Describing Passive Joint Moments with a Nonlinear Viscoelastic Model

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Abstract - The objective of this study was to develop a mathematical model that could be used to describe passive joint properties. A model based upon the Kelvin model for viscoelasticity was implemented because of its ability to describe both the nonlinear elasticity and viscosity with one equation. To determine the model parameters, passive moments at the knee joint of an able-bodied male subject were measured isokinetically at three velocities. To account for the effects of biarticulate muscles, measurements were performed with the hip and ankle joints in different fixed positions. The results showed passive moment curves qualitatively similar to ones found in the literature and that model parameters can be accurately estimated. These results indicate that one equation is able to describe the elastic and viscous properties of a joint. Further testing of spinal cord injured and able-bodied individuals will be performed to estimate model parameters for the remaining lower extremity joints.

I. INTRODUCTION

The resistance imposed by passive joint properties can impede the functionality of Functional Electrical Stimulation (FES) systems during limb *movements*. However, at fixed *postural* positions, the resistance provided by passive properties could aid postural stability by providing disturbance rejection. To better understand the role of passive joint properties, a musculoskeletal model of the lower extremity joints is being developed which will include the passive joint properties of the ankle, knee and hip. Several investigators [1-3] have previously developed models to describe the passive elastic properties or the passive viscoelastic properties of a joint. However no model has represented the elastic and viscoelastic properties with one equation. Consequently, the objective of this study was to develop a single mathematical equation that could be used to describe the passive elastic and viscoelastic properties of a joint. To accomplish this goal a model based upon the Kelvin model for viscoelasticity was developed and fitted to experimental passive moment data measured from an able-bodied subject.

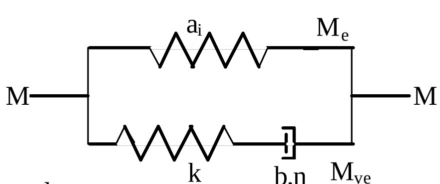

Figure 1.
Mechanical analogue
of the passive moment model

II. METHODS

Model Structure

The classic Kelvin model consists of a spring in parallel with the series combination of a spring and dashpot. To describe passive moments we implemented an angular version of this typically linear-motion model. In addition, the single elastic element and the viscous element were modeled with nonlinear equations. Figure 1 illustrates a mechanical analogue of the passive moment model and the variables used to develop the mathematical equations.

The nonlinear equation for the elastic element was based upon a double exponential function. Due to the past efforts of researchers [4, 5] the following functional form has been developed to describe the moment produced by the nonlinear elastic element:

$$M_{elastic} = a_1 e^{a_2 q + a_3 q_p + a_4 q_d} + a_5 e^{a_6 q + a_7 q_p + a_8 q_d} + a_9$$

q = joint angle

q_p and q_d = proximal and distal joint angles

a_i's are parameters estimated from experimental data

The joint angles of the adjacent joints are included as variables to describe the effects of biarticular muscles on the passive elastic moment developed at a joint.

For the nonlinear viscous element, others [6, 7] have shown that a power function was suitable for modeling the relationship between the joint velocity and the viscous moment. The form included in the current model for the viscous element was:

$$M_{viscosity} = -b \dot{q}_2^n$$

$\dot{q}_2$ = velocity of angle 2

b and n are parameters estimated from experimental data

The equation describes the moment produced by the viscous element as increasing rapidly at low velocities and then plateauing at higher velocities.

The third component of the model was an elastic element that produced a moment linearly related to its change in angle. The form for this element was:

$$M_{linear\ elastic} = -k(q_1 - q_r)$$

q_1 = position of angle 1

q_r = neutral angle for linear elastic element

k is a parameter estimated from experimental data

Based upon these equations, the following mathematical equation was developed to describe the passive moments at a joint:

$$\dot{M} = \dot{M}_e - k\dot{q} + k\left(\frac{M_e - M}{b}\right)^{1/n}$$

The model for the knee joint has 12 parameters. For the nonlinear parameter estimation, the Levenberg-Marquardt algorithm implemented in MATLAB [The MathWorks, Inc., Natick, MA] software was utilized.

Experimental Measurements

The passive moments at the knee joint were measured on a healthy able-bodied male subject. To measure the moments, a Biodex® dynamometer (Figure 2) was used. The dynamometer moved the passive joint through its range of motion (ROM) at a constant velocity. The forces and moments about all three axes were measured using a JR3® transducer attached to the spindle (Figure 2). This provided a means to ensure that the off axis moments were minimal during testing.

To obtain an accurate description of the passive moments developed at the joint several factors had to be taken into account: joint angle and velocity, angles of adjacent joints proximal and distal to the joint of interest, and the length of time the joint had been at a given angle. To account for all of these factors, an experimental protocol was developed that consisted of isokinetic testing of the joint throughout its ROM at three different velocities. At each velocity, the joint was preconditioned through ten cycles and then the passive joint moments were measured for the next five cycles.

Table 1

Knee Joint	Trial No.	Hip Angle (Deg)	Ankle Angle (Deg)	Velocity (deg/s)
	1	3	0	5
	2	3	0	60
	3	3	0	90
	4	42	0	5
	5	73	0	5
	6	73	12	5
	7	73	25	5
	8*	73	25	20

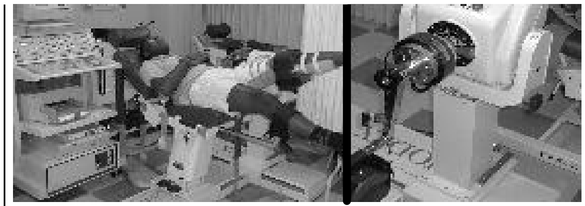

Figure 2. Dynamometer configuration

A cycle consisted of a movement from full flexion to full extension to full flexion or vice versa. The testing ROM and velocities used during the experiment included joint angles and velocities traversed during a sit-to-stand transition. To measure the influence of biarticular muscles crossing a joint, the positions of the proximal and distal joints (i.e. the hip and ankle joints respectively) to the knee joint were varied through a set of fixed joint angles as the knee was rotated. For the experimental trials, the sets of positions used for the adjacent joints were chosen to cover a majority of the ROM encountered during a sit-to-stand transition. The velocities used to rotate the knee and the positions for each of the adjacent joints are included in Table 1. Trial 8 includes an additional testing velocity. The data form this trial was used to examine the ability of the model to reproduce data not used for the parameter estimation.

III. RESULTS

The passive moment measurements obtained along with the model results are illustrated in Figures 3. The experimental results appear to agree with the shapes and magnitudes of passive moment curves found in the literature. The model results are close to experimental results overall, but there are periods where the model diverged more than a Newton-meter from the data. The parameters estimated for the model are given in Table 2.

Figure 3a shows the passive moments at the knee as the hip flexion angle was varied. As hip flexion was increased the passive moment measured at full knee extension increased likely due to increased tension of the hamstring muscles. In Figure 3b, varying the ankle joint angle did not noticeably affect the passive knee

Table 2

a_1	a_2	a_3	a_4	a_5	a_6
2.421	-2.128	1.325	0.056	-43.95	0.165
a_7	a_8	a_9	k	b	n
0.011	0.0009	47.10	306.4	0.789	0.206

moment. In this case, the changes in ankle angle did not significantly influence the tension in the gastrocnemius muscle for it to have an effect on the moment developed at the knee. Lastly in Figure 3c, the effect of varying the velocity of the isokinetic test is demonstrated. Due to the viscous properties of the joint, the amount of hysteresis exhibited by the data is a function of velocity. As shown, there appears to be a large constant component to the hysteresis with a smaller component of the hysteresis affected by the magnitude of the rotational velocity. This supports the use of the power function for describing the viscous component of the model.

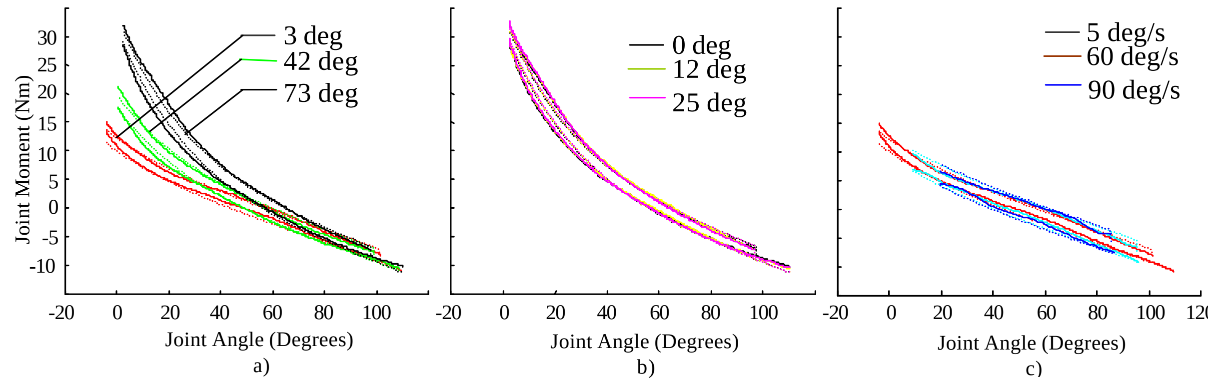

Figure 3. Passive moment variations due to changes in hip angle, ankle angle, and joint velocity respectively. Solid lines represent experimental data and dashed lines represent the fitted curves from the model.

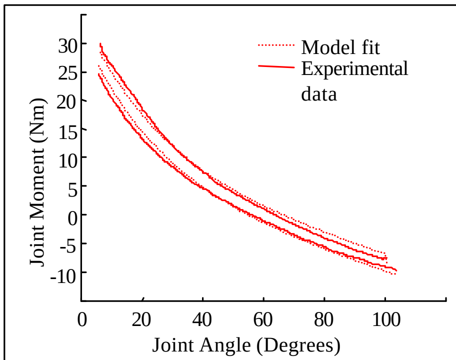

Figure 4. Comparison between data and moments predicted by the model

The predictability of the model is illustrated in Figure 4. The experimental data plotted was not used during the parameter estimation stage. Qualitatively the model results are close to the experimental results, which is very encouraging. Further analysis must be performed however, to quantify the error.

V. CONCLUSIONS

In this study, the goal was to develop a single equation that could be used to describe the viscoelastic nature of passive joint properties. A model based upon the Kelvin model for viscoelasticity was developed and the parameters estimated from experimental data obtained at the knee joint of an able-bodied individual. The results showed that the model parameters could be estimated and that the fit between the model and the data was good.

Future work for this study includes testing additional subjects with and without spinal cord injuries to ensure a robust set of model parameters is found. In addition, testing will be performed to obtain the model parameters for the remaining lower extremity joints.

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REFERENCES

[1] H. Hatze, "A three-dimensional multivariate model of passive human torques adn articular boundaries," *Clinical Biomechanics*, vol. 12, pp. 128-135, 1997.

[2] J. M. Mansour and M. L. Audu, "The passive elastic moment at the knee and its influence on human gait," *J Biomech*, vol. 19, pp. 369-73, 1986.

[3] R. Riener and T. Edrich, "Significance of passive elastic joint moments in FES," *Proceedings of the 2nd Conference of IFESS*, pp. 103-106, 1997.

[4] R. Riener and T. Edrich, "Identification of passive elastic joint moments in the lower extremities," *J Biomech*, vol. 32, pp. 539-44, 1999.

[5] Y. S. Yoon and J. M. Mansour, "The passive elastic moment at the hip," *J Biomech*, vol. 15, pp. 905-10, 1982.

[6] A. Esteki and J. M. Mansour, "An experimentally based nonlinear viscoelastic model of joint passive moment," *J Biomech*, vol. 29, pp. 443-50, 1996.

[7] R. P. Hubbard and K. J. Chun, "Mechanical responses of tendons to repeated extensions and wait periods," *J Biomech Eng*, vol. 110, pp. 11-9, 1988.