Amputation causes limitation of muscle power resources and produces compensating overloading of unserved muscles. These overloadings can be minimized by means of a prosthesis of optimal design, especially by a prosthesis using external power sources.

Components of "propulsion problem" of walking have been determined. These components are to be normalized with help of special devices which incorporate external sources of power and multiple control systems. The main parameters of such sources have been found.

Consider the problem of modelling normal human locomotion using methods of classical mechanics, bio-mechanics and electrophysiology. Some aspects of the solution of this problem are likely to be of importance while selecting, for instance, the power sources for the prosthesis for the lower limbs which enable, to some extent, compensation for the losses of muscles caused by the amputation.

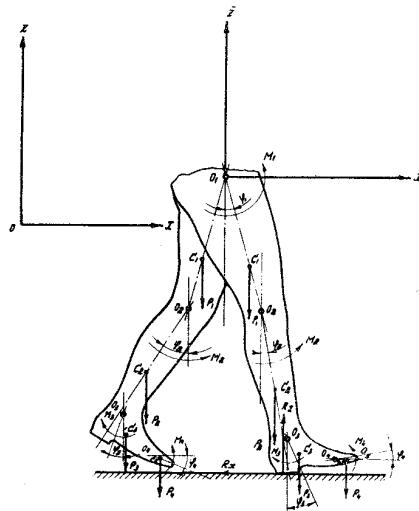


Fig. 1. Model of human extremities

As a model imitating the movements of lower limbs physical pendulum consisting of four limbs has been chosen. The first point of plumb line is a movable one and coincides with the center of hip joint (Fig. 1).

The following notation is applied:

Oxz - reference coordinate system,

$O_1\tilde{x}\tilde{z}$ - movable coordinate system,

$O_1O_2=L_1$, $O_2O_3=L_2$, $O_3O_4=L_3$, $O_4O_5=L_4$ - lengths of femur, shank, front and rear portion of foot respectively,

O_2 - centre of knee joint,

O_3 - centre of ankle joint,

O_4 - centre of metatarsal phalangeal joint,

ϕ_1 , ϕ_2 , ϕ_3 - angles between the axes of joints and vertical line,

ϕ_4 - deviation of the axis of the front portion of the foot from horizontal line (angles ϕ_i have been chosen as generalised coordinates),

$l_i=O_iC_i$ - static radii of segments,

i_i - radii of gyration about axes coming through the joint centres perpendicular to the sagittal plane,

P_i - weights of segments,

R_x , R_z - components of support to foot reaction,

M_i - total moments of muscle forces causing the rotation of the limbs about axes coming through the joint centres perpendicular to the sagittal plane,

X , Z - coordinates of O_1 - first point of pendulum,

L - human height, P - human weight.

It should be noted that moments M_i are the resulting moments of all muscles, and are applied in such a way that cause the movement of the relative joint. The nature of interaction of these muscles is not discussed in this paper. The evaluation of power spent for rotation in relative joints is under consideration here. Unloading walking was only taken into account, and therefore the system to be discussed deals with nonintergrated bonds. Limitations are imposed only on the variations of interjoint angles, and these have been chosen as independent parameters.

The natural movements of extremities are sure to be complicated. Not only muscles located in the leg region are used during walking, but the muscles of trunk and arms are brought into play. To consider the action of the whole muscles the model should be

more complicated, not consisting only of four links but of multiple links. At present there is no principle difficulty in developing a multiple link model. Time only is needed for the collection of the information about the kinetic and dynamic characteristics of the upper parts of body during locomotion.

The problem is likely to be forthcoming in the near future on the basis of steady complication of model, but at present moment we want to concentrate on the useful information which can be obtained from the analysis of the four link system.

Omitting the details of method, the Lagrange equations of the second order, keeping in mind the following remarks, are composed:

O_1 and centre of gravity are closely located, and

$$\ddot{x} = \frac{R_x}{m} = \frac{R_x g}{P}; \quad \ddot{z} = \frac{(-P+R_z)g}{P}$$

These forms of motion have been chosen so that the surface-to-foot reactions can be found directly by simple experiments, but methods for the determination of the acceleration are complicated and are not utilized widely.

In connection with the above, the expression for the surface-to-foot reactions as a fraction of human weight are as follows: $R_x = K_x P$; $R_z = K_z P$

where K_x , K_z are dynamic coefficient in the case of their representation of the linear acceleration of the hip joint (or centre of gravity) and will remain K_{x1} , K_{z1} when they represent surface to foot reaction of one leg. In accordance with the above and after corresponding transformations, the equations of motion are:

$$\begin{aligned} & \alpha_{11} \ddot{\phi}_1 + \alpha_{12} (\ddot{\phi}_2 \cos(\phi_1 - \phi_2) + \dot{\phi}_2^2 \sin(\phi_1 - \phi_2)) + \alpha_{13} (\ddot{\phi}_3 \sin(\phi_1 - \phi_3) - \dot{\phi}_3^2 \cos(\phi_1 - \phi_3)) + \\ & + \alpha_{14} (\ddot{\phi}_4 \sin(\phi_1 - \phi_4) - \dot{\phi}_4^2 \cos(\phi_1 - \phi_4)) + C_{11} (K_x \cos \phi_1 + K_z \sin \phi_1) + \\ & + C_{12} (K_{x1} \cos \phi_1 - K_{z1} \sin \phi_1) = M_1; \end{aligned}$$

$$\begin{aligned} & \alpha_{21} (\ddot{\phi}_1 \cos(\phi_2 - \phi_1) + \dot{\phi}_1^2 \sin(\phi_2 - \phi_1)) + \alpha_{22} \ddot{\phi}_2 + \alpha_{23} (\ddot{\phi}_3 \sin(\phi_2 - \phi_3) - \dot{\phi}_3^2 \cos(\phi_2 - \phi_3)) + \\ & + \alpha_{24} (\ddot{\phi}_4 \sin(\phi_2 - \phi_4) - \dot{\phi}_4^2 \cos(\phi_2 - \phi_4)) + C_{22} (K_x \cos \phi_2 + K_z \sin \phi_2) + \\ & + C_{23} (K_{x1} \cos \phi_2 - K_{z1} \sin \phi_2) = M_2 \end{aligned}$$

$$\begin{aligned}
& -\alpha_{31}(\ddot{\phi}_1 \sin(\phi_3 - \phi_1) - \dot{\phi}_1^2 \cos(\phi_3 - \phi_1)) - \alpha_{32}(\ddot{\phi}_2 \sin(\phi_3 - \phi_2) - \dot{\phi}_2^2 \cos(\phi_3 - \phi_2)) + \\
& + \alpha_{33}\ddot{\phi}_3 + \alpha_{34}(\ddot{\phi}_4 \cos(\phi_3 - \phi_4) + \dot{\phi}_4^2 \sin(\phi_3 - \phi_4)) - C_{33}(K_x \sin \phi_3 - K_z \cos \phi_3) + \\
& + K_{x1}P(z - L_1 \cos \phi_1 - L_2 \cos \phi_2) - K_{z1}Px_1 = M_3;
\end{aligned}$$

$$\begin{aligned}
& -\alpha_{41}(\ddot{\phi}_1 \sin(\phi_4 - \phi_1) - \dot{\phi}_1^2 \cos(\phi_4 - \phi_1)) - \alpha_{42}(\ddot{\phi}_2 \sin(\phi_4 - \phi_2) - \dot{\phi}_2^2 \cos(\phi_4 - \phi_2)) + \\
& + \alpha_{43}(\ddot{\phi}_3 \cos(\phi_4 - \phi_3) + \dot{\phi}_3^2 \sin(\phi_4 - \phi_3)) + \alpha_{44}\ddot{\phi}_4 - C_{44}(K_x \sin \phi_4 - K_z \cos \phi_4) + \\
& + K_{x1}P(z - L_1 \cos \phi_1 - L_2 \cos \phi_2 + L_3 \sin \phi_3) - K_{z1}P(x_1 - L_3 \cos \phi_3) = M_4.
\end{aligned}$$

Coefficients in this system are the combinations of inertia characteristics of the segments. It should be noted that the equation

$$\alpha_{ij} = \alpha_{ji}$$

is correct for the system, and is in accordance with the general features of the theory of swinging. Coefficients α_{ij} can be represented as a matrix which is symmetrical to the main diagonal.

$$\begin{vmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\
\alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\
\alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\
\alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44}
\end{vmatrix}
=
\begin{vmatrix}
J_{O_1Y} + \frac{1}{g}P_{2+3+4}L_1^2; & \frac{1}{g}(P_2L_2 + P_{3+4}L_2)L_1; & \frac{1}{g}(P_3L_3 + P_4L_3)L; & \frac{1}{g}P_4L_4L_1 \\
\frac{1}{g}(P_2L_2 + P_{3+4}L_2)L_1; & J_{O_2Y} + \frac{1}{g}P_{3+4}L_2^2; & \frac{1}{g}(P_3L_3 + P_4L_3)L_2; & \frac{1}{g}P_4L_4L_2 \\
\frac{1}{g}(P_3L_3 + P_4L_3)L_1; & \frac{1}{g}(P_2L_2 + P_4L_3)L_2; & J_{O_3Y} + \frac{1}{g}P_4L_3^2; & \frac{1}{g}P_4L_4L_3 \\
\frac{1}{g}P_4L_4L_1; & \frac{1}{g}P_4L_4L_2; & \frac{1}{g}P_4L_4L_3; & J_{O_4Y}
\end{vmatrix}$$

Coefficients C_{ij} are as follows:

$$\begin{aligned}
C_{11} &= P_1L_1 + P_{2+3+4}L_1; & C_{12} &= PL_1; \\
C_{22} &= P_2L_2 + P_{3+4}L_2; & C_{23} &= PL_2; \\
C_{33} &= P_3L_3 + P_4L_3; & C_{44} &= P_4L_4
\end{aligned}$$

Because the weight, static radius and moment of inertia of any segment of the human body can be expressed as proportions of weight, P , and height, L , all the coefficients α_{ij} and C_{ij} can be given as follows:

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{vmatrix} = \frac{10^{-4}}{g} PL^2 \begin{vmatrix} 47.40 & 21.10 & 1.35 & 0.03 \\ 21.10 & 19.10 & 1.38 & 0.03 \\ 1.35 & 1.38 & 0.50 & 0.01 \\ 0.03 & 0.03 & 0.01 & 0.01 \end{vmatrix}$$

$$C_{11} = 10^{-4} PL \times 270.00$$

$$C_{12} = 10^{-4} PL \times 2380.0$$

$$C_{22} = 10^{-4} PL \times 88.00$$

$$C_{23} = 10^{-4} PL \times 2410.0$$

$$C_{33} = 10^{-4} PL \times 7.05$$

$$C_{44} = 10^{-4} PL \times 0.135$$

In this case the equations turn into algorithms for the determination of the moments of muscle forces M_i on the basis of angular displacements, velocities, and accelerations ($\phi_i, \dot{\phi}_i, \ddot{\phi}_i$), surface to foot reactions (K_x, K_z), coordinates of their application point (X) and coordinate of the movable point of the plumb line (Z). These data have been collected by means of known methods incorporating differential block of analogue machine.

Figure 2 represents the charts of angular displacements, velocities, and accelerations of any leg joint during double stroke.

Figure 3 represents the charts of vertical (K_z) and longitudinal (K_x) components of surface to foot reactions and their total values. These components have been obtained with the help of tensometric dynamo-grapher of CSRIRPP construction.

Using the above data the programme for IBM has been developed, and scanning of moments of muscle forces has been achieved.

Keeping in mind that the period of double stroke is usually 1.216 second and $PL = 11900$ kgm (for some anthropological type) the mean value of the moments we found was as follows:

$$* \quad M_{cp} = \frac{1}{T} \int_0^T dM$$

Using the charts (Fig. 4) the work within the angular displacements can be determined as below:

$$A_i = M_{\text{mean}} \int_{\beta_0}^{\beta_T} d\beta = M_{cp} \beta \Big|_{\beta_0}^{\beta_T}$$

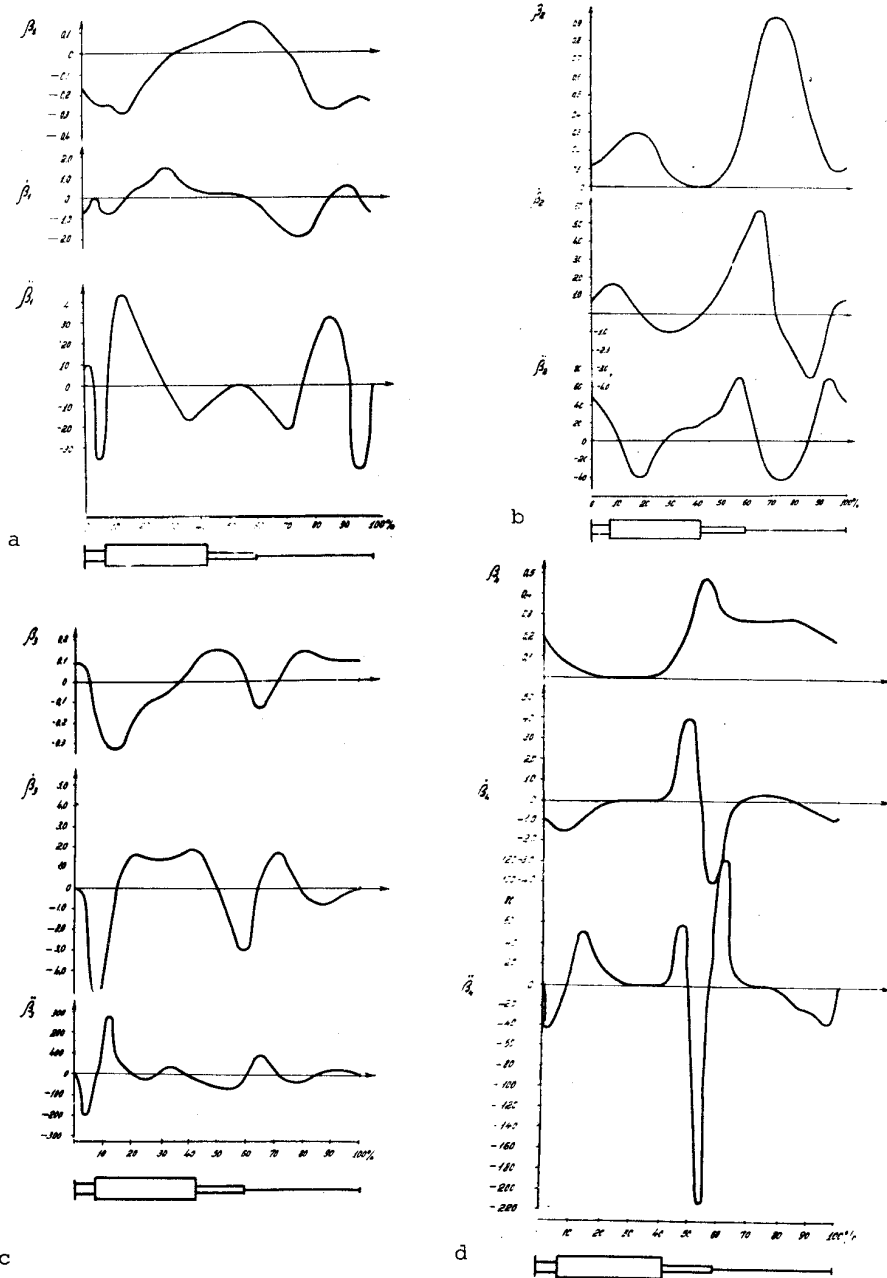


Fig. 2. Angular displacements, velocities and accelerations of big joints of the lower extremity during walking (hip joint, knee joint, ankle joint, metatarsophalangeal joint).

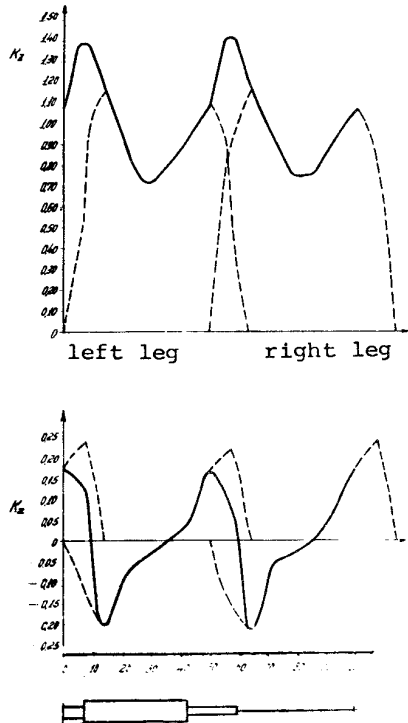


Fig. 3. Vertical (K_{z1}) and longitudinal (K_{x1}) components of surface to foot reaction of each extremity, (K_z, K_x) - total for both extremities.^x

All the data are given in table 1.

Table 1

β_1	β_2	β_3	β_4	M_1 kgm	M_2 kgm	M_3 kgm	M_4 kgm	ΣM_i kgm	A_1 kgm	A_2 kgm	A_3 kgm	A_4 kgm	ΣA_i kgm
1.1	2.2	1.5	1.1	2.36	1.82	1.50	5.02	10.70	2.60	4.00	2.25	5.52	14.37

39% 61% 100%

The vertical dotted line divides the curve of the moments into stance (left) and swing phases. If we assume that 100% is the total moment of the lower limb in one double stroke total muscle moment during stance phase is about 85% and during swing phase about 15%. So, the peak energetic period of the muscle activity is during the stance phase. It should be expected that the muscles are activated in the stance phase to lift the centre of

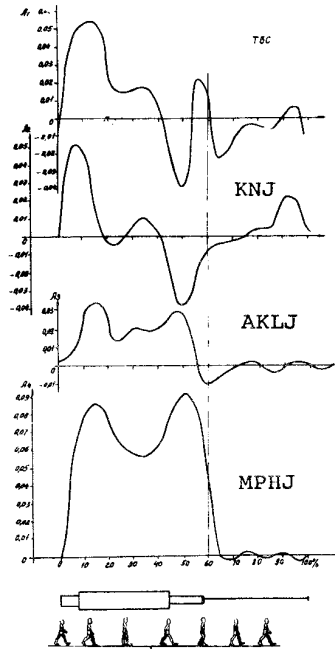


Fig. 4. Scanning of the moments exerted by muscles.

gravity and to accelerate the body in movement forward.

The phenomena that the peak moment and work are generated by the muscles at the time of rotation around the axis through the metatarsophalangeal joint, also is not unexpected. Just before the instant of toe-off the metatarsophalangeal joint is located on the surface and can be compared with the bearing housing. Otherwise, the body can be treated as a rod swinging around the above joint. The rod represents almost the whole body. The lifting of the centre of gravity and moving of the body forward are taking place at this instant. It is not likely that the metatarsophalangeal joint of the human is protruded topmost forward. It is done to make the lever arm of any leg muscle more effective.

Here one more perplexity should be excluded. The meaning of M_i , for instance, $M_{\text{metatarsus}}$ is to be considered as the integrated result of action of all the shank muscles, but not the muscles located only in the region of the metatarsophalangeal joints.

How the power developed by the muscles within the whole cycle of movements can be calculated for each joint. In accordance with the charts the time of action of moments $M_{\text{hip joint}}$ and $M_{\text{knee joint}}$ was determined to be 1.216 seconds while the time of actions of movements $M_{\text{metatarsus}}$ and $M_{\text{ankle joint}}$ is $0.6 \times 1.216 = 0.730$ sec.

The power (W) developed in respective joints is $W_{\text{hip joint}} = 20.9\text{wt}$; $W_{\text{knee joint}} = 32.1\text{wt}$; $W_{\text{ankle joint}} = 30.2\text{wt}$; $W_{\text{metatarsus}} = 70.5\text{wt}$; $\Sigma W_i = 157.7\text{wt}$ - for one limb, 314.4wt for both limbs.

To form a true notion, let us calculate the work spend during walking at a distance of 5 km. Considering the mean length of double stroke is 1.4 meter

$$A = \frac{5000}{1.4} \times 14.37 = 51 \times 10^3 \text{ kqm}$$

It is known that efficiency of muscles of the normal human is 0.2 (for sportsman - 0.4).

Consequently, to restore the power losses during walking the following amount of power is needed:

$$E = 2 \frac{A}{0.2} = 51.2 \times 10^4 \text{ kqm} = 1.40 \text{ kwt/hour}$$

If the normal human being engaged in a simple labour process needs about 4 kwt/hour to restore the power losses throughout the day and night, and we assume this amount as 100%, the power ex-

penditures during walking is about 35%. Our data were compared with the data obtained by the physiologists.

For this integrated electric myograms of 16 leg muscles, which are the main "motor-participants" in walking in one double stroke, have been obtained on ten normal subjects.

The constancy of the experiment was provided by utilization of the same equipment (amplifiers УБП-1-01 integrator of CSRIRPP construction, oscilloscope H-102, standard amplifier supplying the integrator with voltage and giving 50 impulses (i) per second.

Statistical analysis allowed us to determine that the total bioelectric activity of two proximal joints (hip joint and knee joint) is about 39%, but of the two distal ones is 61%, i.e. bioelectric activity distributes like total moments of muscle forces (Fig. 1). Distribution of total bioelectric activity during stance and swing phases has the analog trend, i.e. 79% and 21% respectively. (85% and 15% - in the case of moments).

The natural supposition may arise about the existance of a linear relationship between total bioelectric and mechanical activities of the muscles at the time of normal unloaded walking, it should be noted that this phenomenon was found earlier regarding the single muscle.

The order of the equivalent connection can be found if the mean total bioelectric activity (584) is compared with the total moment of the muscle forces (10.70 kgm) developed within one double stroke. In such a way it has been found that one impulse "i" of bioelectric activity is equivalent approximately to 2 kgm.

For different types of test subjects, this coefficient is used to be equal in the case of walking of 10 normal men - 1.84; for normal extremity of 10 amputees with above knee prosthesis - 2.05; for normal extremity of 10 amputees with below knee prosthesis - 1.92.

The reported facts are likely to be of interest not only as an example of synthesis of mechanics and physiology, but also because they can be useful to the designers of anthropomorphic mechanisms.