ANALYSIS OF ENERGY DEMAND DISTRIBUTION WITH ANTHROPOMORPHIC SYSTEMS

M. Vukobratović, D. Hristić, V. Ćirić, and M. Zečević

Summary

This contribution describes briefly the fundamentals of the synthesis approach of the artificial anthropomorphic robot. A summary of results by other authors who have experimentally obtained some energy information has been given. The algorithmization of mathematical models for obtaining all driving torques in the powered joints of the exoskeleton type anthropomorphic mechanisms is presented. The necessary driving torques for various gait types (prescribed synergies) are determined and the corresponding compensating synergies of the complete anthropomorphic system, exoskeleton plus patient, are calculated. There are discussed some results of energy analysis for two adopted characteristic gait types.

Introduction

In a number of papers last year there has been postulated a new approach to the study of anthropomorphic mechanisms /1, 2, 3, 4, 5, 6, 7, 8/. The purpose has been to synthesize and maintain a stable biped gait upon a programmed ground configuration. The synthesis of the artificial gait has been intended for patients requiring rehabilitation, or healthy men finding themselves in unusual working regimes.

In this approach, the gait types and hierarchical structure of the control system have been borrowed from the men. In other words, there biomechanical data on the way man walks has been used and in doing this, has not been stated why the man walks just in such way. Therefore, in addition to some formal reasons, the artificial gait has not been synthesized starting with the minimization of some criteria such as minimum power consumption. Such a hypothesis, among others, is based on the statement that in different locomotor tasks, there are probably different criteria satisfied as well. However, in general, this would concern a class of particular tasks. Usualy, when any criteria can be spoken about, they cannot be identified because humans walk according to very different algorithms. Moreover, in the

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course of our investigations, we learned that some adopted gait types, which are also encountered often with humans, are rather inadequate from the standpoint of power criterion. However, this statement is only conditional, because it is based on the model with which the gait types "borrowed" from the man are realized by a minimum number of degrees of freedom, whilst for the same task with the natural analogon - man - tens of muscle groups are available. Taking into account these several facts, it is almost unjustifiable to seek the artificial synergy of the anthropomorphic mechanism on the basis of some definite criterion. But, because it is impossible to discover in which way man defines and solves optimality task during his motion, if he solves it at all, is still not enough reason to not state the task of optimization for anthropomorphic machine with a much smaller number of degrees of freedom and quite different type of actuators. As a proof of the extent of nonpurposefulness of the optimization task itself, refer to the work by Chow and Jacobson /9/. They have tried, based on the criterion of minimum energy, to get the driving torques and appropriate trajectories of the legs. However, in order to solve this optimization problem, the authors have made some assumptions that have considerably simplified the original locomotion problem. Thus, the problem of motion has been reduced to the sagittal plane. Further, the problem of dynamic equilibrium has not been solved, and the problem of unknown dynamic reaction at the point of support has been bypassed by its quasi-static formulation. Nevertheless, the leg trajectories that finally represent a possible human gait have been obtained. In our opinion, the work cited has just indicated that in the synthesis of anthropomorphic robots, this procedure cannot be followed. With such simplifications it would be rather difficult to accept the result obtained as an optimal solution. So there remains our initial opinion on the nonpurposefulness of postulating the criteria of the so-called optimal solutions when synthesizing the anthropomorphic gait being performed by a minimum number of artificial actuators.

Based on the approach mentioned at the beginning, the synthesis of artificial synergy is executed in two steps: for some coordinates, a fixed motion program is set and the laws of changing the remaining coordinates are found from the equations of dynamic relations /1, 2, 5, 7/. These relations separate those solutions, that provide system dynamic equilibrium around the zero-moment points* prescribed in advance. Further, from such a narrowed set os solutions, there have been extracted only those, prossessing the repeatability properties. That means, the repeatability conditions of walking are formulated mathematically, undoubtedly representing an actual characteristic under conditions of regular locomotion activities of a man. In their essence, the assumptions have even determined the synthesis method of the artificial anthropomorphic gait. In this way among others, there has been solved a new task of mechanics that by its character belongs neither to the first nor second classical problem of mechanics. That is, in the problem considered, both the accelerations and the torques causing them are unknown.

The next step in the synthesis of the artificial gait was to define the torques developed by drives for the purpose of performing the total synergy of the anthropomorphic mechanism. To define the driving torques we "break" the kinematic chains along the axes of particular joints, and having done this, the driving torques can be regarded as external and determined from the conditions of kineostatic equilibrium.

In this way, all elements for a complete energy analysis of actual anthropomorphic mechanisms have been found. Thus there was realized an automaton that based on particular gait types and other geometric-dynamic initial parameters, gives a complete insight into the dynamic reaction forces, driving torques in all joints of the mechanism, mechanical operations, and appropriate power-all this representing necessary information in the case it is desirable to deal with this problem realistically.

Review of Some Previous Results

Many authors have treated the problem of determining forces and moments of the human locomotor apparatus /10, 11, 12/. The results have been obtained by experimental measurements, using these results directly in the form of data, and/or for further estmation of load. The main intention of these investigations has been to study the load of the human skeleton and muscle groups from the standpoint of changes due to some injuries or pathologic phenomena. Thus for example, there was studied the effect of injury consequences of some leg bones

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* Zero-moment point represents the point at which act the resultant force of dynamic reaction at the contact between the extremity and the ground.

to the advent of pathologic changes in the joints due to the disturbed harmony of normal gait /12/.

All these investigations, generally, have terminated on torques in particular joints. The power demand has been studied by J.B. Morrison /10/ and Peizer /11/. The results obtained for values of torques, show a considerable dispersion, which all authors point-out as the consequence of differences between the measuring devices, methods and the individual difference in gait of test subjects and patients.

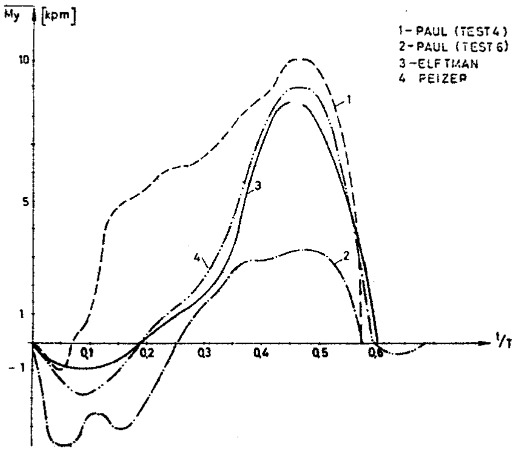


Fig. 1. Ankle joint torque according to some authors

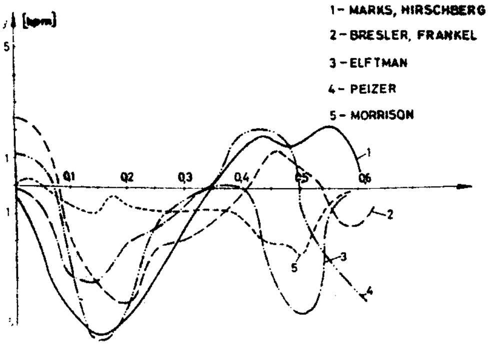


Fig. 2. Knee joint torque according to some authors

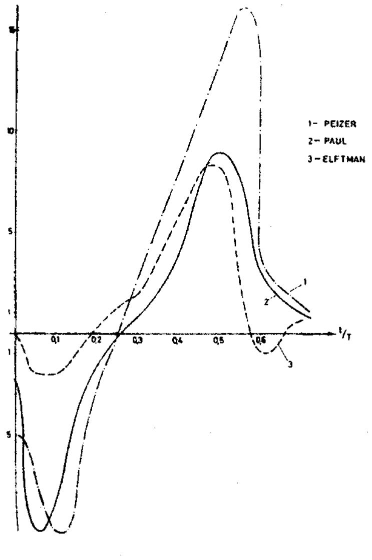


Fig. 3. Hip joint torque according to some authors

Thus Figure 1 shows the torques in the ankle joint, Figure 2 in knee, and Figure 3 in the hip, according to various authors. A great difference in the values of results by particular authors is evident, illustrating an abundance of possible versions of human gait, and at the same time, indicating that it is necessary to record each parti-

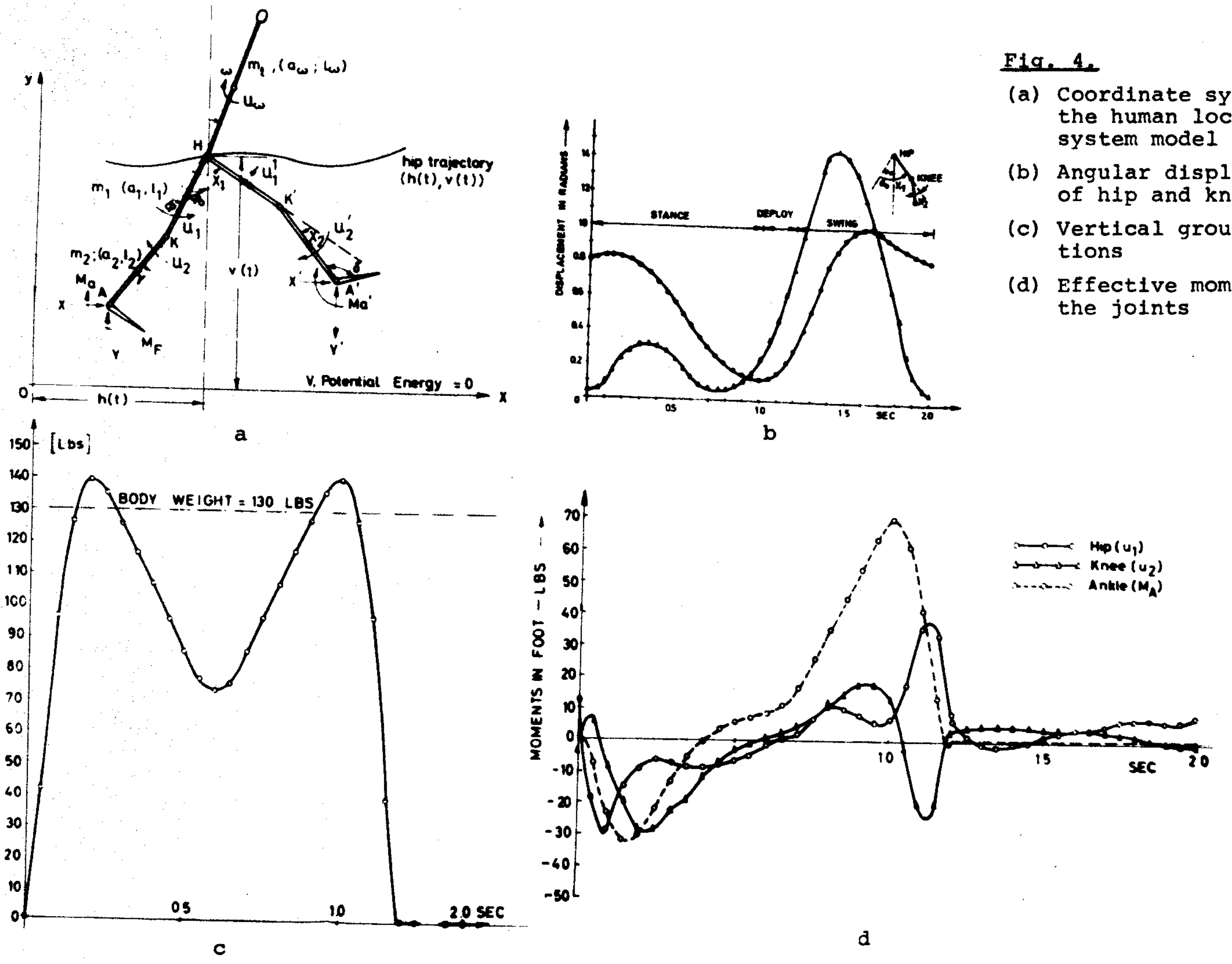


Fig. 4.

(a) Coordinate system for the human locomotor system model

(b) Angular displacement of hip and knee

(c) Vertical ground reactions

(d) Effective moments of the joints

cular case.

On this occasion the work by Chow and Jacobson /9/ that has treated the problem of simplified two-legged locomotion by means of optimal programming should be mentioned. For the sake of information, the results of their analysis are given in Figure 4.

Taking into account the data cited from the listed experiments and computational procedure by Chow and Jacobson, and the fact of great dispersion of results without enough systemtized causes of such dispersion, as well, the need to develop a universal method to collect such information has been pointed-out. The method is computational and based on a general anthrpomorphic model capable of performing all motion types upon programmed terrain configuration. Thus it has been made possible to get systematically, based on adopted types of legs movement (adopted synergy), the information on dynamic reaction forces, driving torques in all active joints, appropriate power and other energy parameters. Here it has to be noted that, without taking into consideration the extent of simplifying the equivalent mechanical model that substitutes for the human natural locomotor system, the results of energy analysis are reliable values. Finally, the results of energy analysis depend upon the character of dynamic reactions at the contact point, foot-ground, and these parameters depend upon the adopted gait type, geometric and dynamic input parameters of the anthropomorphic mechanism and the gait operating regime (stride and cadence).

Theoretical Implementation

As stated in the introduction, the central point in the synergy synthesis of the artificial gait is to set the point of attack of the reaction resulting forces. Those points have been called the zero-moment points /1, 2, 3/, and for them there have been written the equations of dynamic equilibrium.

Let τ be the zero-moment point (ZMP, in short). According to D'Alambert's principle, dynamic relations have the form of:

$$\begin{aligned} &\sum_{i=1}^{n} \left(\bar{\rho}_{\tau i} x (\bar{F}_i + \bar{G}_i) + \bar{M}_{Fi}\right) \bar{e}_x = 0 \\ &\sum_{i=1}^{n} \left(\bar{\rho}_{\tau i} x (\bar{F}_i + \bar{G}_i) + \bar{M}_{Fi}\right) \bar{e}_y = 0 \end{aligned} \qquad (1)$$

where: $\rho_{\tau i}$ - vector from point τ to the mass centre of the i-th element;

F_i, M_{Fi} - main vector and main moment of inertial forces of the i-th element;

e_x, e_y - unit orts of orthogonal axes x, y, through the point τ.

The third equilibrium equation with respect to the point of the friction forces resultant can be written as

$$\sum_{i=1}^{n} (\overline{M}_{Fi} + \overline{\rho}_i x \overline{F}_i)\overline{e}_\xi = 0 \qquad (2)$$

where ρ_i - vector from ZMP to the origin of axis from the surface of contact between foot and ground;

e_ξ - unit vector of axis ξ.

Equations 1 and 2 represent the mathematical model written in extremely general form. In the case when the arms are considered, too, this model is extended with as many second-order differential equations as the "arms" have degrees of freedom. By fixing the arms we keep only three equations of dynamic equilibrium. Since the number of degrees of freedom of the anthropomorphic mechanism is considerably greater than three, the law of changes in the remaining (n-3) coordinates has to be prescribed in order to provide a periodic transfer of legs with period T. The synthesis of artificial synergy is performed as follows: for some coordinates there is given a fixed motion program, and the remaining coordinates are found from the equations of dynamic relations (Eqs. 1 and 2). If from the set of coordinates, ϕ_i, we extract the given coordinates $\tilde{\phi}$ and coordinates computed from dynamic relations, ϕ^*, then the general system can be written as

$$\sum_{i=1}^{n} \overline{c}_i \ddot{\phi}_i^* + \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{d}_{ij} \dot{\phi}_i^* \dot{\phi}_j^* + \overline{g} = 0 \qquad (3)$$

where: c_i, d_{ij} - vector coefficients depending on $\tilde{\phi}$, $\dot{\tilde{\phi}}$, $\ddot{\tilde{\phi}}$, and vector g on $\tilde{\phi}$, ϕ^*, $\dot{\tilde{\phi}}$, $\dot{\phi}^*$.

System (3), with the repeatability conditions (1, 2)

$$\phi_i^*(0) = \pm\phi_i^*(T/2); \qquad \dot{\phi}_i^*(0) = \pm\dot{\phi}_i^*(T/2) \qquad (4)$$

where T is step period, gives the compensating synergy ϕ^* based upon the adopted gait algorithm $\tilde{\phi}$.

After determining the cited synergy, it is quite easy to define dynamic reaction forces according to

$$
\begin{aligned}
F_\xi &= \Big(\sum_{i=1}^{n} \bar{a}_i \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{b}_{ij} \dot{\phi}_i \dot{\phi}_j + \bar{G}_i \Big) \bar{e}_\xi \\
F_x &= \Big(\sum_{i=1}^{n} \bar{a}_i \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{b}_{ij} \dot{\phi}_i \dot{\phi}_j \Big) \bar{e}_x \\
F_y &= \Big(\sum_{i=1}^{n} \bar{a}_i \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{b}_{ij} \dot{\phi}_i \dot{\phi}_j \Big) \bar{e}_y
\end{aligned}
\qquad (5)
$$

Since the entire system synergy is known, the next step is to define the driving torques to be developed by drives in order to realize a complete synergy of anthropomorphic mechanism. To determine these torques, we "break" the kinematic chain along the axes of particular joints. Thus, for the part of system not connected with the ground, the summing torque, in this case the driving torque, can be computed according to:

$$
\bar{\mathfrak{M}}^k = \sum_{i=1}^{n} \bar{c}_i^k \ddot{\phi}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \bar{d}_{ij}^k \dot{\phi}_i \dot{\phi}_j + \bar{g}^k \qquad (6)
$$

where index k indicates the number of joints where the kinematic chai of the mechanism has been "broken".

Further, based on the computed driving torques, it is possible to define both the power of individual actuators, and the total power necessary for the whole system.

$$
P(t) = \sum_{i=1}^{n} f(\mathfrak{M}_i(t), \omega_i(t)) \qquad (7)
$$

Energy Analysis of Anthropomorphic Robot

The results of a theoretical approach to the synthesis of artificial anthropomorphic gait have been verified on a walking mechane of the exoskeleton type (Fig. 5). The equivalent mechanical configuration of the assembly is illustrated in Figure 6. The same figure gives, in tabular form, the geometric parameters and tensors of system mass and inertia. Due to the lack of space, the energy results have been given for joints 1, 2, 3; that is, for the joints of the stance leg only. It should be pointed-out that the results concern the single-support gait; that is, the gait type with which the leg support is changed without overlapping. Such a gait, no doubt, influences the quantitative value of results, keeping in mind the fact that human gait always possesses a shorter or longer double-support phase. However, the change in results in the case of the leg algorithm when single- and double-support phases exchange consecutively, will not be essential with the exception of the moment of the double-support phase itself. Variations are generally expected in the sense of certain reductions in driving torques, and the greatest reduction occurs at the half-step.

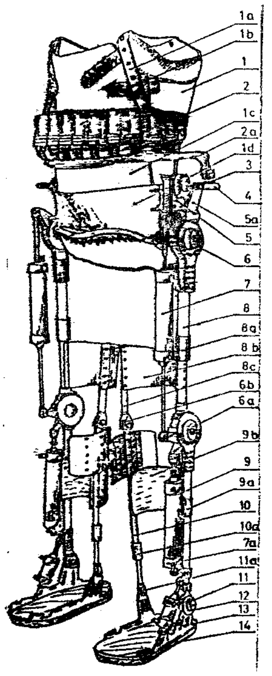


Fig. 5. Biped robot type walking machine

Here are demonstrated energy results based on two characteristics adopted gait types:

A. "Moderate Normal Walk"

Programmed synergy of the lower extremities, the computed synergy of the pelvic part, and the computed compensating motions are given in Figure 7. Three cases of the change in the point at which act the resulting reaction forces for the full step period T = 2 sec, have been considered (zero-moment point shift):

Case I: support at the point vertically below the ankle joint during the entire support phase
Case II: support on the heel lasting 0 to 0.15 sec;
support in Case I, from 0.15 to 1.00 sec
Case III: support on heel in Case II;
support same as in Case I but from 0.15 to 0.60 sec;
support on toes from 0.60 to 1.00 sec.

T.1.

Mass* M and Moment of Inertia* J

[kg, m, sec]

i	m	Jx	Jy	Jz
1	0,18	0,00007	0,00065	0,00052
2	0,38	0,0044	0,0044	0,00045
3	0,98	0,0138	0,0141	0,0035
4	0,83	0,0079	0,0062	0,0071
5	4,7	0,211	0,190	0,031
6	0,98	0,0138	0,0141	0,0035
7	0,38	0,0044	0,0044	0,00045
8	0,18	0,00007	0,00065	0,00052

*) Exoskeleton Included. Weight of Patient Alone: 71 kp.

T.2. Distance $\vec{l}$ [m]

i	l_x	l_y	l_z
1	0	0	0,1
2	0	0	0,400
3	0	0	0,45
4	−0,047	0,1	0,09
5	—	—	—
6	0	0	-0,445
7	0	0	-0 400
8	—	—	—

T.3. Distance $\vec{r}$ [m]

i	r_x	r_y	r_z
1	0,030	0	0,038
2	0	0	0,22
3	−0,033	0	0,33
4	−0034	0,093	0,069
5	0,038	0	0,42
6	−0,032	0	-0,139
7	0	0	-0,192
8	0,030	0	-0,059

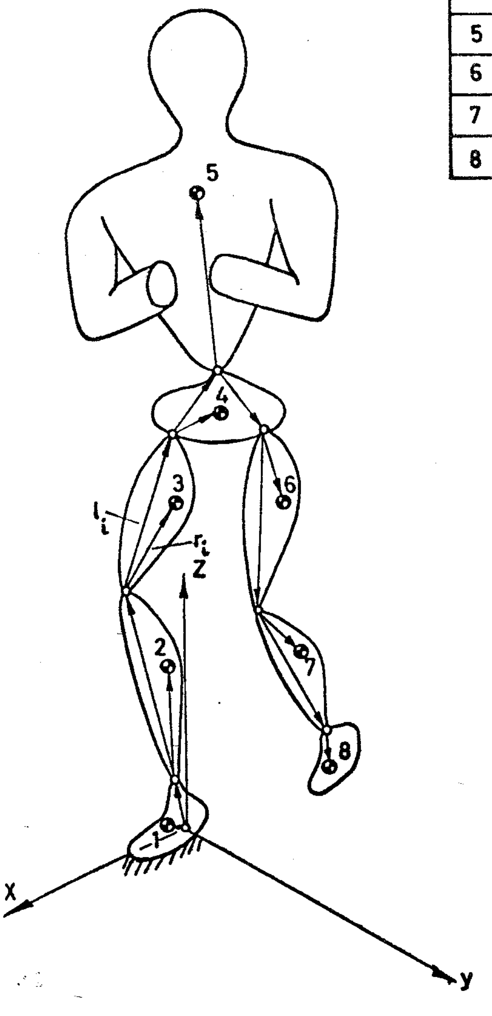


Fig. 6. Anthropomorphic mechanical system with fixed "arms".

SYNTHESIS OF SYNERGY - SYSTEM WITH FIXED ARMS
"MODERATE NORMAL WALK"

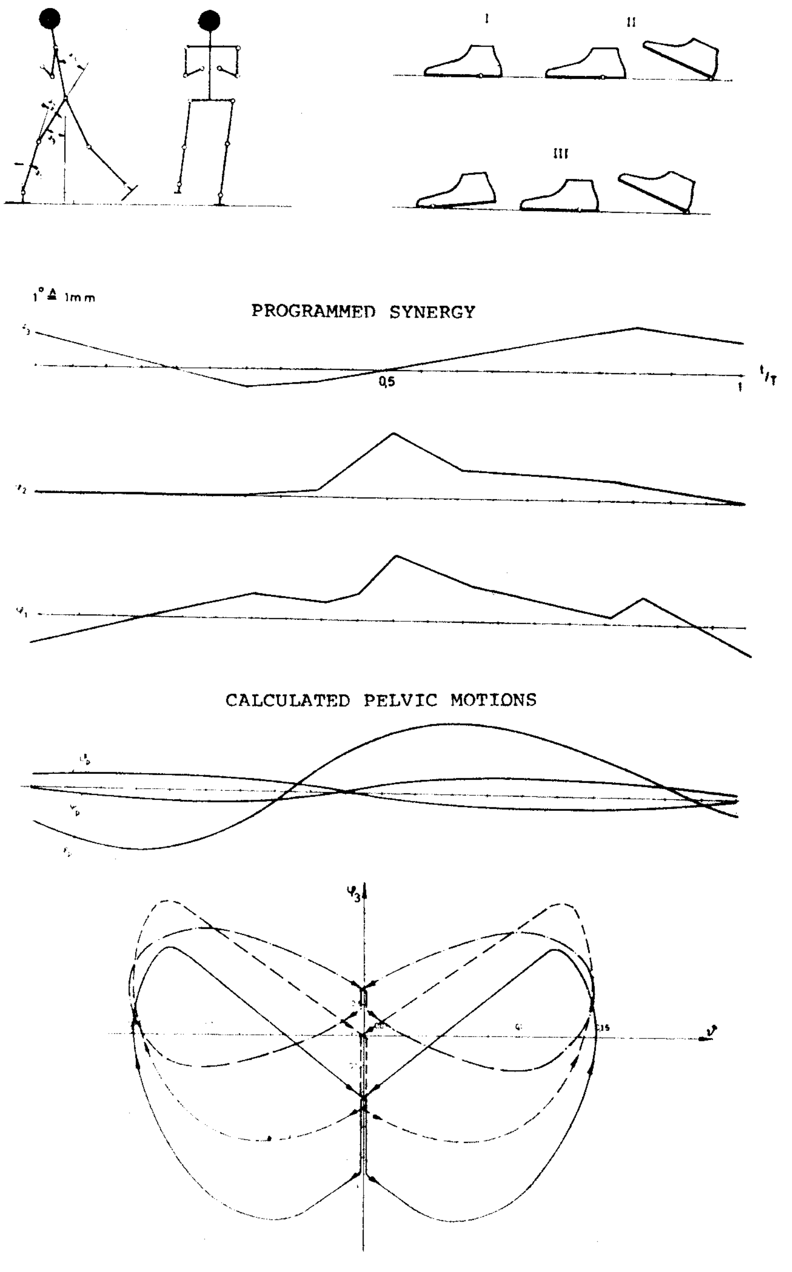
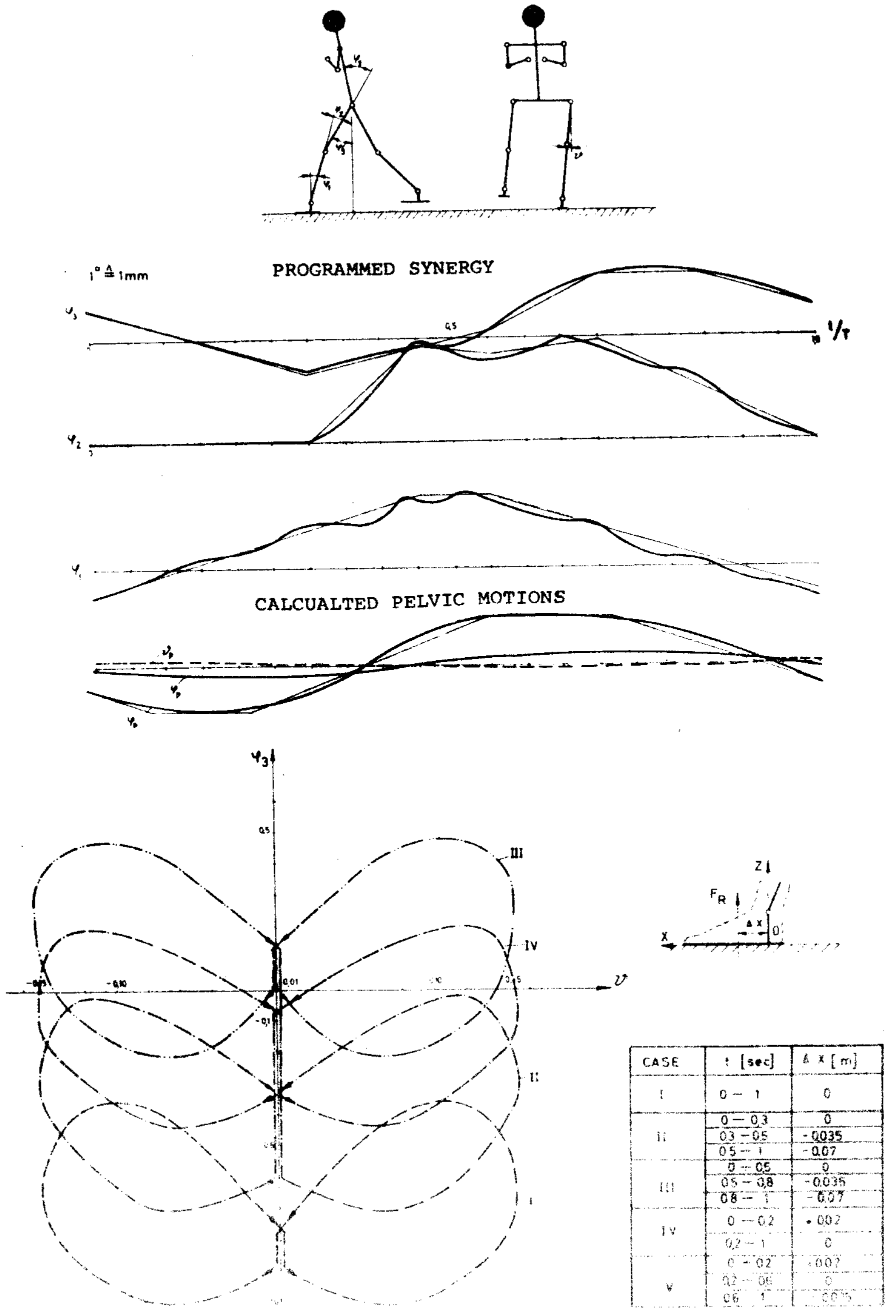


Fig. 7.

SYNTHESIS OF SYNERGY - SYSTEM WITH FIXED ARMS
SPECIAL "FLAT - FOOT" GAIT



CASE	t [sec]	Δ x [m]
I	0 — 1	0
II	0 — 0,3	0
	03 — 05	-0,035
	05 — 1	-0,07
III	0 — 05	0
	05 — 0,8	-0,035
	08 — 1	-0,07
IV	0 — 0,2	+0,02
	0,2 — 1	0
V	0 — 0,2	+0,02
	0,2 — 06	0
	06 — 1	-0,035

Fig. 8.

B. Special "Flat-Foot Gait"

The main data are given in Figure 8. This type of gait is very interesting in that the angle of the ankle joint in the absolute system of coordinates β_1 is always equal to zero. Zero-moment point shift has been taken in five cases:

Case I: support at the point vertically below the ankle joint during the entire support phase.

Case II: support same as in Case I from 0 to 0.30 sec;
support shifted forward (toward otes) by 0.035 m from 0.30 to 0.50 sec;
support shifted futher forward by 0.070 m from 0.50 to 1.00 sec.

Case III: support same as in Case I from 0 to 0.50 sec;
support shifted forward (toward toes) by 0.035 m from 0.50 to 0.80 sec; support shifted further forward by 0.070 m from 0.80 to 1.00 sec.

Case IV: support shifted backwards (toward heel) by 0.020 m from 0 to 0.20 sec;
support same as in Case I from 0.20 to 1.00 sec.

Case V: support shifted backwards as in Case IV from 0 to 0.20 sec;
support same as in Case I from 0.20 to 0.60 sec;
support shifted forward by 0.035 m from 0.60 to 1.00 sec.

With this two gait types (A & B) and their 8 cases, there has been set forth a rather wide family of types of artificial gait. This makes possible the selection of the appropriate synergy for some definite task of the anthropomorphic robot. Conversely, this is proof of general-purpose character and the capability of the mathematical model to produce very easily the desired sets of results, both for different mechanism configurations and for arrays of geometric-dynamic parameters within particular gait algorithms.

In the text to follow, let us analyse and compare both gait types.

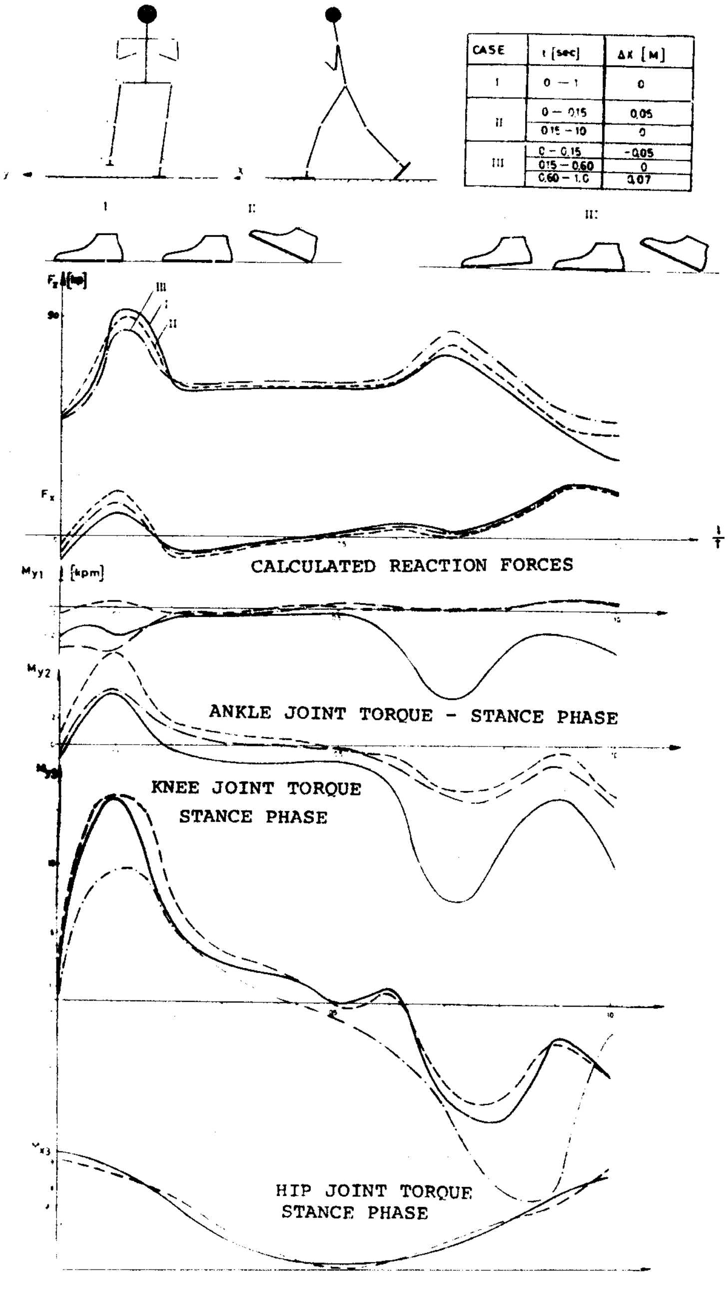
As far as the *computed compensating motions* are concerned, the general and logic conclusion holds that for both gait types the more the ZMP is backwards and remains longer in that position, the solution of compensating motion in sagittal plane (Figs. 7 and 8) are the more "backward", i.e., angle ϕ_3 with its greater portion finds itself in the region of negative values. Conversely, solutions in the frontal plane (angle θ) partically do not depend at all on the gait type and the choice of ZMP trajectories. This is indication for the role of this coordinate (angle) primarily in the sense of transferring the body weight from one leg to the other one during walking.

Reaction forces and driving torques have been given for gait types A and B in Figures 9 and 10. These data are most important for exoskeleton designs the rigidity standpoint under conditions of dynamic load. Reaction forces in the vertical direction (Z-components) attack mostly the main struts, bearings, and corresponding links. Driving torques affect the sizes and quality of material of couplings, torque rods, force levers, universal joints, and the like. If we look at Figure 10 for gait type A, there can be seen a lessening effect of dividing the aupport phase of foot against support subphases. This reflects at most in important values of vertical components of reaction (F_z) and driving torques M_{y1}, M_{y2}, M_{y3}, of the stance leg. Practically, the same conclusion holds for gait type B, as well; however, it is very interesting to recognize the difference between the cases II and III, that are characterized by the same zero-moment point shift but with various time durations of particular phases. Thus, a shorter duration of one and the same phase with support shifted forward (toward the toes) as in Case II, compared with Case III, gives greater values of the component F_z of reaction force, as well as the very important driving torque of the hip (M_{y3}) this happens evidently because of greater values of acceleration. A general conclusion follows that the gait type B ("Flat-Foot") provides dynamically "smoother" gait.

Computed values of power demand for gait types A & B have been given in Figures 11 and 12. Data are given for three servo-drives only. Complete data can be found in /8/. General conclusion on "smoother" gait type B holds here, too, and thus it can be said that this gait type requires, on average, approximately 25% less power for its realization than type A (moderate normal walk). In addition, these diagrams indicate that the job to be performed by the control system and servo-drives of the exoskeleton in the presence of sudden and considerable changes in load is rather difficult.

The types of actuators will not be discussed here. It should be only emphasized that from the standpoint of control (frequency response), the electric motors as the actuators are most suitable for

REACTION FORCES AND DRIVING TORQUES - SYSTEM WITH FIXED ARMS
"MODERATE NORMAL WALK"



CASE	t [sec]	Δx [M]
I	0 — 1	0
II	0 — 0,15	0,05
	0,15 – 1,0	0
III	0 – 0,15	-0,05
	0,15 – 0,60	0
	0,60 – 1,0	0,07

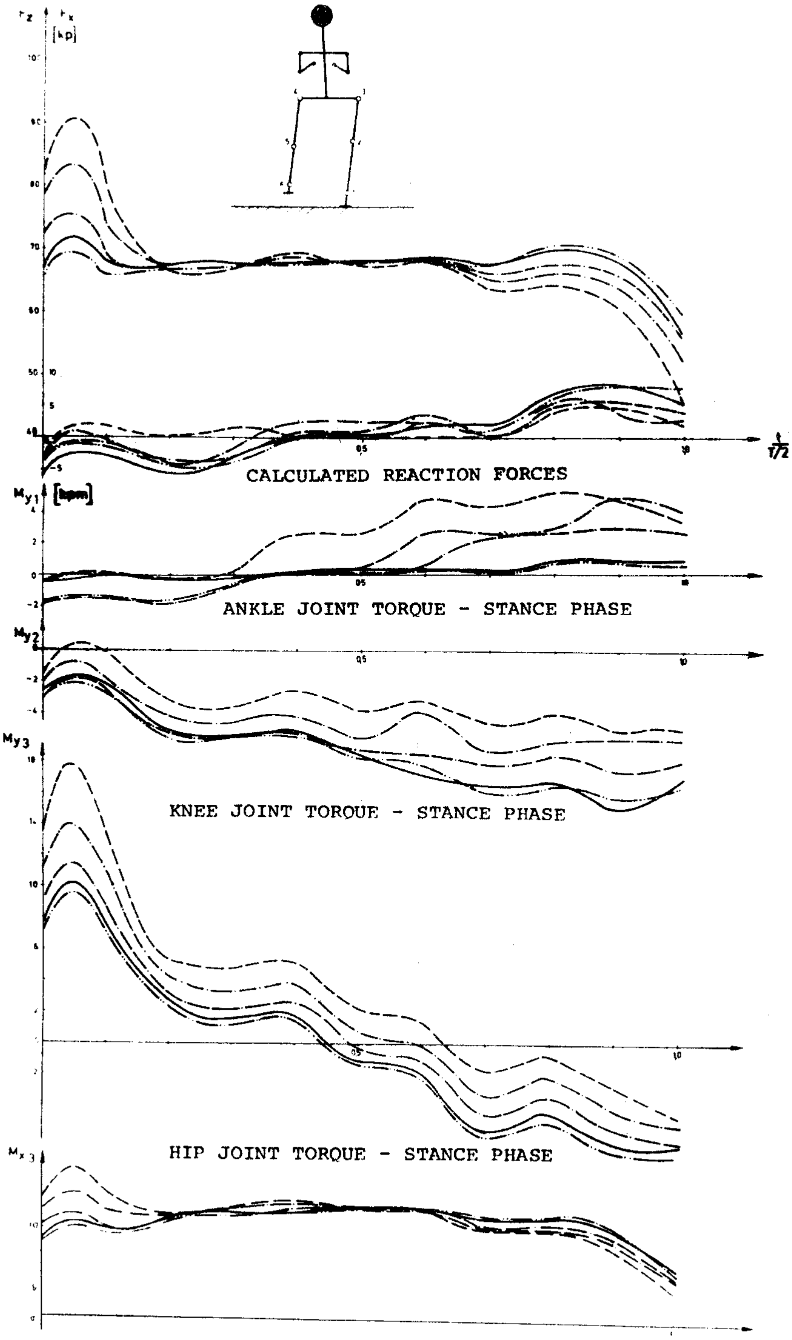
CALCULATED REACTION FORCES

ANKLE JOINT TORQUE - STANCE PHASE

KNEE JOINT TORQUE
STANCE PHASE

HIP JOINT TORQUE
STANCE PHASE

REACTION FORCES AND DRIVING TORQUES - SYSTEM WITH FIXED ARMS
SPECIAL "FLAT - FOOT" GAIT



CALCULATED REACTION FORCES

ANKLE JOINT TORQUE - STANCE PHASE

KNEE JOINT TORQUE - STANCE PHASE

HIP JOINT TORQUE - STANCE PHASE

CALCULATED POWER DEMANDS OF SYSTEM WITH FIXED ARMS
"MODERATE NORMAL WALK"

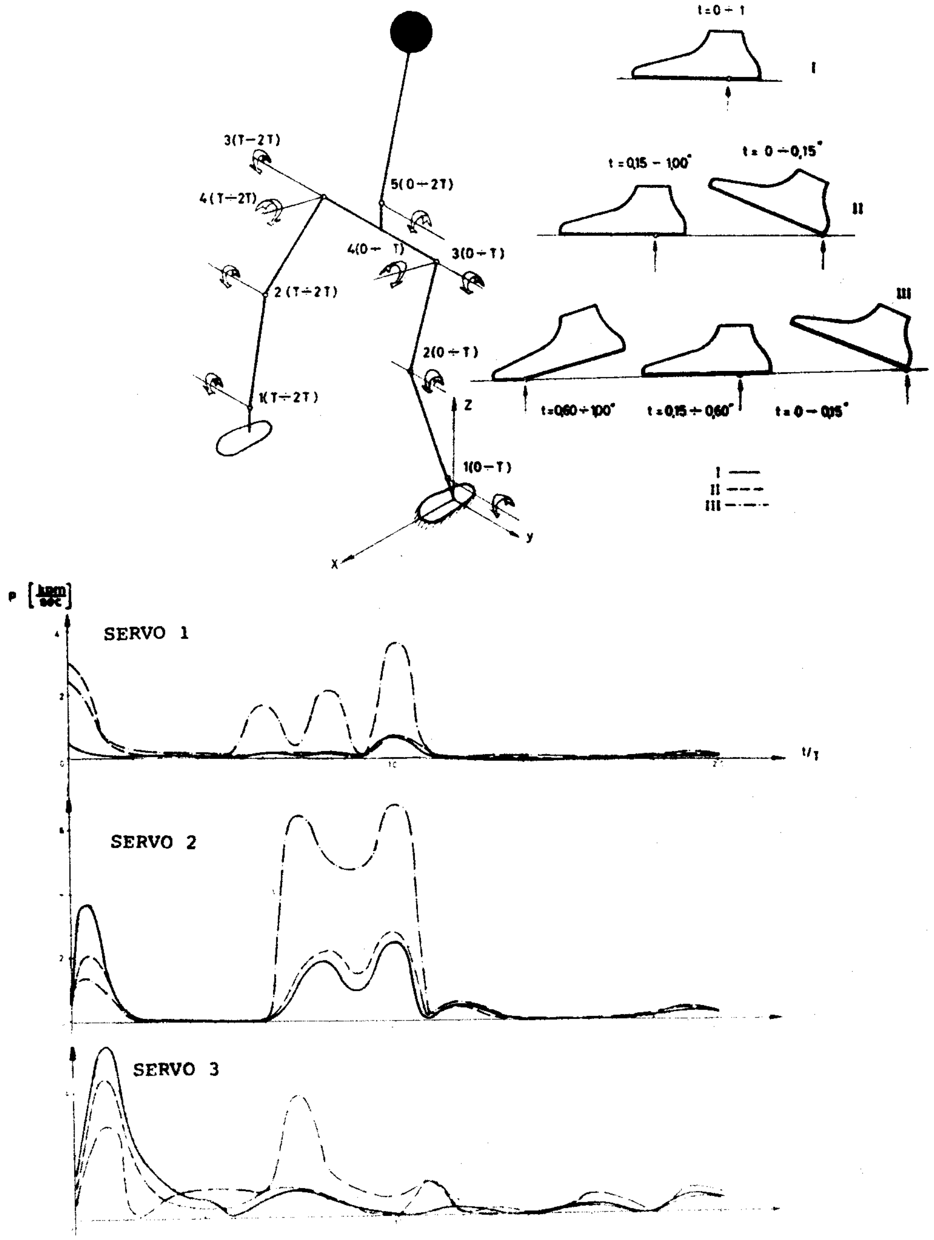
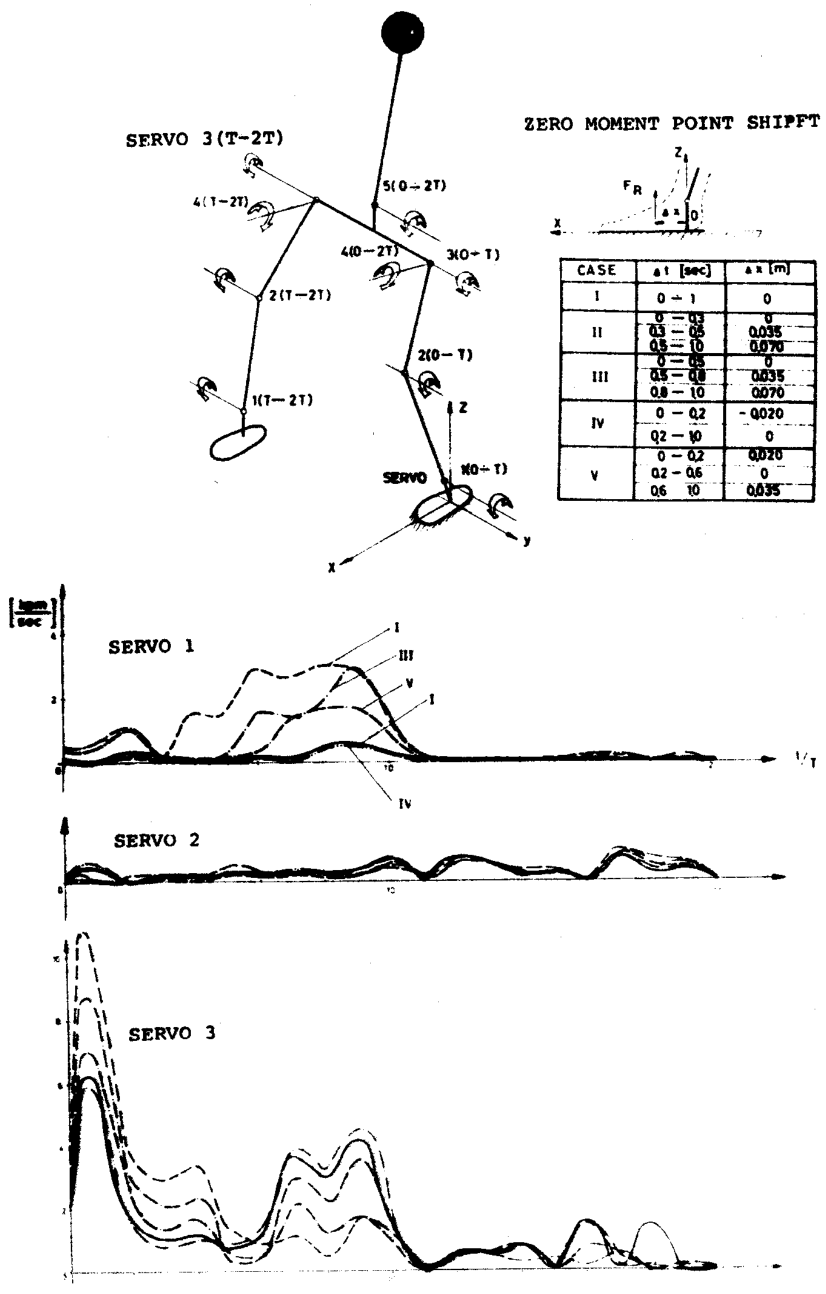


Fig. 11.

CALCULATED POWER DEMANDS OF SYSTEM WITH FIXED ARMS SPECIAL "FLAT - FOOT" GAIT



CASE	Δt [sec]	Δx [m]
I	0 ÷ 1	0
II	0 − 0,3	0
	0,3 − 0,5	0,035
	0,5 − 1,0	0,070
III	0 − 0,5	0
	0,5 − 0,8	0,035
	0,8 − 1,0	0,070
IV	0 − 0,2	- 0,020
	0,2 − 1,0	0
V	0 − 0,2	0,020
	0,2 − 0,6	0
	0,6 1,0	0,035

Fig. 12.

this definite application, though in the present state of technological development, they are still too heavy for the power developed. Thus, for example, for servo 3 (hip) and gait type B there a servomotor of nominal power output of about 0.07 HP is needed. This corresponds to electric (input) power of about 100 W.

Dependencies of driving torques on angular velocities of particular joints are given in diagrams in Figures 13 and 14 for gait types A & B. Such form of this dependence is very adequate for selection of actuators having appropriate power characteristics. Thus the characteristic of actuator of the electro-motor type with reducer must contain the whole diagram (ω). For any type of actuator, its available characteristic (ω) must be greater by its absolute value than the necessary power of the joint at any point.

Conclusion

The objective of this contribution has been to give an insight into the possibility of a computational procedure capable of demonstrating all energy charateristics of the artificial anthropomorphic system by means of input data only in the form of imposed gait types and other geometric-dynamic parameters. We are of the opinion that by such a procedure it is possible to facilitate to a great extent the long-term and troublesome experiments with a great number of subjects. At the same time a considerable systematization of results obtained, and more precise explanations of the changes in energy parameters with the same subjects in different experiments, or with various subjects in similar operating regimes, and the like, can be expected.

Finally, though, in question is only one approach based on a simplified anthropomorphic system, being still capable of meeting the requirements of a human-like gait. The energy parameters obtained in this way could serve as a starting point in the analysis of load distribution in particular muscle groups of the natural human system. In this way, indirectly, it could be possible eventually to identify the criteria developing in different operating regimes of the locomotor system that would make the knowledge of biomechnical processes of this kind more profound.

Besides the anthropomorphic systems being in the focus of attention in our research, it is possible by this method to analyse the energy demands of multi-legged and particularly, of quadruped artificial systems in which the question of dynamic stability still remains

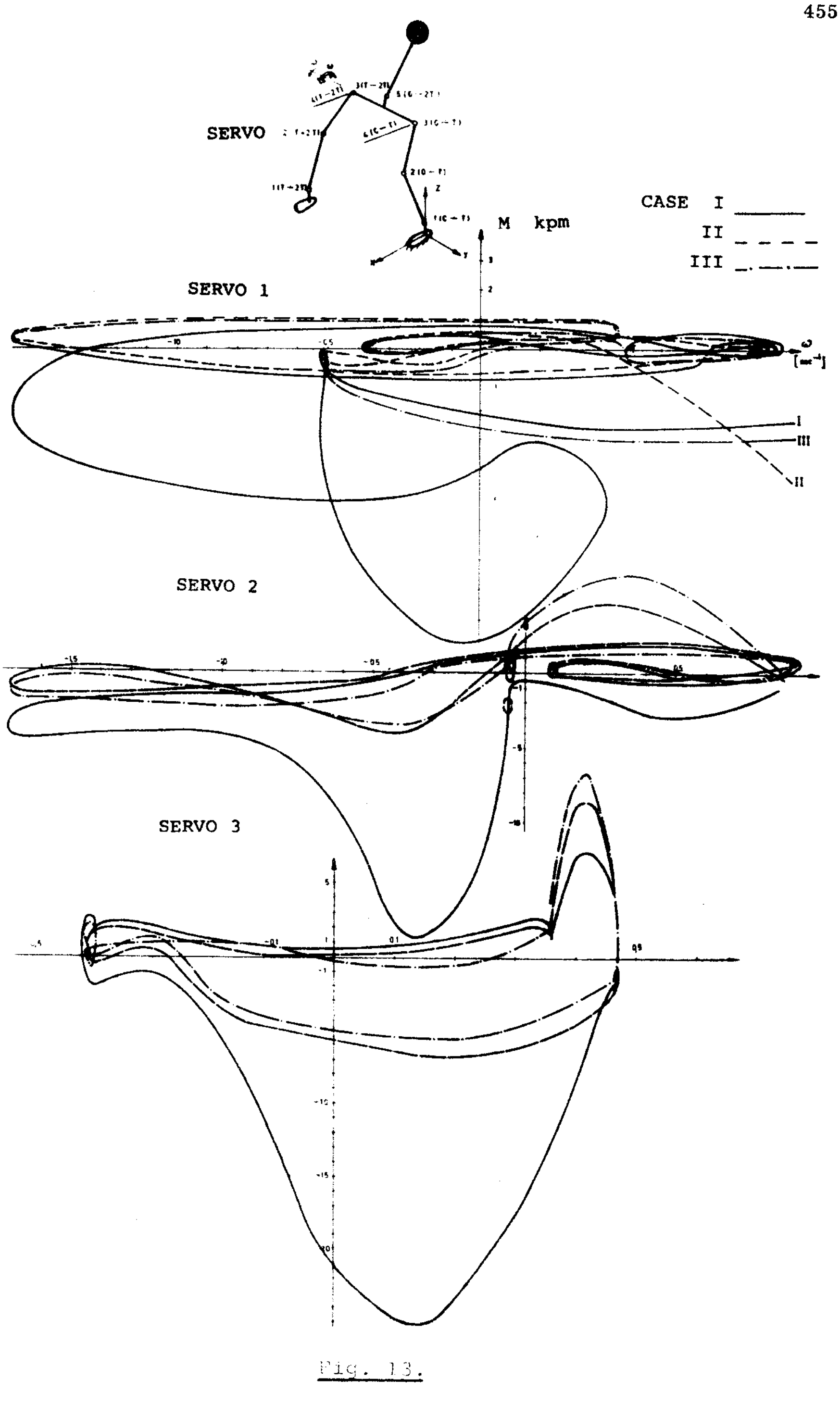


Fig. 13.

DIAGRAMS: DRIVING TORQUES - ANGULAR SPEEDS (M = f/ /)
SYSTEM WITH FIXED ARMS - SPECIAL "FLAT - FOOT" GAIT

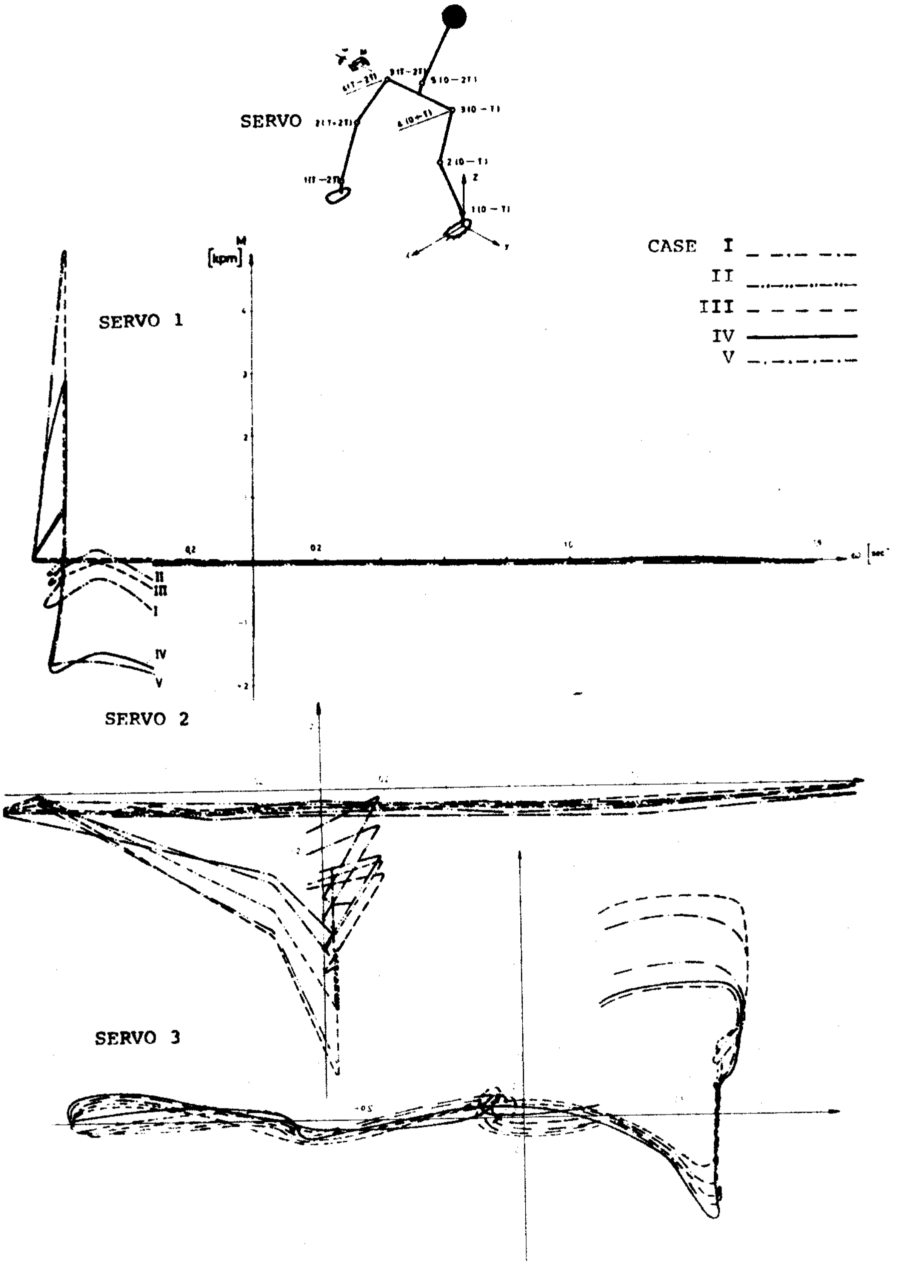


Fig. 14.

a delicate problem /4/.

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