

# The Inverse Dynamics Solutions of Arm-Free Standing for Paraplegics

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## Abstract

*In this paper, an application of the dynamics and control of redundant robotic systems is presented in order to investigate the feasibility of functional electrical stimulation assisted arm-free standing for paraplegics. Through the inverse dynamics analysis of a three dimensional dynamic model of quiet standing, several sets of the minimum number of degrees of freedom (DOF) such that there are unique torque inputs that generate desired kinematic outputs of the model were found. In addition, it was demonstrated that the proposed nonlinear dynamic model could achieve asymptotic stability with only six DOF out of twelve DOF, assuming the remaining six DOF are not actuated. Two controllers, proportional and derivative plus gravity compensation scheme and the computed torque control were proposed. Stability analysis and simulation results suggested that the dynamic redundancy of the biological bipedal stance system allows the selection of an ideal subset of DOF in a particular patient to design a neuroprosthesis for standing.*

## 1. INTRODUCTION

Paraplegics' lost ability to stand can be regained by Functional Electrical Stimulation (FES). Although closed-loop control for arm-free standing has been studied extensively [2], [3], implementing a practical FES system for paraplegia arm-free standing is still limited in use. One of the main reasons is the lack of understanding of the complex dynamics of the double-support phase during standing, i.e. a 3D multi-body and closed-chain mechanism [4]. That is, it is not known how many DOF and

which <sup>1</sup>DOF must be controlled to facilitate the stable standing.

Accordingly, we focused on the application of dynamic modelling and control of closed-chain rigid body systems since they are the mature subjects in the field of robotics. By applying methods used for dynamic modelling in robotics, we developed a nonlinear 3D closed-chain dynamic model of quiet standing. In addition, by applying a novel method of inverse dynamics calculation, i.e. Nakamura's method [8], we found the several sets of minimum number of DOF such that there are unique torque inputs that generate a feasible quiet standing motion of the proposed dynamic model. Finally, by applying the work in Liu *et al.* [6] and Cheng *et al.* [7], we demonstrated that these sets of minimum number of DOF could obtain the asymptotic stability of the proposed nonlinear dynamic model during quiet standing. Simulation results were shown to verify the theoretical results.

## 2. METHODS

### 2.1. Kinematic and Dynamic Modelling

The 3D dynamic model shown in Figure 1 represents quiet standing. The HAT (head-arms-trunk) was modelled as a rigid body with a constant mass and moment of inertia. Each leg consists of 6DOF since the double-support phase can be reasonably approximated by 6DOF for one leg [5].  ${}^wF_o \in \mathbb{R}^{6 \times 1}$  denotes the external force and moment applied to the HAT COM (centre of mass) with respect to the world frame  $\{W\}$ . Denavit-Hartenberg notation [1] was used for kinematic modelling of the legs. The Newton-Euler and Lagrange formulations

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<sup>1</sup> In this paper, DOF was also used to indicate certain degrees of freedom, e.g. "Flexion/extension is the only DOF of the knee joint."

were used for dynamic modelling of the HAT and the legs respectively. The details of the model can be found in [4].

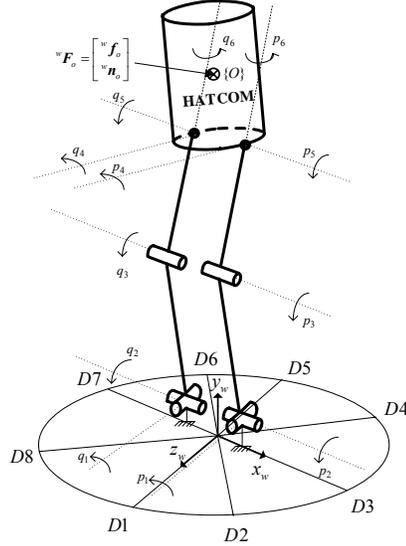


Figure 1. 3D dynamic model of quiet standing shown in the eight directions, D1-D8 ( $q_1, p_1$ : ankle I/E (inversion/eversion),  $q_2, p_2$ : ankle D/P (dorsiflexion/plantarflexion),  $q_3, p_3$ : knee F/E (flexion/extension),  $q_4, p_4$ : hip A/A (abduction/adduction),  $q_5, p_5$ : hip F/E,  $q_6, p_6$ : hip rotation)

## 1.2. Inverse Dynamics

In order to find the inverse dynamic solution of the dynamic model shown in Figure 1, we applied Nakamura's method [8]. This method is based on the Lagrange-D'Alembert principle and is efficient for closed-chain mechanisms that involve passive DOF, i.e. zero torque at passive DOF. By utilizing Nakamura's method, the inverse dynamics solution is obtained as

$$\tau_a = W^T (M_t \ddot{q} + C_t \dot{q} + N_t) \in \mathfrak{R}^{N_a \times 1} \quad (1)$$

where  $W := \begin{bmatrix} I \\ -J_p^{-1} J_a \end{bmatrix} \in \mathfrak{R}^{12 \times N_a}$ ,  $I \in \mathfrak{R}^{(12-N_p) \times N_a}$ :

identity matrix,  $q := [q_a \ q_p]^T \in \mathfrak{R}^{12 \times 1}$ ,  $q_a \in \mathfrak{R}^{N_a \times 1}$ : active DOF,  $q_p \in \mathfrak{R}^{N_p \times 1}$ : passive DOF, i.e. zero torque at the passive DOF,  $N_a$ : number of active DOF,  $N_p$ : number of passive DOF,  $\tau_a \in \mathfrak{R}^{N_a \times 1}$ : torque at active DOF,  $J_a \in \mathfrak{R}^{6 \times N_a}$ : Jacobian matrix with respect to the active DOF,  $J_p \in \mathfrak{R}^{6 \times N_p}$ : Jacobian matrix with respect to the passive DOF,  $M_t(q) \in \mathfrak{R}^{12 \times 12}$ : inertia matrix of the tree-structure, i.e. the structure formed by severing one or more passive DOF in the closed-chain mechanism,  $C_t(q, \dot{q}) \in \mathfrak{R}^{12 \times 12}$ : Coriolis-centrifugal force matrix of

the tree-structure, and  $N_t(q) \in \mathfrak{R}^{12 \times 1}$ : gravity vector of the tree-structure.

According to (1), as long as  $N_a \geq 6$  and  $\text{rank}(J_p) = 6$ , the inverse dynamics solution, i.e.  $\tau_a$ , exists. Table 1 describes the combinations of six DOF, i.e. the minimum number of DOF, which obtained the full rank, 6 in the feasible quiet standing motions. Note that the six combinations of active DOF in Table 1 were obtained based on the assumption that  $q_1, q_6, p_1$ , and  $p_6$  were always passive DOF since these DOF are rarely controlled by FES. Refer to [4] for the details of equation (1).

DOF	Ankle D/P		Knee F/E		Hip A/A		Hip F/E	
LEG	A	B	A	B	A	B	A	B
I		o	o	o		o	o	o
II	o		o	o		o	o	o
III	o	o		o		o	o	o
IV	o	o	o			o	o	o
V	o	o	o	o	o		o	
VI	o	o	o	o		o	o	

Table 1. Six cases of six active DOF combinations

## 1.3 FES Control Strategy

Based on the above results, two controllers for FES-assisted arm-free standing for paraplegic individuals were proposed.

### A. PD plus gravity compensation

Equation (1) can be written as

$$\tau_a = \hat{M}_t \ddot{q}_a + \hat{C}_t \dot{q}_a + \hat{N}_t \quad (2)$$

where:  $\hat{M}_t(q) = W^T M_t W \in \mathfrak{R}^{6 \times 6}$

$$\hat{C}_t(q, \dot{q}) = W^T M_t \dot{W} + W^T C_t W \in \mathfrak{R}^{6 \times 6}$$

$$\hat{N}_t(q) = W^T N_t \in \mathfrak{R}^{6 \times 1}$$

We proposed the following control law to (2)

$$u = -K_p e_a - K_v \dot{q}_a + \hat{N}_t \quad (3)$$

where  $e_a = q_a - q_a^d \in \mathfrak{R}^{6 \times 1}$ : error between the  $q_a$  and the equilibrium point  $q_a^d$  and  $K_p, K_v \in \mathfrak{R}^{6 \times 6}$ : gain matrices.

### B. Computed Torque

We propose the following control law to (2)

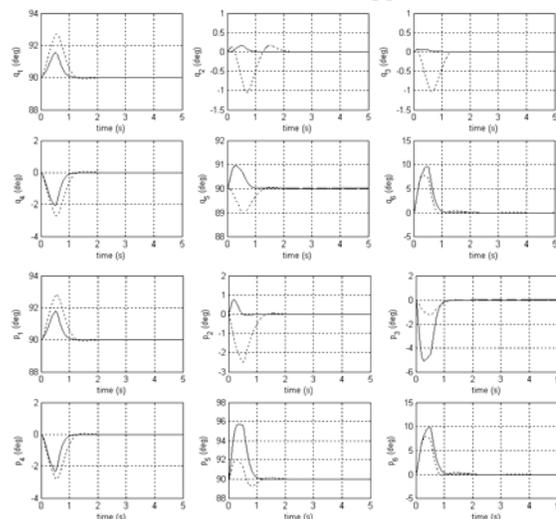
$$u = \hat{M}_t (\ddot{q}_a^d - 2\lambda \dot{e}_a - \lambda^2 e_a) + \hat{C}_t \dot{q}_a + \hat{N}_t \quad (4)$$

where  $\lambda = \lambda I \in \mathfrak{R}^{6 \times 6}$  is a gain matrix.

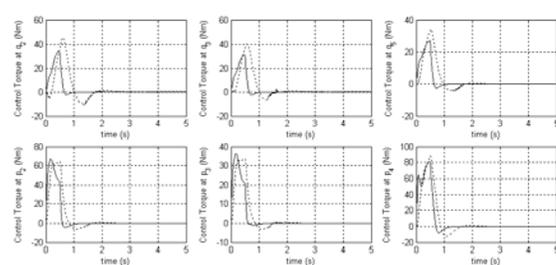
Refer to [4] for the stability analyses of the above proposed control laws, (3) and (4).

### 3. RESULTS

In this section, simulation was performed for the proposed controllers. The dynamic model shown in Figure 1 was assumed to be a paraplegic individual whose weight and height were 66.7 kg and 1.72 m, respectively. In addition, only 6DOF (case VI in Table 1) were assumed to be controlled. Case VI was chosen because cases V and VI generated the least amount of torque amongst the six cases [4]. Furthermore, the paraplegic individual was assumed to be in the upright posture initially. Then, the subject was perturbed by a sudden external force, 100 N, at the HAT COM of the subject in the direction of D6 (see Figure 1). The gravity effect of the movements of the head, arms, and the upper-body bending were considered as external disturbances. Therefore, the following external disturbance force was used for simulation,  ${}^wF_o = [-70.7 \ 0 \ -70.7 \ 10 \ 10 \ 10]^T$  (N, Nm) for 0.5s. The total simulation time was 5 s. The gain was  $K_p = 1000 \text{ I (Nm/rad)}$ ,  $K_v = 300 \text{ I (Nm/rad/s)}$ , and  $\lambda = 10 \text{ I (Hz)}$ . As shown in Figure 2, the body returned to the initial upright posture in about 2 s after the above external disturbance was applied.



(a) Joint variable trajectories



(b) Control torque at active DOF

Figure 2. Simulation results (solid line: computed torque, dashed line: PD plus gravity compensation)

### 4. DISCUSSION AND CONCLUSIONS

In this paper, several critical problems in the field of FES-assisted arm-free standing for paraplegic individuals were investigated. We approached the solutions of these issues through investigation of the dynamic modelling and control of closed-chain robotic systems. Through the inverse dynamics analysis of the proposed dynamic model, we proposed six cases of 6DOF combinations such that there are unique torque inputs that could generate the feasible body motion during quiet standing. Simulation results demonstrated that controlling only 6DOF out of 12DOF could obtain asymptotic stability. This result implies that the dynamic redundancy is in fact an advantage to practical implementation of a FES system for paraplegic standing. That is, it is not necessary to control all DOF in the lower limbs in order to obtain stable standing of paraplegic individuals, but only six DOF can be controlled, thus allowing for one to choose the feasible DOF sets to be controlled depending on the individual.

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