

STABILITY AND DYNAMIC CONTROL OF CERTAIN
TYPES OF BIPED LOCOMOTION

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Summary

The results of the research reported in this paper suggest some effective control concepts which can assure stable biped locomotion relative to a biped model with six degrees of freedom. With an adequate selection of the body coordinate system, the linearized form of nonlinear equations of motion can be completely decoupled into six independent modes, each associated with one degree of freedom. Linear control laws are derived for each independent mode for continuous control inputs to the nonlinear system. In addition, effective step length control have been applied both in the direction of motion and orthogonally to it. The combination of continuous and discrete-time controls with nonlinear control of the gait period results in a high degree of overall stability of biped locomotion. Some examples of the computer simulation are given to demonstrate the proposed control concepts.

Introduction

Several studies relating to the problem of biped locomotion pointed out considerable difficulties associated with its stability and control concepts which would enable stable locomotion for a large class of disturbances. These difficulties originate from the complexity of the dynamic model of a multi-linkage system with several degrees of freedom for which the derivation of equations of motion is not a trivial task. However, some interesting approaches have been given in order to overcome the complexity of the nonlinear model such as algorithmic controls proposed by Vukobratović /2, 5/ and derivation of controls based on the linearized model at the singular point of equilibrium /3/. Furthermore, some ideas used in the legged locomotion studies could eventually be applied for biped locomotion controls /1, 4/.

The earlier control concepts of biped locomotion considered only "marching locomotion", in which the function of legs has been predetermined with a constant step length and a constant gait pe-

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riod. The disturbances were eliminated by body action so as to maintain the cyclic requirement of locomotion. The basic idea of this work, to the contrary, is to maintain stable locomotion under disturbances by adequate motion of legs. The step length control concept, along with local altitude and attitude controls, resulted from this concept which, in essence, resembles the behaviour of living bipeds. As in the previous work on locomotion, the notion of massless legs has been retained in order to simplify the derivation of new control concepts. The effect of this assumption, however, has still to be determined.

Some of concepts presented here have been derived in an earlier paper for a planar model /6/, and are extended here to a six-degree-of-freedom model in three dimensional space.

Mathematical Model for a Biped

During locomotion, the biped model goes through several phases of motion which are characterized by the selected type of gait. Its dynamics can be defined by equations of motion for a particular phase. Since it is desired to derive new control concepts, some assumptions have been made towards simplification of the model.

The model consists of a rigid body with mass m and moments of inertia J_{xx} , J_{yy} and J_{zz} in the three dimensional space. A pair of massless legs is attached to the body. The notion of a variable leg length with an active force generator is used, effectively replacing the knee function of a massless leg /6/.

The hypothetical biped with six degrees of freedom is shown in Figure 1. The singular gait has been selected in which only one leg is in the stance phase at a given time. The foot transmits the reaction forces of the ground and no slippage is permitted between the foot and the ground.

The model altitude is controlled by force (F^1) acting along the leg, and the three torques M_1^1 , M_2^1 , and M_3^1 applied at the hip control the attitude of the body in sagittal and lateral planes, and its rotation around the body major axis, respectively. A suitable coordinate system has been selected, similar to the one of the human vestibular apparatus. It is a spherical coordinate system as shown in Figure 1.

Undisturbed biped locomotion without particular constraints is considered in the sagittal plane (YZ plane), in which position of the hip and the body are defined by the leg length r , angles ϕ_1

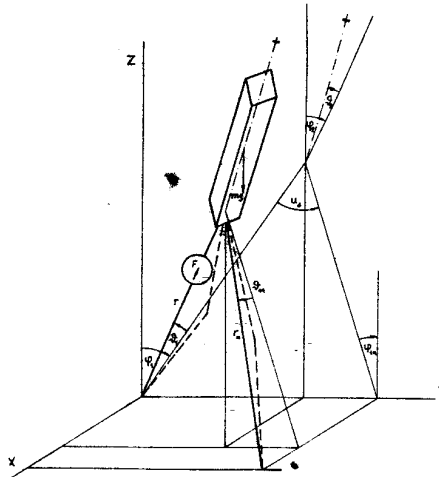


Fig. 1.

and ϕ_2 , and the distance of the body's center of gravity from the hip 1. Under lateral disturbances, the biped model moves outside the sagittal plane. Its position in the lateral direction is defined by angles θ_1 and θ_2 .

The equations of motion are nonlinear, with one second order differential equation for each degree of freedom:

$$f(q, \dot{q}, \ddot{q}, Q) = 0 \quad (1)$$

where vectors q and Q contain generalized coordinates and generalized forces, respectively.

$$q = [r \ \phi_1 \ \phi_2 \ \theta_1 \ \theta_2 \ \psi]' \quad (2)$$

$$Q = [F \ M_1 \ M_2 \ M_3]' \quad (3)$$

The vector of generalized forces is reduced to unit mass $Q = Q^1/m$. The equations of motion were derived by the Lagrangian equation of motion.

The point of unstable equilibrium defined by $q = [r_0 \ 0 \ 0 \ 0 \ 0 \ 0]'$ and $\dot{q} = 0$ is one of the most interesting states of the system. During normal locomotion, the model either goes through this state or passes closely by it. Therefore, let us analyse the equations of motion around this point.

Linearized equations of motion (1) can be given in the following form

$$\dot{x} = \begin{bmatrix} A_1 & & & & & & & & & & & & & 0 \\ & A_2 & & & & & & & & & & & & & \\ & & A_1 & & & & & & & & & & & & \\ & & & A_2 & & & & & & & & & & & \\ & & & & A_1 & & & & & & & & & & \\ & 0 & & & & A_2 & & & & & & & & & \\ & & & & & & A_1 & & & & & & & & \\ & & & & & & & A_2 & & & & & & & \\ & & & & & & & & A_1 & & & & & & \\ & & & & & & & & & A_2 & & & & & \\ & & & & & & & & & & A_1 & & & & \\ & & & & & & & & & & & A_2 & & & \\ & & & & & & & & & & & & A_1 & & \\ & & & & & & & & & & & & & A_2 & \\ & & & & & & & & & & & & & & A_1 \end{bmatrix} x + \begin{bmatrix} B_1 & & & & & & & & & & & & & & & & 0 \\ & B_2 & & & & & & & & & & & & & & & & \\ & & B_3 & & & & & & & & & & & & & & & \\ & & & B_2 & & & & & & & & & & & & & & \\ & & & & B_3 & & & & & & & & & & & & & \\ & & & & & B_2 & & & & & & & & & & & & \\ & & & & & & B_3 & & & & & & & & & & & \\ & & & & & & & B_2 & & & & & & & & & & \\ & & & & & & & & B_3 & & & & & & & & & \\ & & & & & & & & & B_2 & & & & & & & & \\ & & & & & & & & & & B_3 & & & & & & & \\ & & & & & & & & & & & B_2 & & & & & & \\ & & & & & & & & & & & & B_3 & & & & & \\ & & & & & & & & & & & & & B_2 & & & & \\ & & & & & & & & & & & & & & B_3 & & & \\ & & & & & & & & & & & & & & & B_2 & & \\ & & & & & & & & & & & & & & & & B_3 & \\ & & & & & & & & & & & & & & & & & B_2 \\ & & & & & & & & & & & & & & & & & & B_3 \end{bmatrix} u \quad (4)$$

with x as the state vector

$$x = [r - r_0 \quad \dot{r} \quad \phi_1 \quad \dot{\phi}_1 \quad \phi_2 \quad \dot{\phi}_2 \quad \theta_1 \quad \dot{\theta}_1 \quad \theta_2 \quad \dot{\theta}_2 \quad \psi \quad \dot{\psi}]' \quad (5)$$

and u as the control vector

$$u = Q - Q^* \quad (6)$$

where Q^* is the value of generalized forces at the point of unstable equilibrium. The matrix blocks in Equation 4 are

$$B_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}', \quad B_2 = \begin{bmatrix} 0 & -c \end{bmatrix}', \quad B_3 = \begin{bmatrix} 0 & d \end{bmatrix}' \quad (7)$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ b^2 & 0 \end{bmatrix} \quad (8)$$

with $b = g/(l+r)$, $c = (J_{xx} + r_0 l^2)/J_{xx} r_0$, and $d = (r_0 + l)/J_{xx} r_0$.

The eigenvalues of the matrix A_2 are equal to b and $-b$ indicating instability of the system.

It is evident from Equation 4 that the linearized system can be partitioned into six modes each associated with one degree of freedom, owing to favourable selection of the state coordinates. The control matrix, however, indicates coupling between some modes. In spite of this, four completely uncoupled subsystems appear, describing the motion of the body along its leg, in the sagittal plane, in the lateral direction, and around the body major axis.

The independent subsystems suggest independent control loops, each containing only the state variables associated with the corresponding mode. Furthermore, it is clear that in addition to the four control components of u , additional controls are necessary which would enable the model to walk with step length as one of the controllable quantities.

Different types of controls are to be studied as follows:

- body altitude control
- body attitude control
- step length control

The first two are continuous and the last one is time-discrete, with the sampling interval T which is equal to one half-cycle of locomotion.

Linear Feedback Control Concepts

As mentioned previously, linearized equation of motion decouple into six independent modes, out of which two pairs are coupled via control inputs to form two fourth order subsystems each with a single control input. The control input for the first of fourth order subsystems is the torque $u_2 = M_1 - M_1^*$ which determines the body attitude in the sagittal plane. The second mode is this subsystem, corresponding to the matrix A_2 , describes the hip motion during one step period. At discrete time intervals, the model exchanges legs in the supporting phase, thus shifting the model coordinate system into a new position with respect to the earth coordinate system. The change affects only the hip components on state vector. Its magnitude is controlled by step control mechanism which is of a discrete type. The discrete step control both in the sagittal and in the lateral direction is essential to produce the forward motion with a average velocity v_0 . Between two successive steps, continuous inputs are applied for the body attitude and altitude controls.

Linear feedback laws have been derived according to the parameters of the linearized model using pole assignment techniques for continuous and time-discrete controls.

Altitude and Attitude Controls

The body altitude control is maintained by the length control which supplies the system with energy lost at the moment of leg switching. From equations of the first mode

$$\dot{x}_r = A_1 x_r + B_1 u_1 \quad (9)$$

we can derive a linear feedback law

$$u_1 = h_1 x_1 + h_2 x_2 \quad (10)$$

which gives the poles of the closed loop subsystem λ_1 and λ_2 with the following selection of feedback constants

$$h_1 = -\lambda_1 \lambda_2 \quad h_2 = \lambda_1 + \lambda_2 \quad (11)$$

The body attitude controls are continuous. They are controlling position of the body with respect to the angles ϕ_2 , θ_2 , and

ψ via control torques u_1 , u_2 , and u_3 respectively.

In the sagittal direction, the mode is defined by

$$\dot{x}_{s2} = A_1 x_{s2} + B_3 u_2 \quad (12)$$

With the linear feedback law

$$u_2 = h_5(x_5 - \alpha) + h_6 x_6 \quad (13)$$

we can assign the closed loop poles to λ_5 and λ_6 if

$$h_5 = -\lambda_5 \lambda_6 / d \quad h_6 = (\lambda_5 + \lambda_6) / d \quad (14)$$

The angle α defines inclination of the body in the forward direction during locomotion.

In the lateral direction we have

$$u_3 = h_9 x_9 + h_{10} x_{10} \quad (15)$$

with the control constants equal to

$$h_9 = -\lambda_9 \lambda_{10} / d \quad h_{10} = (\lambda_9 + \lambda_{10}) / d \quad (16)$$

Linearized equation of rotation around the body major axis is

$$\dot{x}_\psi = A_1 x_\psi + B_1 u_4 \quad (17)$$

Again, linear control law is applied

$$u_4 = h_{11} x_{11} + h_{12} x_{12} \quad (18)$$

in order to assign closed loop poles to λ_{11} and λ_{12} by satisfying

$$h_{11} = -\lambda_{11} \lambda_{12} \quad h_{12} = \lambda_{11} + \lambda_{12} \quad (19)$$

It should be noted that for this particular control, the existence of a foot is necessary in order to transmit the reaction forces of the ground.

All the closed loop poles may have negative real parts for stable locomotion.

Step Length Controls

Biped locomotion results in leg switching at the discrete time intervals, thus providing the system with a variable structure /7/. Periodical changing of the bipes structure may be represented as time-discrete periodical switching of the hip angle

ϕ_1 in the sagittal plane as given in the following form

$$\dot{x}_{s1} = A_2 x_{s1} - B_4 u_s(kT) \delta(t-kT) \quad (20)$$

where $B_4 = \begin{bmatrix} 1 & 0 \end{bmatrix}'$ and $\delta(t)$ is a Dirac delta function. In spite of the coupling of this mode with the mode given in Equation 12, its unstable eigenvalues have not been affected because of the autonomous type of feedback (Eq. 13). The instability of the hip motion is essential for producing the desired velocity. It can be stabilized, through, by the step angle control u_s at the discrete time intervals.

Replacing Equation 20 with an equivalent discrete form gives

$$x_{s1}(k+1) = A_s x_{s1}(k) - B_s u_s(k) \quad (21)$$

where the matrix A_s has the following elements: $a_{11}=a_{22}=\cosh bT$, $a_{12}=(1/b)\sinh bT$, $a_{21}=b^2 a_{12}$, $b_1=a_{11}$, and $b_2=a_{21}$. A suitable linear form of $u_s(k)$ is

$$u_s(k) = h_3(x_3(k) - \Delta) + h_4(x_4(k) - \beta v_0/r_0) + \Delta \quad (22)$$

where Δ is the desired total step angle, and β is a constant.

Closed loop poles λ_3 and λ_4 of the stable discrete system 21 have to be chosen within the unit circle. Corresponding feedback gains have to be set to

$$\begin{aligned} h_3 &= 1 - \lambda_3 \lambda_4 \\ h_4 &= ((1+\lambda_3 \lambda_4) \cosh bT - \lambda_3 - \lambda_4) / (b \sinh bT) \end{aligned} \quad (23)$$

In the lateral direction, a discrete system can be derived in a similar form as system 21. The lateral step angle u_l is given in the linear form

$$u_l(k) = h_7 x_7(k) + h_8 x_8(k) + \gamma (\dot{x}_x(Y) - v_x) / r_0 \quad (24)$$

where γ is constant, \dot{x}_x is desired trajectory in the XY plane, and p_x is position of the foot. The last term of Equation 24 is designed to meet the requirement of walking on the predetermined path.

Time optimal lateral control has been selected for this linearized discrete system to accelerate the transition process in the case of disturbances. The feedback constants are therefore

$$h_7 = 1 \quad h_8 = (1/b) \coth bT \quad (25)$$

We could obtain them also by setting $\lambda_7 = \lambda_8 = 0$ into Equation 21.

By these control concepts, stable locomotion can be achieved

on a certain path in the three dimensional space.

Stability and Control of Nonlinear System

For larger excursion from the point of unstable equilibrium, the nonlinear equations of motion apply. Autonomous linear controls for each mode are expected to be effective at least for small deviations of the state vector from the equilibrium point, and for a small step size. Autonomous action of continuous controllers has to be supported by additional terms Q^* in the nonlinear region according to Expression 6, which are called dynamic compensating algorithms.

Leg length control has to compensate the weight of body mass with

$$F^* = g \quad (26)$$

The leg action must be compensated with the torque which would keep the body in the state $\phi_2 = 0$ and $\theta_2 = 0$, $\dot{\phi}_2 = 0$ and $\dot{\theta}_2 = 0$ regardless on what the leg action would be. From the nonlinear equations of motion, it is easy to derive compensating torques

$$M_1^* = \frac{Fr l \cos \theta_1 \sin \phi_1 (r \cos \theta_1 + 1)}{(r \cos \theta_1 + 1 \cos \phi_1) (r + 1 \cos \theta_1)} \quad (27)$$

$$M_2^* = \frac{Fr l \sin \theta_1}{r + 1 \cos \theta_1} \quad (28)$$

providing that $\psi = 0$ and $\dot{\psi} = 0$ with $M_3^* = 0$. With these dynamics compensating algorithms and linear continuous controls, we can achieve a stable motion of the body within the step period.

The unstable hip motion is stabilized for the nonlinear system with linear feedback controls described by Equations 22 and 24. But in this case the hip state vector components undergo a nonlinear transformation because of the geometric configuration of the legs in the moment of switching /6/ as can be realized from Figure 1.

The gain stability is improved largely by nonlinear control of the sampling interval. From observation of human behaviour, we can conclude that, in the case of a large disturbance, a biped should switch the legs earlier than in the gait half-period T . The switching period T' is therefore a function of the hip state vector components and its reference value. In the simple form, this

relation could be given by nonlinear law

$$T' = \begin{cases} (t-t_k) u (t-t_k-T) \\ (t-t_k) u (/x_3/-x_3^*) \\ (t-t_k) u (/x_7/-x_7^*) \end{cases} \quad (29)$$

where $T' = t_{k+1} - t_k$ when one of the step function first change its value to 1.

Larger disturbances will be eliminated in shorter step periods, thus increasing the stability of the biped model.

Simulation Results

The applicability of the derived control concepts can be examined by the nonlinear system simulation. Three types of stability will be considered in this simulation,

1. Body stability
2. Biped gait stability
3. Biped path stability

Solution of nonlinear equations of motion (Eqs. 1), with altitude and attitude continuous controls, and with discrete step angle control in sagittal and in lateral directions, gives the behaviour of the system for very large disturbances.

The effectiveness of derived control concepts and the biped body stability along with stability of gait, has been tested in the following example. The model with parameters similar to those of a human body started its motion from the position of a runner in a footrace with nearly inverted body ($\phi_2 = 3$ rad). The sequence of stages of locomotion is given in Figure 2. After two subsequent steps, the body has reached almost the normal vertical position. Moreover, after the third step additional, disturbances occurred in the form of stairs. Three steps on the stairs neither influenced the body stability nor the stability of gait considerably, although each step was 0.3 m high. No lateral disturbances were assumed. The main control parameters were $\lambda_1 = -10$, $\lambda_2 = -20$, $\lambda_3 = \lambda_4 = 0.1$, $\lambda_5 = \lambda_6 = \lambda_9 = \lambda_{10} = -10$, $\lambda_7 = \lambda_8 = 0$, $T = 0.6$ sec, and $v_0 = 1.5$ m/sec.

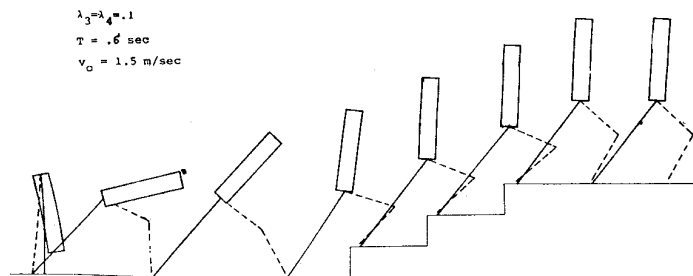


Fig. 2.

The body path stability of the six degree of freedom nonlinear model was tested in the case when the model was forced to walk on a line under initial lateral disturbance. The body was bent in a lateral direction for $\theta_2 = 57^\circ$ with remaining components of the state vector equal to zero. The required velocity was $v_0 = 1.5$, $T = 0.5$ sec, and the poles of the linearized closed loop systems were set to the same value as in the previous case except for $\lambda_4 = 0.2$.

In the Figure 3a the transition process, regarding the step angle in both principal directions of motion, is shown. Body attitude control brought the body into the upright position within two steps. By the foot action, the lateral disturbance was eliminated almost completely in twelve successive steps. The model was brought on line as shown in the Figure 3b, and continued locomotion with a stable singular gait.

A study of pole selection for lateral controls has been made by making the control not to be time optimal regarding the linearized subsystem. In figure 4 we can see an oscillatory behaviour of the lateral step control angle $u_1(k)$, which in return influenced the forward step angle $u_s(k)$ and the body path in XY plane developed a shape similar to a sine curve. The body was only algorithmically controlled in this particular case, with the lateral initial position of the hip $\theta_1 = 0.2$ rad and $\dot{\theta}_1 = 0.5$ rad/sec. The poles of the lateral step control loop were set to be $\lambda_7 = 0.1$ and $\lambda_8 = 0.2$, the required velocity was $v_0 = 1.5$ m/sec, and the time period $T = 0.7$ sec. It is interesting to note that the path in Figure 4 reminds one of a person walking under the influence of alcohol.

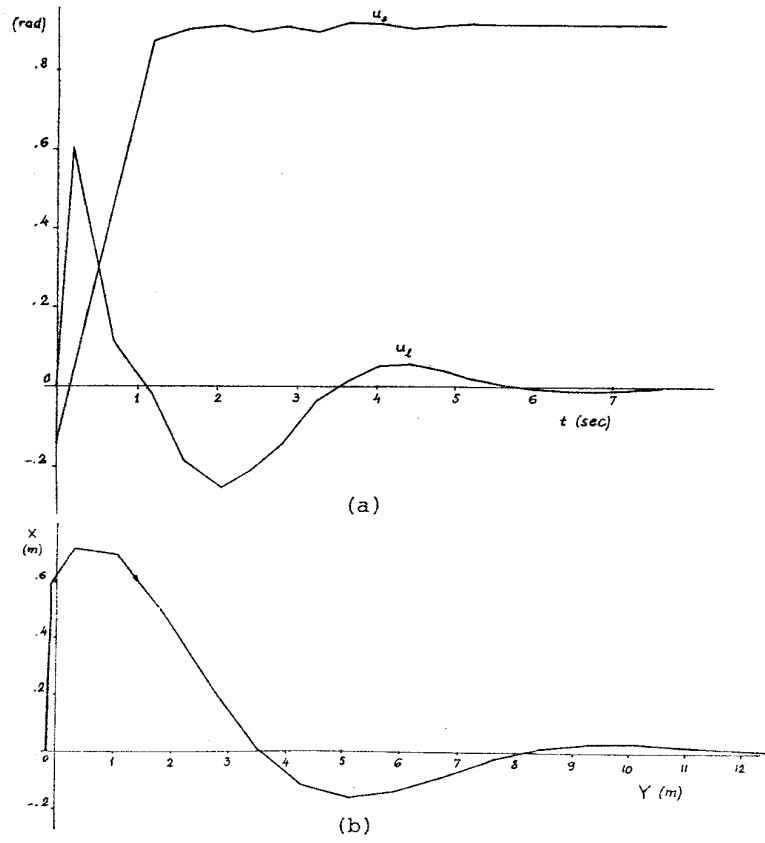


Fig. 3.

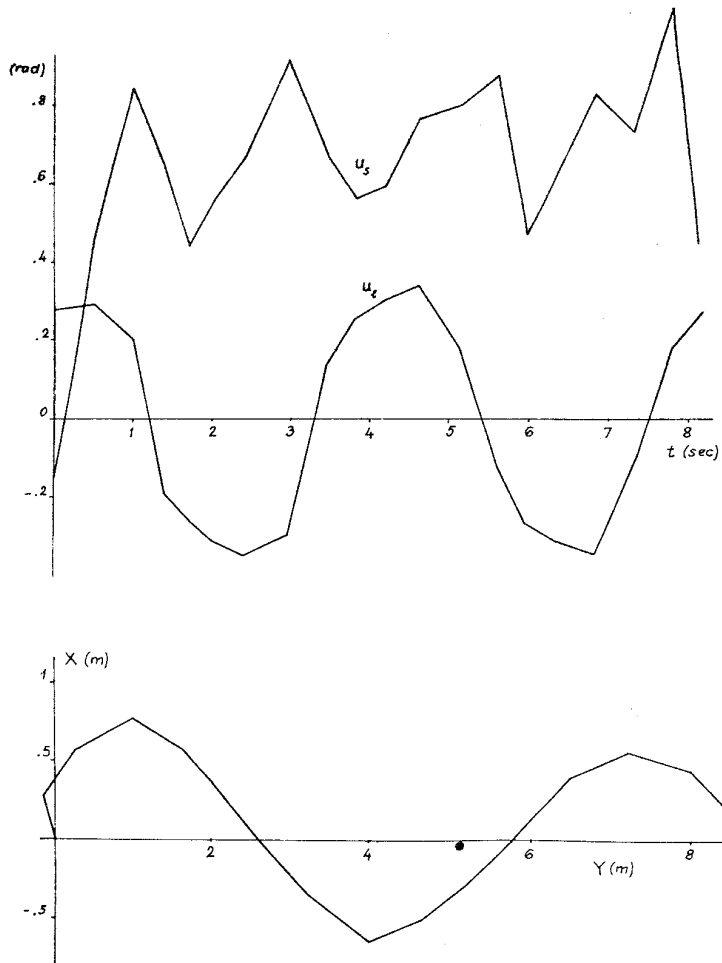


Fig. 4.

Conclusion

Linear continuous controls, combined with dynamic algorithm controls of the body altitude and attitude, together with linear time-discrete step length controls, have been successfully applied in the control of a nonlinear model with six degrees of freedom. Computer simulation of the model with the derived controls for a large initial disturbance and for walking on the stairs demonstrated a high degree of body stability and the stability of gait. Moreover, the body path stability has been achieved by time optimal feedback gains selection for the lateral step length control. Six autonomous linear control loops, associated with one degree of freedom each, have proven to be, together with dynamic compensation algorithms a very effective control concept for biped locomotion control. In addition, the nonlinear time period control has greatly increased overall stability of the model under consideration.

Different types of gaits and ideal combination of the control parameters remain yet to be determined. It is hoped, however, that these results may eventually attain a practical application in orthotic and prosthetic systems, as well as in legged-locomotion machines.

Appendix: Nonlinear Equations of Motion

$$\ddot{r} - l\ddot{\phi}_2 R_2 + l\ddot{\theta}_2 (L_3 - R_3) - r\dot{\phi}_1^2 \cos^2 \theta_1 - l\dot{\phi}_2^2 R_1 + 2l\dot{\phi}_2 \dot{\theta}_2 R_4 - r\dot{\theta}_1^2 - l\dot{\theta}_2^2 (L_4 + R_1) + g \cos \theta_1 \cos \phi_1 = F$$

$$r^2 \ddot{\phi}_1 \cos^2 \theta_1 + lr\ddot{\phi}_2 R_1 - lr\ddot{\theta}_2 R_4 + 2r\dot{r}\dot{\phi}_1 \cos^2 \theta_1 - r^2 \dot{\phi}_1 \dot{\theta}_1 \sin 2\theta_1 - 2lr\dot{\phi}_2 \dot{\theta}_2 R_3 - lr_2 R_2 (\dot{\phi}_2^2 + \dot{\theta}_2^2) - gr \cos \theta_1 \sin \phi_1 = -M_1 + M_3 (L_2 - R_5)$$

$$-lr\ddot{R}_2 + lr\ddot{\phi}_1 R_1 + (J_{xx} + l^2 \cos^2 \theta_2) \ddot{\phi}_2 + lr\dot{\theta}_1 R_6 + 2l\dot{r} (\dot{\phi}_1 R_1 + \dot{\theta}_1 R_6) + lr\dot{\phi}_1^2 R_2 - 2lr\dot{\phi}_1 \dot{\theta}_1 R_5 + lr\dot{\theta}_1^2 R_2 - l^2 \dot{\phi}_2 \dot{\theta}_2 \sin 2\theta_2 - gl \cos \theta_2 \sin \phi_2 = M_1$$

$$lr\ddot{\phi}_2 R_6 + r^2 \ddot{\theta}_1 + lr\ddot{\theta}_2 (L_1 + R_7) + 0,5 r^2 \dot{\phi}_1^2 \sin 2\theta_1 + 2r\dot{r}\dot{\theta}_1 + lr\dot{\phi}_2^2 R_5 - 2lr\dot{\phi}_2 \dot{\theta}_2 R_8 - lr\dot{\theta}_2^2 (L_2 - R_5) - gr \sin \theta_1 \cos \phi_1 = -M_2 + M_3 \cos \theta_2 \sin (\phi_2 - \phi_1)$$

$$\begin{aligned}
& l\ddot{r}(L_3-R_3) - l\ddot{r}\dot{\phi}_1 R_4 + (l^2 + J_{YY})\ddot{\theta}_2 + l(\ddot{\theta}_1 + 2\dot{r}\dot{\theta}_1)(L_1 + R_7) - \\
& - 2l\dot{r}\dot{\phi}_1 R_4 + l\dot{r}\dot{\phi}_1^2 R_3 + 0,5l^2\dot{\phi}_2^2 \sin 2\theta_2 + 2l\dot{r}\dot{\phi}_1 \dot{\theta}_1 R_8 + \\
& + l\dot{\theta}_1^2 (-L_3 + R_3) - gl \sin \theta_2 \cos \phi_2 = M_2
\end{aligned}$$

$$\ddot{\psi}_{ZZ} = M_3$$

where

$$R_1 = \cos \theta_1 \cos \theta_2 \cos (\phi_2 - \phi_1)$$

$$R_2 = \cos \theta_1 \cos \theta_2 \sin (\phi_2 - \phi_1)$$

$$R_3 = \cos \theta_1 \sin \theta_2 \cos (\phi_2 - \phi_1)$$

$$R_4 = \cos \theta_1 \sin \theta_2 \sin (\phi_2 - \phi_1)$$

$$R_5 = \sin \theta_1 \cos \theta_2 \cos (\phi_2 - \phi_1)$$

$$R_6 = \sin \theta_1 \cos \theta_2 \sin (\phi_2 - \phi_1)$$

$$R_7 = \sin \theta_1 \sin \theta_2 \cos (\phi_2 - \phi_1)$$

$$R_8 = \sin \theta_1 \sin \theta_2 \sin (\phi_2 - \phi_1)$$

$$L_1 = \cos \theta_1 \cos \theta_2$$

$$L_2 = \cos \theta_1 \sin \theta_2$$

$$L_3 = \sin \theta_1 \cos \theta_2$$

$$L_4 = \sin \theta_1 \sin \theta_2$$

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