OPTIMAL LOCOMOTION PATH PLANNING OF LEGGED MECHANISMS

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Summary

Optimal control of locomotion of legged mechanisms is extremely complex, and therefore difficult to synthesize. However, a concept based on hierarchical multi-level structuring of the control system offers an oportunity to realize a near optimal form of legged locomotion. Within that scope, a planning problem of optimal legged locomotion paths has been considered. This report shows how that problem, which cannot be solved by the classical calculus of variations, can be stated as a problem of finding optimal paths through a graph and how efficiently it can be solved. The procedure described has been evaluated with reference to particular examples that have been dealt with in this study. Having considered different aspects involved, it was concluded that the described procedure is suitable for implementation.

Introduction

A legged system is a linked mechanism which has many degrees of freedom, and therefore it is a multivariable dynamic system. The locomotion control task is also complex. It has two goals: first, to move legs in order to produce walking; second, to simultaneously maintain the mechanism in an upright state.

The objective of optimal control is to solve the locomotion task while at the same time optimizing a prescribed criterion, for instance, energy or time. However, different obstacles and terrain boundaries as well as constraints on the articulation, actuation, and energy resources make the locomotion task even more difficult. To those acquainted with the theory of optimal control and its practical capabilities it should be clear that the problem of synthesis of optimal control of legged locomotion is extremely difficult. Efforts in that direction have been made, but to date limited progress has been achieved /1/.

However, a new concept based on hierarchical multi-level structure of control systems of multi-linked mechanisms /2/, offers practical possibilities for synthesizing a near optimal legged locomotion control. According to this concept, the locomotion control system has a functional hierarchical multi-level structure in which the optimal path planning takes place at one of the highest control levels. The optimal path is directed as a

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reference input to the lower control levels, which organize the locomotion of the mechanism by means of functional movements of legs, whilst the upright state is being maintained.

The subject of this consideration is the synthesis of optimal path along which the legged mechanism performs a quasi-stationary locomotion, from an initial to the desired location, with minimum energy expenditure. It will be shown how the optimal path planning is reduced to determination of the optimal path through a graph, and how efficiently it can be solved by extended dynamic programming /3/.

Problem Statement

The scope of this paper is reduced to the synthesis of optimal paths of the legged mechanism, along which locomotion can be regarded as quasi-stationary. This assumption is based on the fact that a legged mechanism reaches its economical locomotion speed in the first few steps, i.e. on a very short initial part of the locomotion trajectory.

Expression for the power consumption for legged locomotion is very complex /4/, because locomotion consists of several coordinated movements of the legs and the remaining parts of the mechanism.

Due to such unsteady movements of the links, the power consunption is not constant during the stepping cycle. However, assuming a stationary legged locomotion, the average power consumption can well be approximated by the following expression:

$$P = B + F\,v + k(\alpha)\,G\,v\,\sin\alpha$$

where:

- P - is the power consumption,
- B - is the power for maintaining the mechanism upright,
- F - is the force intensity equivalent to friction in the joints,
- v - is the intensity of the locomotion speed,
- G - is the weight of the mechanism,
- α - is the elevation angle of the locomotion path,
- $k(\alpha) = \begin{cases} 1 & \text{if } \alpha > 0 \\ 0 & \alpha \le 0 \end{cases}$ - is the stepwise function.

Therefore, during legged locomotion, power is consumed to maintain the mechanism in the upright state, and to overcome the resisting friction. If the mechanism ascends, an additional amount of power is consumed to overcome the gravitational force. However, the potential energy accumulated by the mechanism, cannot be recuperated practically, which is also the case for legged locomotion of living systems.

The energy consumed by the legged mechansim along the particular path can be obtained by integration:

$$E = \int_S \left(B\ dt + F\ ds + k(\alpha)\ G\ dz\right) \qquad (2)$$

where:

$dt = \frac{1}{v} ds$ - is the time increment,

$ds = \sqrt{dx^2+dy^2+dz^2}$ - is the increment of the travel led path,

dx, dy, dz - are the increment of x, y and z coordinates respectively,

S - is a selected path.

Path along which the legged locomotion is to be performed lies on the terrain described by the following general expression:

$$X = \{(x,y,z) \mid (x,y) \in D,\ z=h(x,y)\} \qquad (3)$$

where:

$(x,y,z) = r$ - is the terrain point,

(x,y) - is the image point of the terrain base,

D - is the domain of the terrain base by which the terrain passability is defined,

$z = h(x,y)$ - is the function which describes the terrain surface.

The problem of optimal legged locomotion path planning is stated as follows:

Among all feasible paths which lead through the terrain X, from a starting point r_s to the desired terminal point r_t, select such a path(s) along which the locomotion energy expenditure assumes its minimum values,

$$E_{min} = \min_S \int_S \left(B\ dt + F\ ds + k(\alpha)\ G\ dz\right) \qquad (4)$$

Problem Solving

The planning problem of the optimal legged locomotion path belongs to the class of problems in the calculus of variations.

But, that problem cannot be solved by the classical methods because the subintegral function of Expression 4 is discontinuous, and the terrain passability is constrained while the function of the terrain surface can be very complex.

However, the stated problem can be defined as a problem of optimal path finding through a graph and then as such solved successfully.

For that reason the domain D by which the terrain passability is defined has to be discretized and described by the finite set $D = \{(x,y)\}$. Such discretization is realistic because the terrain recording, in effect, is done by measuring the altitudes $z = h(x,y)$ in isolated points (x,y) of the terrain base. The distribution of points on the terrain base can be regular or irregular. Irregular distribution are inefficient for the processing, whilst the regular ones are suitable for it. There are two regular distributions: one is based on the square mosaic; the other is based on the equilateral triangle mosaic.

After discretization, the terrain becomes described by the finite set $X = \{(x,y,z)\}$.

A number of path networks can be structured over such discretized terrain. Only those networks, for which the terrain points are path-connected to their adjacent points are good approximations to the network over the continuous terrain surface. However, the regular path networks are the most suitable for the processing. The basic regular networks are:

(1) the equilateral mosaic network

(2) the square mosaic network

(3) the equilateral triangle mosaic network.

The path network over the terrain X, can be described by the graph $G = (X, \Gamma^{-1})$ in the following way:

Each point $r_n = (x,y,z)_n$ of the discretized terrain is represented by a node x_n. In this way the set of the nodes is thus defined,

$$X = \{x_1, x_2, \ldots, x_n, \ldots, x_{n_{max}}\} \tag{5}$$

Each point of the terrain can be approached only from these adjacent points which are path-connected to it. Hence, each node x_n of the graph can be adjoined by a set of adjacent nodes x_{n_i}, by which its inverse mapping is defined,

$$\Gamma^{-1}(x_n)=\{x_{n_1},x_{n_2},\dots,x_{n_i},\dots,x_{n_{i_{namx}}}\},n=1,2,\dots,n_{max} \quad (6)$$

Therefore, Expression 6, define the inverse mapping Γ^{-1} of the graph.

Different obstacles, if present, reduce the terrain passability. The original set of the terrain points is reduced by those points which are occupied by the obstacles. In effect, it implies deletion of the corresponding nodes from the set X, and the relevant modification of the inverse mapping Γ^{-1} of the graph G.

In addition, it will be assumed that path links which connect the adjacent terrain points are approximated by strainght lines. The expression for energy consumption for locomotion directed along a link (i.e. along an arc) can be readily derived from Expression 2,

$$E(r_n,r_{n_i}) = R\Delta s^n_{n_i} + k(\Delta z^n_{n_i})\ G\Delta z^n_{n_i} \quad (7)$$

where:

$r_n = (x,y,z)_n$ - is the terminal point of the arc,

$r_{n_i} = (x,y,z)_{n_i}$ - is the initial point of the arc,

$\Delta z^n_{n_i}$ - is the height ascent

$\Delta s^n_{n_i} = \sqrt{(\Delta x^n_{n_i})^2 + (\Delta y^n_{n_i})^2 + (\Delta z^n_{n_i})^2}$ - is the link length,

$R = \frac{1}{v} B + F$ - is the resultant force,

$$k(\Delta z^n_{n_i}) = \left.\begin{array}{ll} 1 & \text{if } \Delta z^n_{n_i} > 0 \\ 0 & \qquad\quad\ \le 0 \end{array}\right\}$$ is the stepwise function.

Locomotion is assumed to be performed at constant speed, the economical speed being optimal. Hence the resultant force R remains constant. As a consequence, the energy consumed along a path arc, in maintaining upright state and in overcoming friction is proportional to its length. If the mechanism ascends, an additional amount of energy is consumed. That extra amount of energy is proportional to the height of the ascent.

The energy consumption along a path arc per weight unit of the legged mechanism is chosen as the cost of mapping of the initial node of the arc into its terminal node, $t(x_n,x_{n_i}) =$

$= \frac{1}{G} E(r_n, r_{n_i})$. In this way every inverse mapping set $\Gamma^{-1}(x_n)$ is adjoined by the relevant set of mapping costs $T(x_n)$,

$$T(x_n) = \{t_{n_1}, t_{n_2}, \ldots, t_{n_i}, \ldots, t_{n_{i_{nmax}}}\} \quad n=1,2,\ldots,n_{max} \qquad (8)$$

The above set of expressions represents the mapping costs T of the graph.

The problem of optimal locomotion path planning of the legged mechanism can now be stated as a problem of optimal path finding through a graph:

A graph is given by the set of nodes X, the inverse mapping Γ^{-1}, and the mapping costs T, $G = (X, \Gamma^{-1}, T)$. Find such a path through the graph which starts in the node x_s and terminates in the specific node x_t, for which the total costs of mapping assume its minimum value.

This problem could have been solved by any one of the methods known in the theory of graphs. However, the optimal legged locomotion path planning in particular examples was done by the extended dynamic programming method, an efficient method which was elaborated by the author of this report /3/.

Examples

In order to evaluate the described procedure, the path planning of the optimal legged locomotion has been exercised. For that reason a program has been written in FORTRAN IV for the IBM 360/44 computer according to he described procedure. Its flow chart is given in Figure 1.

A legged mechanism has been chosen for which resultant force/weight ratio (Eq. 7) is

$$\frac{F}{G} = 0.2$$

The passable domain of the terrain is assumed to be defined by the following expression,

$$D = \{(x,y) \quad x_{min} \leq x \leq x_{max}, \; y_{min} \leq y \leq y_{max}\} \qquad (9)$$

Firstly, the domain D was discretized with respect to the square mosaic and the following finite set was obtained

$$D = \{(x_i, y_j)\} \qquad (10)$$

where:

$$x_i = x_{min} + (i-1)\Delta, \quad i=1,2,\ldots,i_{max},$$

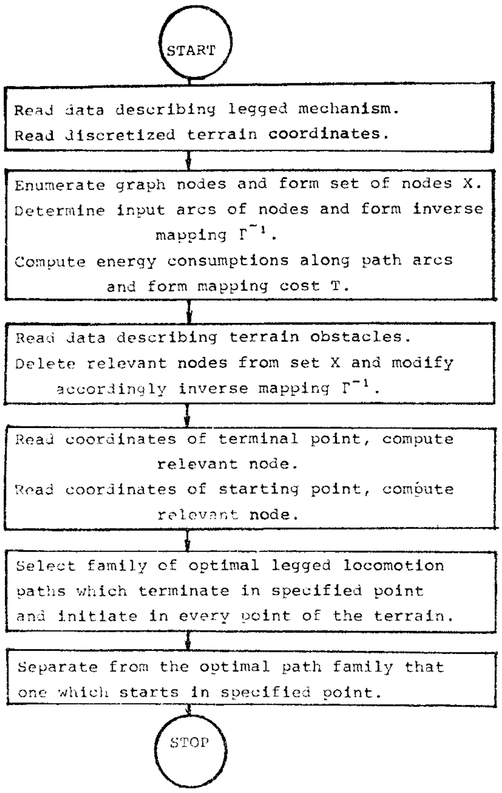

Fig. 1.

$$y_j = y_{min} + (j-1)\Delta, \quad j=1,2,\ldots,j_{max},$$

$$i_{max} = 1 + \mathrm{int}\left(\frac{x_{max} - x_{min}}{\Delta}\right)$$

$$j_{max} = 1 + \mathrm{int}\left(\frac{y_{max} - y_{min}}{\Delta}\right)$$

Δ denotes the discretization interval.

From all regular path networks which can be structured, the square mosaic network has been chosen for its simplicity. Each point of the discretized terrain in this case is surrounded by the four adjacent points at most, which assume equidistant locations,

(x_i, y_j, z_{ij}) surrounded by $(x_{i-1}, y_j, z_{i-1,j})$ if $i-1 \geq 1$

$$(x_{i+1}, y_j, z_{i+1,j}) \text{ if } i+1 \le i_{max}$$
$$(x_i, y_{j-1}, z_{i,j-1}) \quad j-1 \ge 1 \qquad (11)$$
$$(x_i, y_{j+1}, z_{i,j+1}) \quad j+1 \le j_{max}$$

The nodes corresponding to the terrain points were enumerated according to the expression:

$$n = i + (j-1)\, i_{max} \qquad i=1,2,\ldots,i_{max} \quad j=1,2,\ldots,j_{max} \qquad (12)$$

The terrain presented in Figures 2 and 3 by the contour lines has been discretized accordingly. The following parameter values were used:

$\Delta = 20$ m

$i_{max} = 20$

$j_{max} = 20$

The point $(x_2, y_{11}, z_{2,11})$ was specified as the terminal point x_t.

The two starting points x_{s_1} and x_{s_2} were specified, one of which was $(x_{19}, y_3, z_{19,3})$, the other being $(x_{17}, y_{16}, z_{17,16})$. The optimal legged locomotion paths were computed across the terrain without obstacles first of all. The optimal solutions are presented in a graphical form in Figure 2. Then optimal paths were computed across the terrain with two obstacles. They are given in Figure 3. In both cases the computation of optimal paths lasted approximately 1.5 seconds.

Additionally, the domain D is discretized with reference to the equilateral triangle mosaic. Discretization intervals applied to x and y coordinates were Δ and $\frac{\sqrt{3}}{2}\Delta$, respectively. The equilateral triangle mosaic was taken as the most suitable regular path network. Each node in such a net is path-connected to six adjacent nodes at most which are located equidistantly /3/. Specific discretization was done with the following parameter values,

$\Delta = 24.5$ m

$i_{max} = 16$

$j_{max} = 19$

The point $(x_2, y_{11}, z_{2,11})$ was chosen as the terminal point.

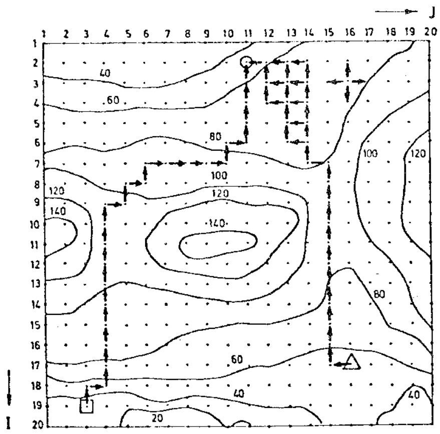

Fig. 2.

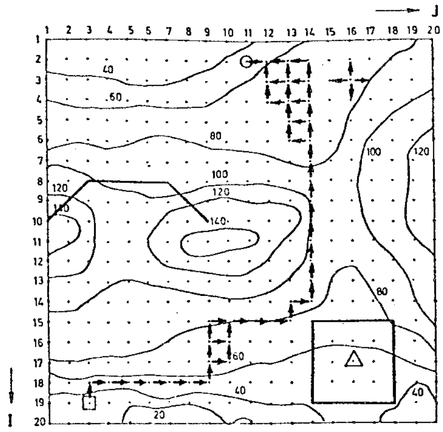

Fig. 3.

The two points, $(x_{15}, y_3, z_{15,3})$ and $(x_{14}, y_{15}, z_{14,15})$ were specified as the starting points of locomotion paths. In the first case, the optimal legged locomotion paths were computed for the obstacle free terrain. The optimal paths are shown in Figure 4. Then, the optimal paths were computed for the terrain with two obstacles. The solutions obtained are presented in Figure 5. In both cases computation was very efficient as before, it took only about 1.5 seconds.

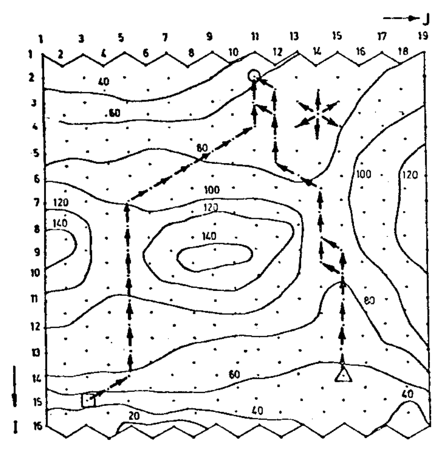

Fig. 4.

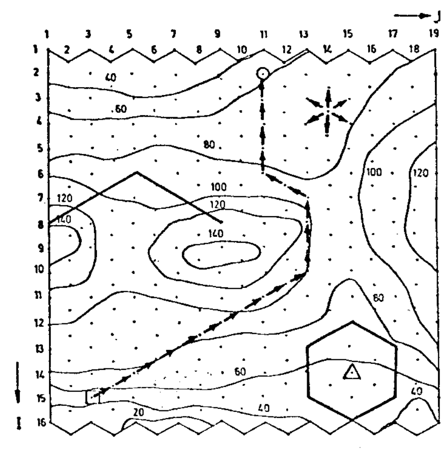

Fig. 5.

Conclusion

The problem of optimal legged locomotion path planning has been considered with the assumption that locomotion of the mechanism is opposed by the friction in the joints, that the potential energy accumulated by the mechanism cannot be recuperated, and that the energy expended in locomotion is chosen as the optimization criterion. Locomotion has been considered across the constrained terrain. Also, the terrain with obstacles has been treated.

It has been shown how the problem, which cannot be solved by the classical calculus of variations, can be defined as the problem of optimal path finding through the graph and how efficiently it can be solved by the extended dynamic programming method.

The described solving procedure remains unchanged even for the case when another integral criterion of optimality was chosen.

However, an essential question one could raise is whether the described procedure of optimal legged locomotion path planning can be implemented in a hierarchically structured multilevel control system.

It is apparent that the described procedure may be executed efficiently only by a digital computer whose main memory is determined chiefly by the data capacity. If the terrain is discretized into n_{max} points, and if a regular path network is structured, in which each node is connected with i_{max} adjacent nodes at the most, then for the data $(3\ i_{max} + 6)n_{max}$ memory words are required /3/.

For the case when the terrain was discretized into 20x20 points ($n_{max} = 400$) with respect to the square mosaic ($i_{max} = 4$), 7200 memory words were needed for the data, whilst for the discretization into 16x19 points ($n_{max} = 304$) with respect to the equilateral triangle mosaic ($i_{max} = 6$), 7269 words were required.

If the discretization of the terrain into 400 points is assumed as a satisfactory approximation, then the required capacity of the memory does not exceed even those capacities offered by the basic configurations of the cheap mini-computers. However, due to the efficiency of the extended dynamic programming method, the optimal path planning by means of a mini-computer whose memory cycle is 1 μs would not take more than a second.

Since the optimal legged locomotion path planning can be performed by a cheap mini-computer in the time which is considerably shorter than the time of locomotion, the described procedure may be regarded as suitable for implementation.

References

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