Finite Element Modeling Validation of Energy-Optimal Electrical Stimulation Waveform

Jezernik S [1] and Sinkjaer T [2]

[1] BERATA AG, Engineering Consulting, Schäferweg 16, 4057 Basel, Switzerland (CH)
[2] Center for Sensory-Motor Interaction, Aalborg University, Fredrik Bajersvej 7D, 9220 Aalborg, DK

jezernik@control.ee.ethz.ch, ts@smi.auc.dk

Abstract

In this study we have validated our earlier theoretical result that an exponentially rising current stimulation waveform enables excitation of nervous and muscular tissue at lowest possible energy. The validation was performed via 3D FEM (finite element modeling) of an upper limb, surface stimulation electrodes, and a 1D FEM of a nerve fiber located within the upper limb. It can be concluded that the influence of surface stimulation and body inhomogenities produces only a distortion of the amplitude and spatial distribution of the extracellular potential generated by the injected current, however, that an appropriate exponentially rising waveform generates the same spatially localized peak membrane potential as the corresponding rectangular stimulation – but at a substantially lower stimulation energy. At the end of this article we also present an electronic circuit diagram that realizes the energy-optimal stimulation.

1. INTRODUCTION

Energy considerations play an important role in design of implanted biomedical systems like neural prostheses, especially when these systems are battery-powered. Energy requirement of such systems can be reduced by exciting the nervous tissue with the least possible energy that is sufficient to generate propagating action potentials. Based on an analytical nerve membrane model and optimal control theory of dynamical systems, an energy-optimal stimulation current waveform for electrical excitation of nerve fibers was derived in [1]. Optimal stimulation waveforms for non-leaky and leaky membranes were calculated, whereby the first case led to rectangular current pulses, and the second case to an exponentially rising stimulation current waveforms. Some experimental evidence that the rectangular pulses might not be energy-optimal was provided in [2]. The mathematical formula for the energy-optimal stimulation waveform is the following one:

$$u^*(t) = \frac{V_{THR}\, g_m}{\sinh\left(\frac{g_m t_F}{C_m}\right)} e^{\frac{g_m}{C_m} t}$$

where V_{THR} stands for the threshold membrane voltage, t_F for the waveform duration (final stimulation time), u(t) for the stimulation current, and g_m and C_m for the membrane conductance (at resting potential) and capacitance respectively. This formula for the current stimulation waveform is optimal when neglecting the second spatial derivative of the membrane potential term in the equation for nerve fiber membrane excitation. When formulating the optimality problem without neglecting the spatial derivatives, the standard variational calculus (Lagrange/Hamilton method) cannot be used since the functional J to be minimized is additionally a function of the spatial coordinate x:

$$J[u(t)]_{(x)} = \int_{t_0}^{t_F} \left\{ H\left(V_{(t,x)}, \frac{\partial^2 V_{(t,x)}}{\partial x^2}\right) + \dot{\lambda} V_{(t,x)} \right\} dt + \left[\lambda V_{(t,x)}\right]_{t_0}^{t_F}$$

where λ(x,t) is the Lagrange multiplier and H the Hamiltonian corresponding to our dynamical system (nerve fiber excitation equation; $V(t_0)=0$, $V(t_F)=V_{THR}$). However, by considering the problem only in the neighbourhood of the x coordinate point x_0 corresponding to the minimal distance between the stimulation electrode (cathode) and the nerve fiber, and by approximating the variation in J term produced due to the dependency of H on $\partial^2 V/\partial x^2$ by $\Delta \cdot \delta u$, the optimal current stimulation waveform becomes:

$$u^*(t) = \left[\frac{V_{THR}\, g_m}{\sinh\left(\frac{g_m t_F}{C_m}\right)} + \tilde{\Delta} G_a \right] e^{\frac{g_m}{C_m} t}$$

Thus, the (approximate) optimal waveform is still a rising exponential waveform.

The goal of the reported work was thus to validate the theoretical results obtained in [1] - which were additionally extended as described above - in FEM simulations of excitation of the nerve membrane with the use of full/correct equation containing the term $G_a \cdot \partial^2 V/\partial x^2$, and also in a more realistic FEM model of electrical stimulation using surface electrodes. The latter simulations were used to confirm that an exponentially rising stimulation waveform is superior (i.e. requires substantially less energy) than the square pulse stimulation waveform even in case of surface stimulation.

2. METHODS

First, the nerve membrane excitation in a 1-dimensional FEM model was studied with square wave pulses and with the optimal, exponentially rising waveform at 200 and 600 µs duration, and electrode-nerve fiber distance of 1 mm. Next, also the nerve membrane excitation following surface electrode stimulation was simulated. Quasi-static electrodynamic (Maxwell) equations were used to calculate a 1-dimensional term $\partial^2 V_e/\partial x^2$ following injection of constant surface electrode current density. The interelectrode distance was 10 cm, and the nerve fiber was located at a depth of 2 cm. Via allowed decoupling of the time and spatial dependency for frequencies < 100 kHz, the 3D simulations yielded the spatial course of the activation function term that was then subsequently used in the 1D simulations of the nerve fiber excitation, which was then modulated with the exponential time function according to the above equations. The FEM simulations were performed with the FEMLAB® software (Comsol AB) with the simulation time-step of 10 µs. The focus of the analysis was on the comparison of the time-evolution of the nerve membrane voltage in case of square wave and exponentially rising current stimulation waveforms.

The 1D FEMLAB model of a nerve fiber is shown in Fig.1. The used nerve membrane excitation equation parameters were: G_a=5e-4 S, g_m=30.4 mS/cm^2, C_m=2 µF/cm^2, and ρ_e=300 Ωcm. The used nerve fiber length was 100 mm, and the FEM mesh had 241, 419, or 481 nodes, whereby the node density was higher in the region around x_0 (maximum of the activation function). Simulations with 10, 100 and 1000 times higher G_a were performed as well in order to study the effect of G_a on the obtained solution (G_a = intra-axonal conductivity).

The 3D arm model with surface electrodes is shown in Fig.2. The arm length was 30 cm, the circular cross-section had a radius of 5 cm. The nerve fiber was assumed to lie 2 cm beneath the electrodes. The used spatial resolution along the x-axis was 0.3 mm.

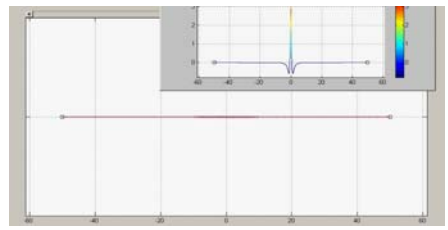


Fig.1: 1D FEMLAB model of a nerve fiber.

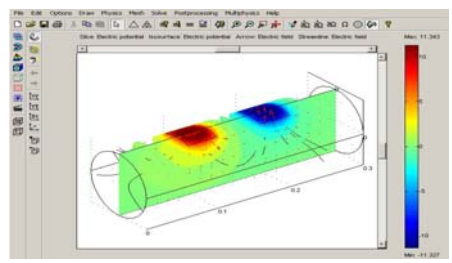
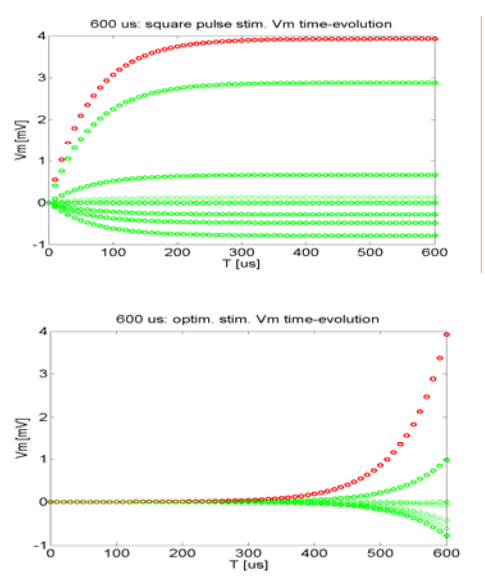


Fig.2: 3D FEMLAB model of an upper limb with surface electrodes.

3. RESULTS

The results regarding the energy savings from [1] were confirmed in the FEM simulations. For example, the optimal stimulation waveform necessitated at 600 µs stimulation duration 74.5% less energy than the square pulse stimulation. At 200 µs, the energy gain was 56.8%. The results from the 1D nerve fiber model FEM simulations are shown in Fig.3A/B,

A



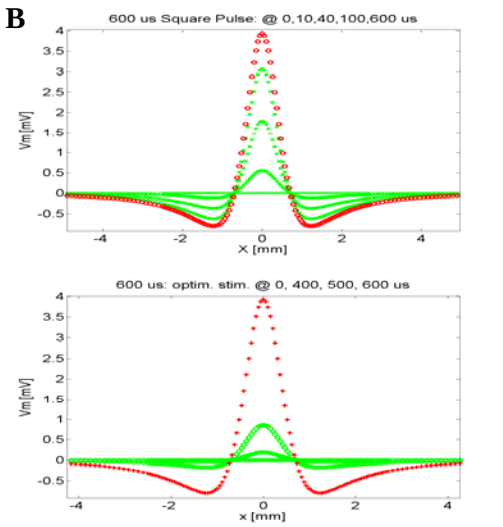


Fig.3: A (previous page): 1D FEM model simulations of the square-wave and rising exponential stimulation: time-courses. B: The corresponding spatial distribution of the membrane voltage at different fixed times.

whereby the time-courses along different spatial locations x_i are plotted in case of square pulse stimulation in the top, and in case of optimal stimulation waveform in the bottom graph of Fig.3A. The time-course at location x=0 is shown in red (top signal trace). In both cases, the final membrane voltage level of 4 mV was reached. Fig.3B on the other hand shows spatial membrane voltage distribution at different time points t_i (snapshots), again for the square wave (top graph) and the exponentially rising waveform (bottom graph). In the latter case we can see that a substantial depolarization is achieved first close to the final stimulation time, however, both types of stimulation lead to the same final depolarization (difference $<10^{-5}$ mV when nominal parameters were used). The effects of changing the G_a parameter by several orders of magnitude were negligible. The top graph in Fig.4 shows the FEM simulation results with surface stimulation, when optimal stimulation was used (snapshots), and the bottom graph compares the obtained membrane voltage distribution when stimulating in an optimal way (red solid lines) vs. square-pulse stimulation (blue circles). Again, the two waveforms led to almost identical result, whereby the exponentially rising waveform saved lots of energy. The depolarization and hyperpolarization peaks were located below the electrodes. Finally, Fig.5 depicts an electronics scheme that can be used to realize exponentially rising stimulation. A voltage generator source on the left generates triangular voltage sweeps

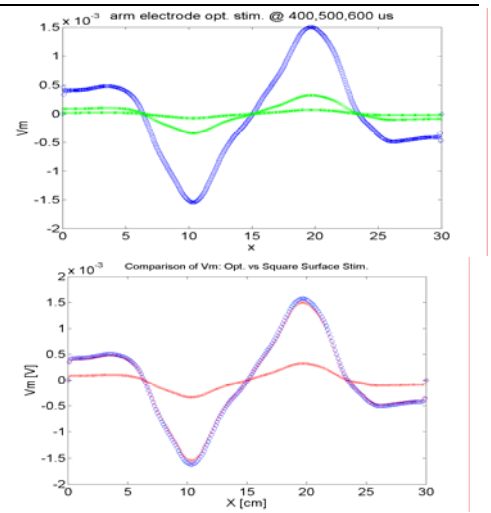


Fig.4: 3D FEM model surface electrode stimulation simulation coupled with the 1D FEM model simulation: spatial distribution of the membrane voltage.

leading to exponentially rising diode currents that are sensed by the operational amplifier with the use of a shunt resistor. The OPAMP is driving a push-pull transistor stage in a linear manner, whereby the feedback of the R_{shunt} voltage makes sure that the current through the load has the same waveform as the OPAMP +input voltage. The OPAMP should be realized by discrete parts due to needed high supply voltage levels (>200 V).

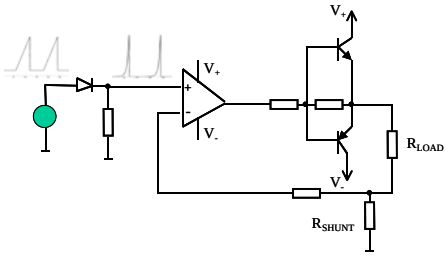


Fig.5: Stimulator electronics scheme for exponentially rising stimulation waveforms.

4. DISCUSSION AND CONCLUSIONS

The energy-optimality of exponentially rising stimulation was confirmed in the FEM simulations. Next step will be an experimental validation thereof.

References

[1] Jezernik S and Morari M. Energy-optimal electrical excitation of nerve fibers. *IEEE TBME*, vol. 52(4): 740-743, April 2005.

[2] Bennie SD, Petrofsky JS, Nisperos J, Tsurudome M, Laymon M. Toward the optimal waveform for electrical stimulation of human muscle. Eur J Appl Physiol, Vol.88, pp.13-19, 2002.

Acknowledgements

The authors would like to acknowledge the use of the geometrical arm model provided by Dr. T.Keller.