Proceedings of the Eighth International Symposium on ECHE, 1984, Dubrovnik

METHOD FOR AUTOMATIC (COMPUTER) FORMING OF MATHEMATICAL MODELS AND FUNCTIONAL MOVEMENT SYNTHESIS

Borovac B., Surla D. and Konjović Z.
University of Novi Sad, Yugoslavia

Abstract

In this paper the method for automatic (computer) forming mathematical models and functional movements synthesis of active spatial mechanisms consisting of open and closed kinematic chains has been presented shortly.

Algorithm for correcting nominal trajectories has been presented, so system in simulation can perform tracking of the nominal trajectories exactly. Example of simulation has been done on an antropomorphic configuration for producing artificial walk.

1. Introduction

In last decade at the Robotics Department of "M.Pupin" Institute from Belgrade automatic methods for forming dynamic models of active spatial mechanisms have been developed. These algorithms got a form of general procedure for computer forming mathematical models of spatial mechanisms of arbitrary complexity, and a couple of methods have been developed based on recurrent relations.

In this way synthesized models are basic for the synthesis of functional movements of system, as well as for synthesis of control algorithms (1 - 5). For functional movements synthesis the half-inverse Vukobratović's method has been used which can been described as follows (4). One part of system has prescribed trajectories which satisfy some functional requests. The dynamics of the rest of the system has to be solved in such a way that this "open" dynamics keep system in dynamic equilibrium with requested boundary conditions and appropriate dynamic links.

In this way the needed functional movements could be given without any simplification or linearization of the system and also specific way of reduction of dimensionality of system has been attained.

In the simulation has been requested that system tracks the already synthesyzed nominal trajectories. All degrees of freedom are "open" (there is no prescribed dynamics), and at perturbed regimes stabilization is done by local and global feedbacks at powered degrees of freedom. Control of moving of unpowered degrees of freedom is done via powered degrees of freedom.

2. Mathematical model of the system

Mathematical model of the system consists of two parts: model of mechanical configuration and model of actuators. Mechanical configuration can consist of some open or close kinematic chains in the form (6, 7):

$$S^M: \quad P = H(q)\cdot\ddot{q} + h(q, \dot{q}) \tag{2.1}$$

where is P - n dimensional vector of generalized forces, H - (n×n) dimensional inertia matrix, h-n dimensional vector of Coriollis, gravitational forces and other influences, q-n dimensional vector of coordinates of degrees of freedom, n - number of degrees of freedom.

Mathematical model of actuator is given in the form:

$$S^i: \quad \dot{x}^i = A^i x^i + f^i P^i + b^i\cdot N(u^i) \tag{2.2}$$

where x^i-n dimensional state vector of i-th subsystem, A^i, P^i, $N(u^i)$ - (3×3) system matrix, generalized force and nonlinearity of saturation type (input) of i-th actuator, respectivelly. f^i and b^i are n^i - dimensional force and control distribution vector.

Suppose m(≤n) degrees of freedom are powered, while (n-m) are unpowered. Model of overall system can be given by uniting S^M and S^i models. In this purpose, united (overall) actuator model can be written as:

$$\dot{x}_c = A\cdot x_c + F\cdot P_c + B\cdot N(u) \tag{2.3}$$

where: $x_c = (x_c^{1T}, x_c^{2T}, \ldots, x_c^{mT})^T$ state vector of actuator system, $B = \text{diag}(b^i)$, $F = \text{diag}(f^i)$, $N(u) = (N(u^1), N(u^2), \ldots, N(u^m))^T$.

For separating coordinates in q which belong to powered degrees of freedom (q_c), from these which belong to unpowered ones (q_N) (n×n) matrix transformatic T_1 is applied so (8):

$$T_1\cdot q = (q_c^T \; q_N^T)^T \tag{2.4}$$

Applying (2.4) on (2.1) gives:

$$P_c = H_{cc}\cdot\ddot{q}_c + H_{cN}\cdot\ddot{q}_N + h_c \tag{2.5}$$

$$P_N = H_{Nc}\cdot\ddot{q}_c + H_{NN}\cdot\ddot{q}_N + h_N \tag{2.6}$$

where:

$$T_1\cdot H\cdot T_1^{-1} = \left[\begin{array}{c|c} H_{cc} & H_{cN} \\ \hline H_{Nc} & H_{NN} \end{array}\right] \quad \text{and} \quad T_1\cdot h = \begin{bmatrix} h_c \\ \hline h_n \end{bmatrix}, \; H_{cc}, H_{cN}, H_{Nc} \text{ and } H_{NN}$$

are matrices of appropriate dimensions, P_c and P_N are generalized forces of powered and unpowered degrees of freedom, respectively. It is supposed that P_N are known in advance. Introducing transformation T such that

$$\dot{q}_c = T\cdot x_c \tag{2.7}$$

it can be given

$$\dot{x}_c = A\cdot x_c + F(I_m - H_p\cdot T\cdot F)^{-1}\{H_p[T\cdot A\cdot x_c + T\cdot B\cdot N(u)] + H_{cN}H_{NN}^{-1}(P_N - h_N) + \\ + h_c\} + B\cdot N(u) \tag{2.8}$$

$$\ddot{q}_N = H_{NN}^{-1}\, P_N - H_{Nc}\cdot T[A\cdot x_c + F(I_m - H_p\cdot T\cdot F)^{-1}(H_p\cdot T\cdot A\cdot x_c + H_p\cdot T\cdot B\cdot N(u) + \\ + H_{cN}\cdot H_{NN}^{-1}(P_N - h_N) + h_c] + B\cdot N(m) - h_N\} \tag{2.9}$$

where: $H_p = (H_{cc} - H_{cN}\cdot H_{NN}^{-1}\, H_{Nc})$, i_m is unit matrix of aproppriate dimension.

3. Functional movements synthesis, nominal control and simulation

Based on mechanical part of mathematical model (2.1), has been synthesized functional movements of active spatial mechanisms. One part of the system has prescribed dynamics. From matrix H and vector H (which are formed automatically), certain coefficients have been taken out and form the reduced system of differential equations. By solving this system with requested boundary conditions, dynamics of the rest of the system is obtained.

Let dynamics of the whole mechanical part of the system be written as: q_{c_o}, q_{N_o}, $\dot{q}_{c_o}$, $\dot{q}_{N_o}$, $\ddot{q}_{c_o}$, $\ddot{q}_{N_o}$.

United system (2.7), (2.3) can be written in the compact form as follows:

$$\dot{x} = \hat{A}(x) + \hat{B}(x)\cdot N(u) \tag{3.1}$$

where is $x = (x_c^T, q_N^1, \dot{q}_N^1, \ldots, q_N^{n-m}, \dot{q}_N^{n-m})^T$ state given vector of united system, $x_c = (x^{1T}, x^{2T}, \ldots, x^{mT})^T$ where $x_c^i = (q_c^i, \dot{q}_c^i, \phi^{iT})^T$ for $i = 1,2,\ldots,m$. Vector $\phi^i(+)$ is of dimension (n^i-2) and consists of the rest of the state vector (except q_c^i and $\dot{q}_c^i$) of the i-th powered degree of freedom. $\hat{A}(x)$ is vector function of order NS, $\hat{B}(x)$ is matrix of size (NS×m), where NS is the order of the whole system.

Let suppose nominal control can be obtained by solving (2.2) if the nominal trajectories q_{c_o}, q_{N_o}, $\dot{q}_{c_o}$, $\dot{q}_{N_o}$, $\ddot{q}_{c_o}$, $\ddot{q}_{N_o}$ are known, and these solutions are $\phi_o^1, \phi_o^2, \ldots, \phi_o^m$ and u_o. But, integration of the system (3.1) with initial conditions $x(to)=x_o$ and nominal control u_o, does not provide that system tracks nominal dynamics q_{c_o}, q_{N_o}, $\dot{q}_{c_o}$, $\dot{q}_{N_o}$, $\ddot{q}_{c_o}$, $\ddot{q}_{N_o}$. This has been caused by cumulative error of numerical integration because of matrix inversion of different dimensions.

On Fig. 1. the flow chart for the algorithm has been shown for nominal control correction for tracking the nominal dynamics.

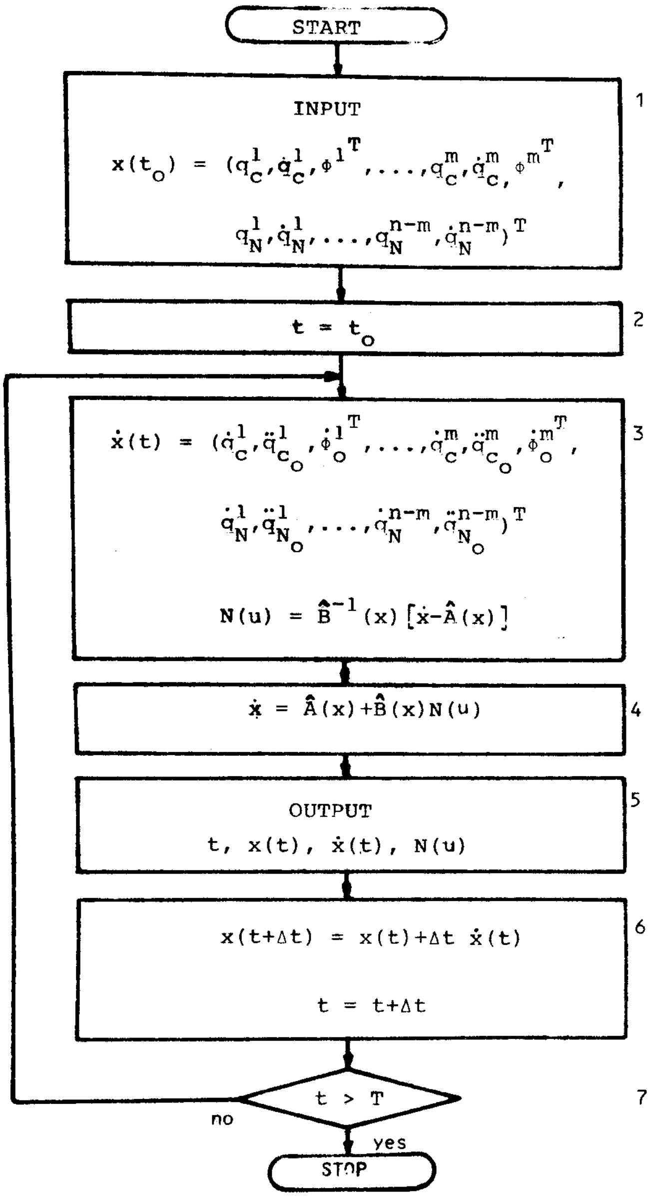


Fig. 1. Flow-chart of the algorithm for nominal control correction

In block 3, at each integration step, correction of the N(n) has been done, based on the $\ddot{q}_{c_o}$, $\ddot{q}_{N_o}$, $\dot{\phi}^1_o$, $\dot{\phi}^2_o,\ldots,\dot{\phi}^m_o$.

In this way not only cummulative error of integration because of matrix inversion of different dimensionality has been avoided, but also all round-off errors.

Numerical integration of system (3.1) with initial conditions $x(0)=x_o$, and nominal control N(u) (memorized at block 5) provide exact tracking of dynamic nominal trajectories memorized at block 5.

Simulation of moving the overall system (3.1) has been done on digital computer. System was requested to track nominal trajectories (memorized at block 5). At the level of perturbation, correction are introduced via local and global feedbacks, so control structure can be presented as (9)

$$u^i(t) = u^{o^i}(t) + u^{L^i}(t) + u^{G^i}(t) \qquad (3.2)$$

where $u^o(t)$, $u^L(t)$, $u^G(t)$ and $u(t)$ are nominal, local, global and total control, while index "i" denatos the corresponding actuator. Local feedback is introduced by state vector of certain actuator, while global can be done by: 1) moments (generalized forces) in mechanism joints, 2) reaction forces by mechanism and environment.

4. Example of active spatial mechanism for producing artificial antropomorphic gait

In this part application of the general algorithm for functional movements synthesis will be shown, applied on the active spatial mechanism for producing artificial antropomorphic gait (10-12). Fig. 2 shows an illustrative kinematic scheme of the antropomorphic system. The ball joints are replaced by apropriate number of kinematic pairs of the fifth class.

Mechanism parameters in Fig. 2. are:

r_{o_1} - starting position of the first segment in absolute coordinate system,

e_1 - unit vector of the first joint axis with respect to absolute coordinate system.

The further vectors are given with respect to internal coordinate system of each segment.

$\tilde{e}_i$ - unit vector of joint axis

$\tilde{r}_{i,i}$ - vector from the center of the "i"-th joint to the center of the gravity of the "i"-th segment

$\tilde{r}_{i,i+1}$ - vector from the center of the i+1-st joint to the center of the "i"-th segment, of some chain.

Kinematic scheme of the antropomorphic system could be divided into three open kinematic chains. First chain represent the

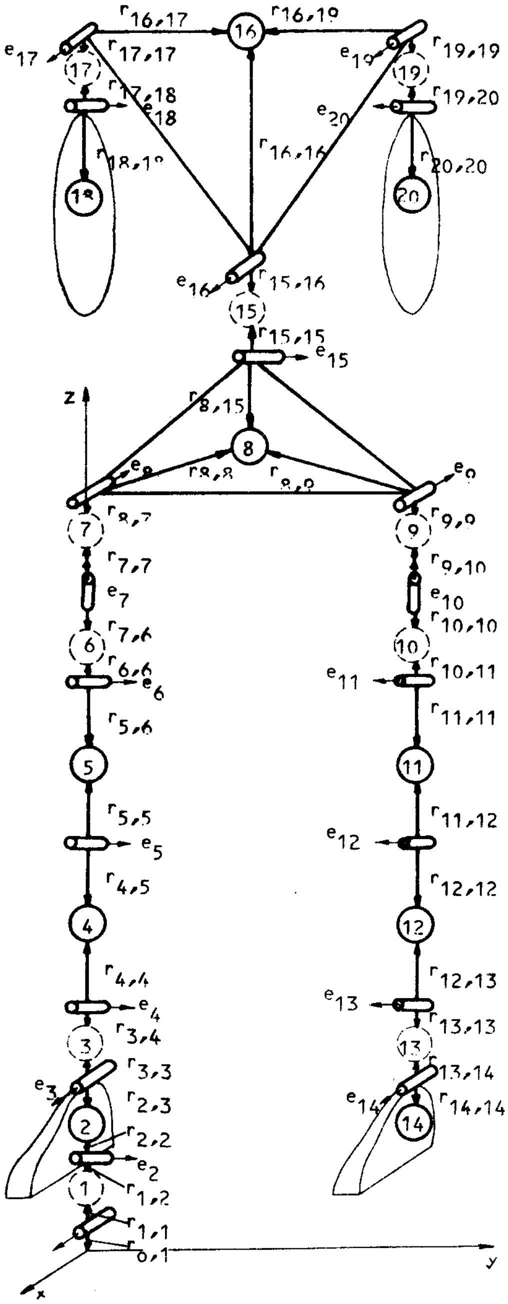


Fig. 2. Kinematic scheme of the arthopomomorphic walking system

legs (segments 1-14), the second upper part of the body and left arm (segments 15-18), and the third one, right arm (segments 19-20).

All input data about the configuration are in one matrix.

Beside these informations about the mechanism some other are also necessary: number of segments, number of chains, number of basic chain (chain which is connected to the first segment), number of basic segment (segment from wich next segment continues), type of segment (body or cane), specificity (if axis e^i of the "i"-th joint, and vector $r_{i-1,i}$ in assembling phase coincides), masses and moments of inertia, internal coordinates and their derivatives.

During walking, system supports at one leg, after that at both, and at the other one, so legs are forming open and closed kinematic chains. According to input data algorithm calculates matrix $H(q)$ and vector $h(q, \dot{q})$ from model (2.1). This model describes mechanism at single support phase.

Suppose, at the double support phase on the other feet are acting unknown forces R^* and moment M^*, which are characteristic of the closed chain support:

$$\sigma = (\sigma_1, \sigma_2, \ldots \sigma_6)^T = (R_x^*, R_y^*, R_z^*, M_x^*, M_y^*, M_z^*)^T \tag{4.1}$$

Model (2.1) for this case is

$$H \cdot \ddot{q} + B \cdot \sigma + h = P \tag{4.2}$$

where matrix H and vector h are formed as in single support phase, but matrix B is multiplied by all elements which depend of σ. Matrix B is formed simultaneously with H and h.

Denote known generalized coordinates as $\{q_o\}$, unknown as $\{q_x\}$. In this case to the model (2.1) or (4.2) are requested boundary conditions:

$$\left\{\begin{array}{c} q_x \\ \hline \dot{q}_x \end{array}\right\}_T = [W] \left\{\begin{array}{c} q_x \\ \hline \dot{q}_x \end{array}\right\}_o . \tag{4.3}$$

Transformation matrix $[W]$ gives connection between state vector at begining and at the end of integration interval T. Under assumption that the perturbations are small, the starting conditions correction is given in the form:

$$\left\{\begin{array}{c} \Delta q_x \\ \hline \Delta \dot{q}_x \end{array}\right\}_o = ([U]-[W])^{-1} ([W] \left\{\begin{array}{c} q_x \\ \hline q_x \end{array}\right\}_o - \left\{\begin{array}{c} q_x \\ \hline q_x \end{array}\right\}_T) \tag{4.4}$$

where $[U]$ is sensitivity matrix with submatrix expressed as Jacobian´s.

In the single support phase algorithm is automatically adjusted for solving the model (2.1) with boundary conditions (4.3), but at the double support phase for system (4.2) with boundary conditions (4.3).

For standard numerical intergration methods it is necessary to calculate right - hand sides of differential equations. Calculation of right-hand sides of differential equations which describe motion of unknown part of the system is realized automatically, just based on information about: prescribed dynamics, additional degrees of freedom (necessary condition for closing the first chain) and known generalized forces.

Correction of initial conditions is based on global and local iterative procedure. Global use gradient method applied on performance index formed as norm of deviation from boundary conditions (4.3) according to matrix [W].

Local iterative procedure is based on correction of these conditions according to (4.4).

In this example the dynamics of the low extremities was prescribed and hands were fixed on the "chest" of the antropomorphic mechanism. "Open" or compensating dynamics consist of moving in frontal (angle θ) and saggital (angle ψ) plane and keep system in dynamic equilibrium. For prescribed dynamic parameters S, T and p are introduced. Parameter S is used for adjusting size, and T for duration of one step, what enables us to control speed of walking and to keep the same type of gait. Parameter p denotes duration of double support phase corresponding to full step.

5. Numerical results

In this paragraph a part of results obtained for antropomorphic configuration shown on Fig. 2 will be presented. Dashed line denotes centers of gravity of fictive segments (introduced just for representing a joint with two or three degrees of freedom with a set of joints of the fifth class), and sizes of vectors $\tilde{r}_{i,i}$ and $\tilde{r}_{i,i+1}$ of these segments are small (10^{-8}m). Other parameters of the mechanism are same as in (4). The joints: 4, 5, 6, 11, 12, 14, 15, 16, (Fig. 2) are powered with DC Globe Industries Division motors of TRW INC type 102A200-8. Joints 1 and 2 are degrees of freedom between foot and floor, and the rest of unpowered joints are considered as "frozen".

Based on nominal dynamics before correction and actuator mathematical model (2.2), nominal control N(u(t)) has been calculated i.e. voltage on DC motor poles. With such nominal control and for undisturbed initial conditions integration of system (3.1) has been done. Euler's method has been used with integration step 0,01875 (s). On Fig. 3 and 4 has been shown deviations of simulation results from nominal; on Fig. 3 are shown diferences of moments under foot (in saggital and frountal plane which should be zero), and differences in moments for knee and hop joints. On Fig. 3a,b,c are shown differences of position, speed and acceleration for joints: 4 (ankle), 6(hip), 15 and 16 (conpensation movements of the trunk). Simulation results prove that not only functionality of the movements has been disturbed, but also instability of the system occurs. Such dynamics cause the ZMP (zero moment point) to move out of area covered with the supporting foot. That means that such nominal trajectories before correction do not allow the system to track them (in the case without initial disturbances). After correction, system follows them exactly.

Correction was done according to algorithm on Fig. 1 on computer EI H6/63 in single precision with 7 digits mantissa. Nominal dynamics before and after correction coincide, at least on 4 significant digits.

6. Conclusion

For more precise study of control structures of active spatial mechanisms such nominal control has to be synthesized that system can follow the nominal trajectories exactly for the undisturbed initial conditions. Nominal control calculated by direct solving actuator subsystems (2.2) does not satisfy such condition. It has been cause by cummulative error of numerical integration because of matrix inversion of different dimensionality. Algorithm for correcting nominal control so that system can track nominal dynamics, has been given in Fig. 1. In this way cummulative error has been avoided, so nominal trajectories before and after corrections differ at each discrete point just for the matrix inversion error. Such nominal provides exact tracking of the system.

7. References

(1) Stepanenko Yu., "Dynamics of Spatial Mechanisms" (in Russian), Monograph, Mathematical Institute, 1974, Beograd.

(2) Juričić D., Vukobratović M., "Mathematical Modelling of Bipedal Walking System" ASME Publication 72/WA/Bhf-13.

(3) Vukobratović M., Stepanenko J., "Mathematical Models of General Anthropomorphic Systems", Mathematical Biosciences 17, 1973.

(4) Vukobratović M., "Legged Locomotion Robots and Anthropomorphic Mechanisms" (in Russian), "Mir", Moscov, 1976.

(5) Stepanenko Yu., Vukobratović M., "Dynamics of active Articulated Spatial Mechanisms in Form of Open Loop Chains", Mathematical Biosciences, Vol. 23, No 1/2, 1970.

(6) Vukobratović M., Potkonjak V., Dynamics of Manipulation Robots: Theory and Application, Monograph, Springer-Verlag, 1982.

(7) Vukobratović M., Stokić D., Control of Manipulation Robots: Theory and Application, Monograph, Springer-Verlag, 1982.

(8) Vukobratović M., Stokić D., "Dinamičko upravljanje manipulatorima" Monograph, Institut "M.Pupin", Beograd 1979.

(9) Vukobratović M., Stokić D., "Significance of Force-Feedback in Controlling Artificial Locomotion-Manipulation Systems", IEEE Trans. on Biomedical Engineering, Vol. BMF-27, No 12, Dec. 1980.

(10) Surla D., "Contribution to Synthesis of Algorithms for Automatic Forming Models of Dynamics of Anthropomorphic Systems", Applied robotics", Beograd, 1981.

(11) Surla D., Konjović Z., "A General Algorithm for the System of Functional Motion of the Active Spatial Mechanisms" Proc., XXVI, ETAN Subotica, Yugoslavia, 1982.

(12) Surla D., Konjović Z., "Dynamic Models of Spatial Active Mechanisms of Open and Closed Configuration", First Soviet-Jugoslav Symposium on Applied Robotics, February 1983, Moscow.

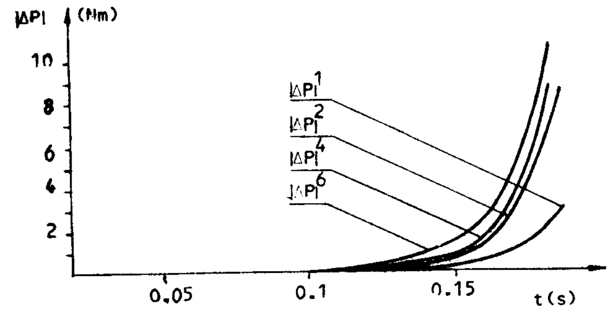
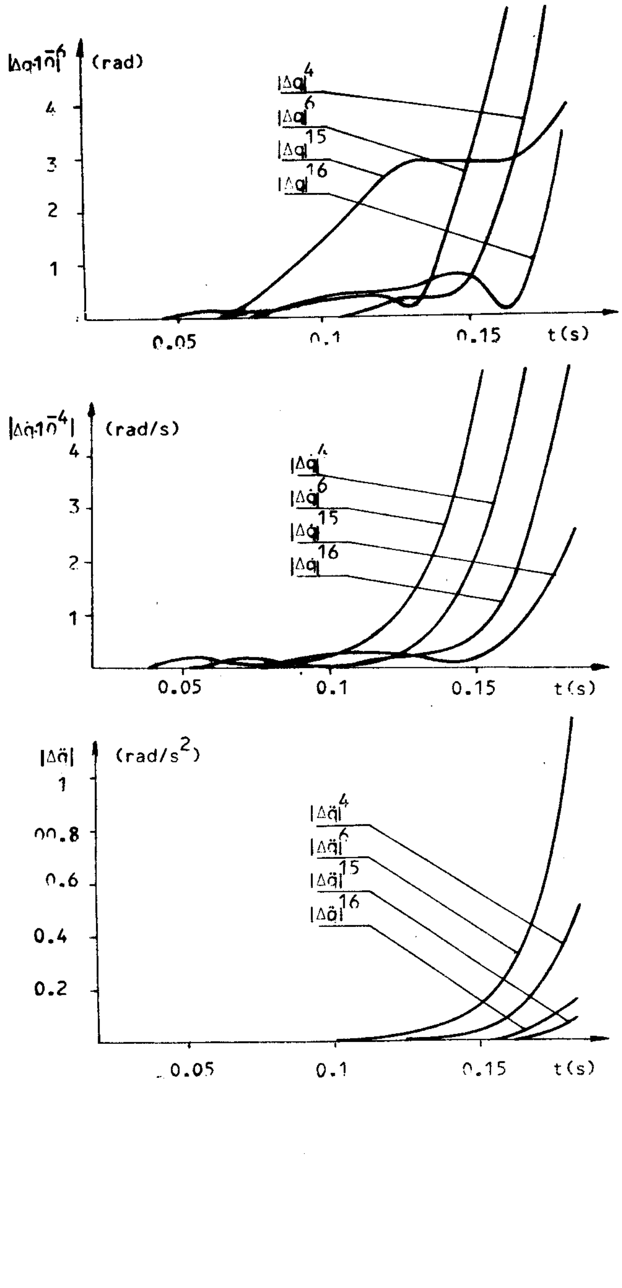


Fig. 3.



(a)

(b) Fig. 4.

(c)